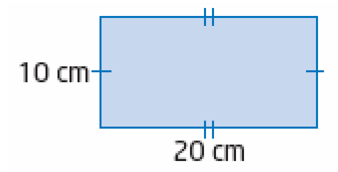
Chapter 9 **Optimizing Measurements**

Chapter 9 Get Ready

Chapter 9 Get Ready **Question 1** **Page 476**

a)

$$\begin{aligned} P &= 2w + 2l \\ &= 2 \times 10 + 2 \times 20 \\ &= 20 + 40 \\ &= 60 \end{aligned}$$

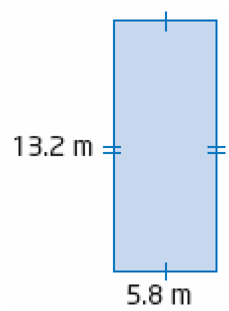


$$\begin{aligned} A &= lw \\ &= 10 \times 20 \\ &= 200 \end{aligned}$$

The perimeter is 60 cm, and the area is 200 cm^2.

b)

$$\begin{aligned} P &= 2w + 2l \\ &= 2 \times 5.8 + 2 \times 13.2 \\ &= 11.6 + 26.4 \\ &= 38 \end{aligned}$$

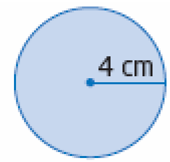


$$\begin{aligned} A &= lw \\ &= 13.2 \times 5.8 \\ &= 76.56 \end{aligned}$$

The perimeter is 38 m, and the area is 76.56 m^2.

Chapter 9 Get Ready		**Question 2	Page 476**

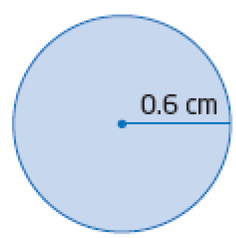
a) $C = 2\pi r$
$= 2 \times \pi \times 4$
$\doteq 25.1$



$A = \pi r^2$
$= \pi \times 4^2$
$\doteq 50.3$

The circumference is approximately 25.1 cm, and the area is approximately 50.3 cm^2.

b) $C = 2\pi r$
$= 2 \times \pi \times 0.6$
$\doteq 3.8$



$A = \pi r^2$
$= \pi \times 0.6^2$
$\doteq 1.1$

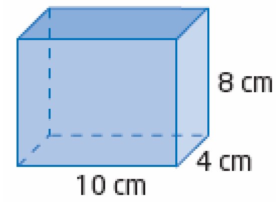
The circumference is approximately 3.8 cm, and the area is approximately 1.1 cm^2.

Chapter 9 Get Ready **Question 3** **Page 476**

a) $V = lwh$

$= 10 \times 4 \times 8$

$= 320$



$SA = 2A_{bottom} + 2A_{sides} + 2A_{front}$

$= 2(4 \times 10) + 2(4 \times 8) + 2(8 \times 10)$

$= 80 + 64 + 160$

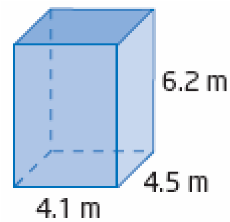
$= 304$

The volume is 320 cm^3, and the surface area is 304 cm^2.

b) $V = lwh$

$= 4.1 \times 4.5 \times 6.2$

$= 114.39$



$SA = 2A_{bottom} + 2A_{sides} + 2A_{front}$

$= 2(4.1 \times 4.5) + 2(4.5 \times 6.2) + 2(4.1 \times 6.2)$

$= 36.9 + 55.8 + 50.84$

$= 143.54$

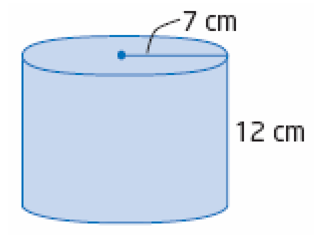
The volume is 114.39 m^3, and the surface area is 143.54 m^2.

