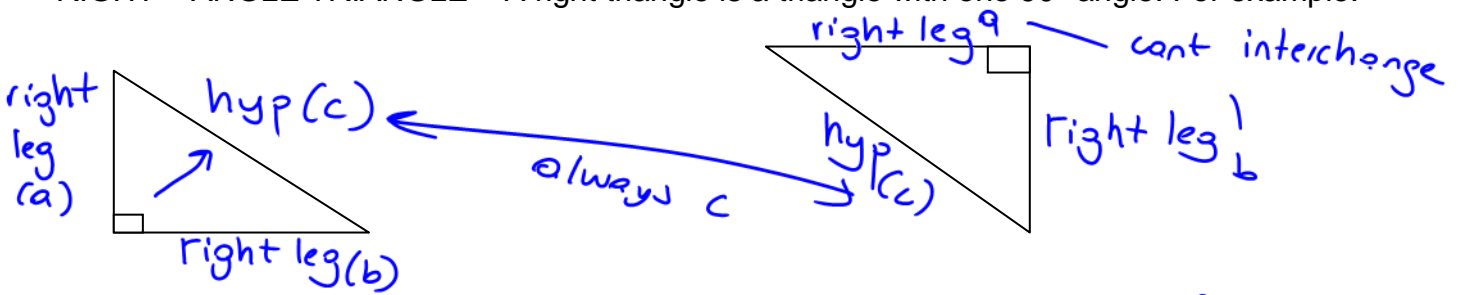


Pythagorean Theorem

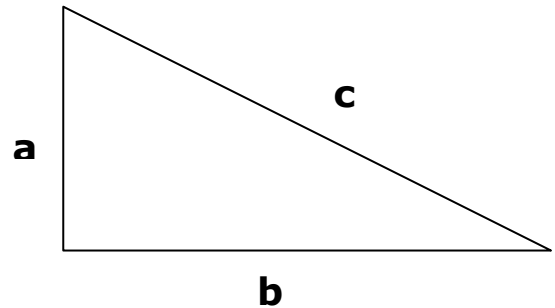
'RIGHT - ANGLE TRIANGLE' - A right triangle is a triangle with one 90° angle. For example:



In any right-angle triangle the side opposite to the right angle is called the HYPOTENUSE
Label the Hypotenuse in each diagram above.

Pythagorean Theorem Formula: In any right- angled triangle, the square of the hypotenuse is equal to the sum of the squares on the other two sides.

$$a^2 + b^2 = c^2$$

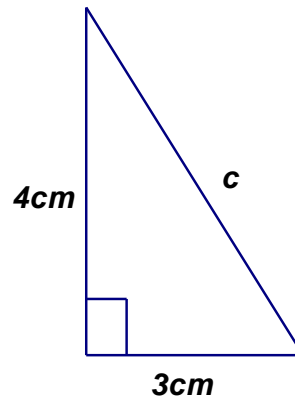


Solving for the Hypotenuse:

Example 1:

Find the side length of the hypotenuse:

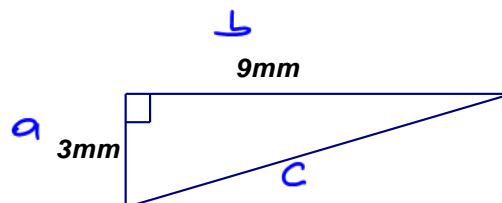
$$\begin{aligned} a^2 + b^2 &= c^2 \\ \underline{4^2 + 3^2} &= \underline{c^2} \\ \underline{16 + 9} &= \underline{c^2} \\ \underline{25} &= \underline{c^2} \\ \sqrt{25} &= \sqrt{c^2} \end{aligned}$$



Example 2: $5 = c$ or $-5 = c$
 $\therefore c$ is 5

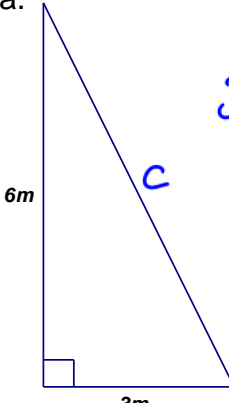
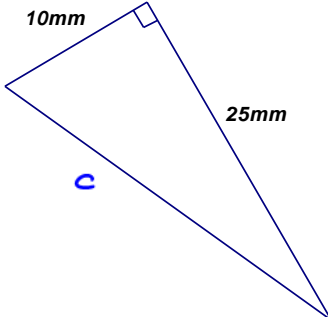
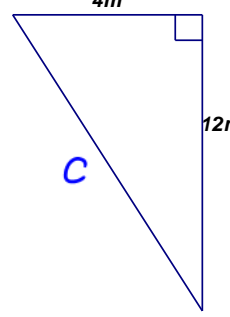
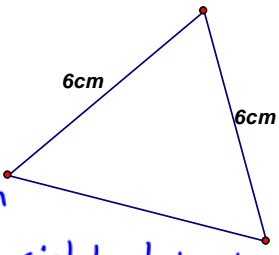
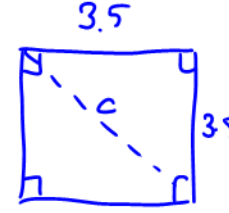
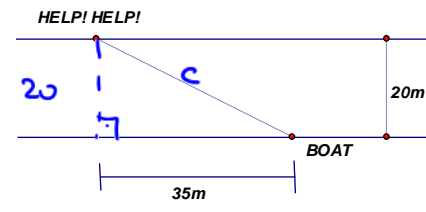
Find the side length of the hypotenuse.

$$\begin{aligned} 3^2 + 9^2 &= c^2 \\ 9 + 81 &= c^2 \\ 90 &= c^2 \\ 9.5 &= c \end{aligned} \quad \therefore c \text{ is } 9.5 \text{ mm.}$$



Practice: Pythagorean Theorem – Finding the Hypotenuse

Find the length of each hypotenuse (round to one decimal place where necessary)

<p>a.</p>  $6^2 + 3^2 = c^2$ $36 + 9 = c^2$ $\sqrt{45} = \sqrt{c^2}$ $6.7 = c$ <p>$\therefore c$ is 6.7m.</p>	<p>b.</p>  $10^2 + 25^2 = c^2$ $100 + 625 = c^2$ $\sqrt{725} = \sqrt{c^2}$ $26.9 = c$ <p>$\therefore c$ is 26.9mm</p>
<p>c.</p>  $4^2 + 12^2 = c^2$ $16 + 144 = c^2$ $\sqrt{160} = \sqrt{c^2}$ $12.6 = c$ <p>$\therefore c$ is 12.6m.</p>	<p>d.</p>  <p>You cannot use pythagorean theorem because it is not a right triangle</p>
<p>e. My neighbour has a square vegetable garden which is 3.5m by 3.5m. He wants to put a walkway diagonally through the garden to make it easier to get the veggies in the middle. How long will his walkway be?</p>  $(3.5)^2 + (3.5)^2 = c^2$ $12.25 + 12.25 = c^2$ $\sqrt{24.5} = \sqrt{c^2}$ $4.9 = c$ <p>\therefore It is 4.9m.</p>	<p>f. An emergency boat is on one side of a waterway and there are cries of help on the other side. The waterway is 20m wide and the boat is about 35 m down the water way from the people in need. What is the distance the boat must travel if they go directly (diagonally) to help?</p>  $20^2 + 35^2 = c^2$ $400 + 1225 = c^2$ $\sqrt{1625} = \sqrt{c^2}$ $40.3 = c$ <p>\therefore It's 40.3m.</p>
<p>ANSWERS: a. 6.7m, b. 17.3mm 26.9mm, c. 12.6m, d. a right triangle?, e. 4.9m, f. 40.3m</p>	

Pythagorean Theorem – Solving for a side, other than the hypotenuse

To find a side other than the hypotenuse, you can use the same formula or this 'rearranged formula'

$$a^2 + b^2 = c^2$$

$$-b^2 \quad -b^2$$

$$a^2 = c^2 - b^2$$

Example 1:

Find the value of the missing side:

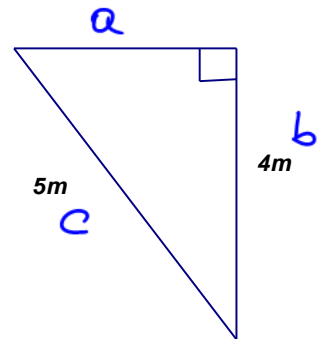
$$a^2 = c^2 - b^2$$

$$a^2 = 5^2 - 4^2$$

$$a^2 = 25 - 16$$

$$\sqrt{a^2} = \sqrt{9}$$

$a = 3$ ∴ It's 3m.



Examples: Find the length of each missing side length:

$$a^2 = 15^2 - 8^2$$

$$a^2 = 225 - 64$$

$$\sqrt{a^2} = \sqrt{161}$$

$$a \approx 12.7$$

$$a^2 = (7.52)^2 - (3.85)^2$$

$$a^2 = 56.5504 - 14.8225$$

$$\sqrt{a^2} = \sqrt{41.7279}$$

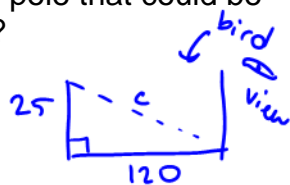
$$a \approx 6.5$$

A box is 120 cm long and 25cm wide. What is the length of the longest ski pole that could be packed to lie flat in the box?

$$c^2 = 25^2 + 120^2$$

$$\sqrt{c^2} = \sqrt{15025}$$

$$c \approx 122.6$$



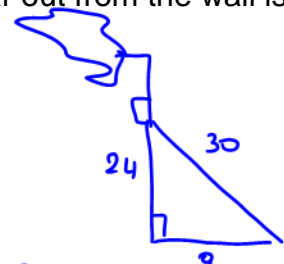
∴ You can fit 122.6 cm long pole.

The window of a burning building is 24 metres above the ground. A ladder that is 30m is angled to reach the window and the base is out from the wall. How far out from the wall is the ladder?

$$a^2 = 30^2 - 24^2$$

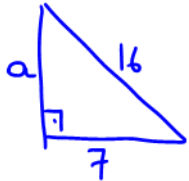
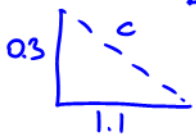
$$a^2 = 324$$

$$a = 18$$



∴ It's 18 m out from the wall.


Practice: Pythagorean Theorem - Draw a diagram to help you identify the triangle and the hypotenuse.

<p>1. A 16 m long ladder leans against a house. The foot of the ladder is 7m from the house. Find the height of the ladder from the ground, correct to the nearest tenth of a meter.</p> <p>$a^2 = 16^2 - 7^2$ $a^2 = 207$ $a = 14.4$ ∴ It's 14.4 m high</p> 	<p>2. Can an umbrella, 1.3m long be packed flat into a box 1.1m by 0.3m? Give reasons for your answer.</p> <p>$c^2 = (0.3)^2 + (1.1)^2$ $c^2 = 1.3$ $c = 1.1$ ∴ No you cannot because the umbrella is longer than 1.1m.</p> <p style="text-align: right;"><i>bird @ view</i></p> 
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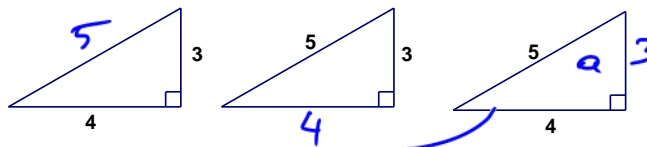
3. When on hike, Jean cuts diagonally across a large rectangular field, 1.6km by 3.0 km, instead of keeping to the sidewalks. What distance does she save by doing this?

$c^2 = (1.6)^2 + (3)^2$
 $c^2 = 11.56$
 $c = 3.4$

∴ She saves $(3 + 1.6) - 3.4 = 1.2m$.



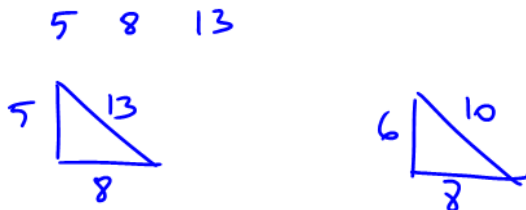
4. A Pythagorean Triple consists of three **positive integers** a , b , and c , such that $a^2 + b^2 = c^2$. A well known example (3, 4, 5). This means that for any of the triangles below, you can easily identify the value of the third side (notice the hypotenuse MUST be 5, the longest side).



a) Confirm that this is a Pythagorean Triple by solving for the missing side in on of the triangles above.

$a^2 = 5^2 - 4^2$
 $a^2 = 9$
 $a = 3$

b) Play around and try to find another Pythagorean Triple. There is a formula for finding Pythagorean triples. Go online and check it out sometime.

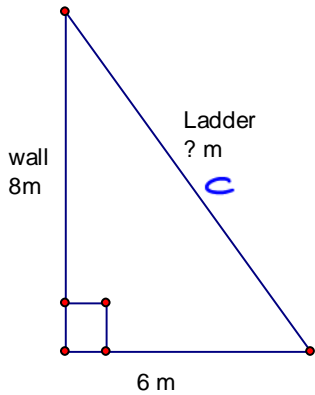


ANSWERS: 1. 14.4m, 2. No, max 1.14m long umbrella, 3. 1.2km, 4. varies

A Pythagorean Puzzle: Free the Hostage

Part A

A friend of yours from math class is being held hostage by the UFPRLM (The United Front of People who are Resisting the Learning of Math). He is in the second floor of a house. You must go save him, so that math class can live on. You managed to get into his room and unite him. You hear the captors approach the room so you must escape out of a window. Fortunately, you planned for this possibility. You brought a ladder, and propped it up against the window before you went in. (see the diagram below). The window is 8m above the ground, and you placed the foot of the ladder 6m from the wall. How long is the ladder?



$$c^2 = 6^2 + 8^2$$

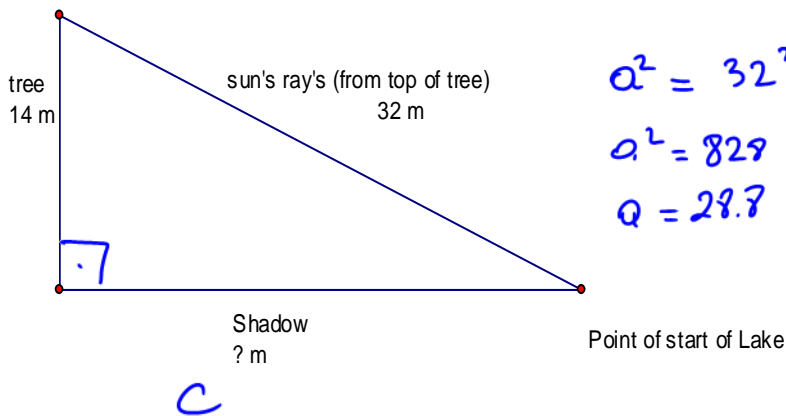
$$c^2 = 100$$

$$c = 10$$

∴ The ladder is 10m long.

PART B

After climbing down the ladder, you and your friend must run across a field. Unfortunately, you spot the captors looking out of the window to find you. In order to avoid being spotted, you must hide behind a tree. However, you don't want to lose time in your escape. Hence, you crawl along the shadow that is caused by the tree blocking the sun, until you reach a lake. (See the diagram below) Use the information given to find how far you must crawl (i.e. find the length of the shadow). Round to one decimal place



$$a^2 = 32^2 - 14^2$$

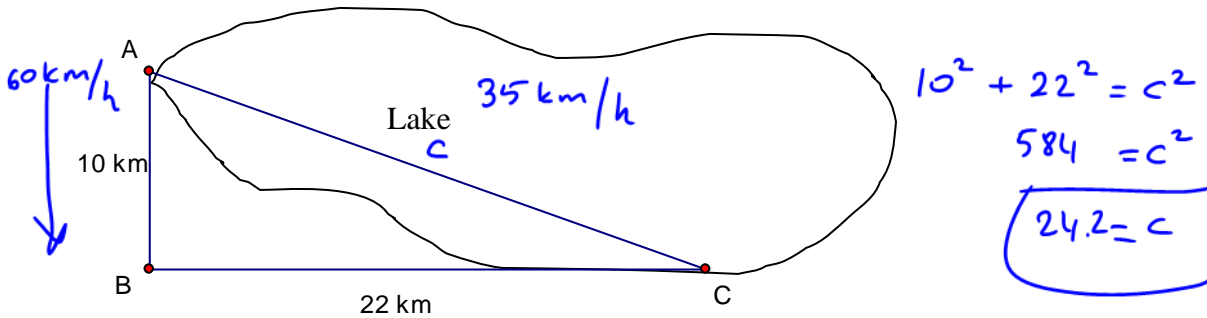
$$a^2 = 829$$

$$a = 28.7$$

A Pythagorean Puzzle: Free the Hostage Continued...

PART C

You have finally made it to the last part of your escape. You are at the edge of a lake at point A. you must end up at point C. (See the diagram below). You have a choice. You may drive from point A to point B, then from Point B to Point C or take a boat from Point A to Point C. Once you make it to Point C, you and your friend are saved, and can continue on to math class. The car travels at 60 km per hour (because of course you obey the law and don't speed), and the boat travels at 35km per hour.



Answer the following questions:

- a) How far (in kms) is the car ride?

$$10 + 22 = 32 \text{ km}$$

- b) How far (in kms) is the boat ride?

$$24.2 \text{ km}$$

- c) You will need to take the fastest route. Determine how long (in hours) each trip will take, then state which method of transportation is the best one to use. ($time = \frac{distance}{speed}$)

Boat	Car
$time = \frac{24.2}{35}$	$time = \frac{32}{60}$
$time = 0.69$	$= 0.53$
\therefore Travelling with car is faster.	

ANSWERS: A) 10m, B)28.8m, C)a. 32km, b.24.1km, c. 0.53hrs vs. 0.69hrs, faster to drive