Chapter 9 Get Ready Question 4 Page 476

a) $V = \pi r^2 h$

$= \pi \times 7^2 \times 12$

$\doteq 1847$



$SA = 2\pi r^2 + 2\pi rh$

$= 2\pi \times 7^2 + 2\pi \times 7 \times 12$

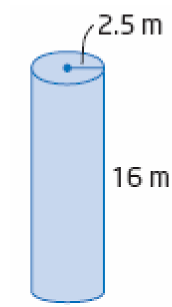
$\doteq 836$

The volume is approximately 1847 cm^3, and the surface area is approximately 836 cm^2.

b) $V = \pi r^2 h$

$= \pi \times 2.5^2 \times 16$

$\doteq 314$



$SA = 2\pi r^2 + 2\pi rh$

$= 2\pi \times 2.5^2 + 2\pi \times 2.5 \times 16$

$\doteq 291$

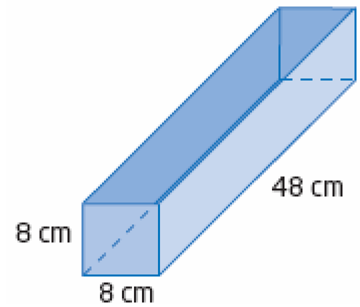
The volume is approximately 314 m^3, and the surface area is approximately 291 m^2.

Chapter 9 Get Ready **Question 5** **Page 477**

a) $V = lwh$

$= 48 \times 8 \times 8$

$= 3072$



$SA = A_{sides} + A_{bottom}$

$= (2(8 \times 48) + 2(8 \times 8)) + (8 \times 48)$

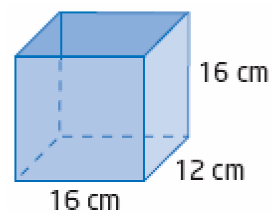
$= 768 + 128 + 384$

$= 1280$

The volume is 3072 cm^3, and the surface area is 1280 cm^2.

$V = 16 \times 12 \times 16$

$= 3072$



$SA = A_{sides} + A_{bottom}$

$= (2(16 \times 16) + 2(12 \times 16)) + (16 \times 12)$

$= (512 + 384) + 192$

$= 1088$

The volume is 3072 cm^3, and the surface area is 1088 cm^2.

b) The volumes of the two boxes are equal.

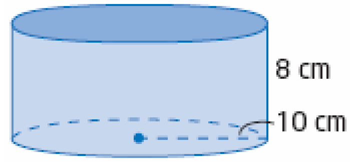
c) The second container requires less material.

Chapter 9 Get Ready **Question 6** **Page 477**

a) $V = \pi r^2 h$

$= \pi \times 10^2 \times 8$

$\doteq 2513$



$SA = \pi r^2 + 2\pi rh$

$= \pi \times 10^2 + 2\pi \times 10 \times 8$

$\doteq 817$

The volume is approximately 2513 m^3, and the surface area is approximately 817 m^2.

$V = \pi r^2 h$

$= \pi \times 5^2 \times 32$

$\doteq 2513$

$SA = \pi r^2 + 2\pi rh$

$= \pi \times 5^2 + 2\pi \times 5 \times 32$

$\doteq 1084$

The volume is approximately 2513 m^3, and the surface area is approximately 1084 m^2.

b) The volumes of the two containers are equal.

c) The first container requires less material.

Chapter 9 Section 1: Investigate Measurement Concepts

Chapter 9 Section 1 Question 1 Page 482

a) The question asks you to investigate the dimensions of rectangles that you can form with a perimeter of 24 units.

b) Answers will vary. A sample answer is shown.

Begin with one grid square as the width and nine grid squares as the length. Then, increase the width by one square and decrease the length by the same amount to draw a series of rectangles with a perimeter of 24 units.

Rectangle	Width (units)	Length (units)	Perimeter (units)	Area (square units)
1	1	11	24	11
2	2	10	24	20
3	3	9	24	27
4	4	8	24	32
5	5	7	24	35

Chapter 9 Section 1 Question 2 Page 482

a) The question asks you to investigate the dimensions of rectangles that you can form with a perimeter of 20 units.

b) Answers will vary. A sample answer is shown.

Begin with one toothpick as the width and nine toothpicks as the length. Then, increase the width by one toothpick and decrease the length by the same amount to construct a series of rectangles with a perimeter of 20 units.

Rectangle	Width (units)	Length (units)	Perimeter (units)	Area (square units)
1	1	9	20	9
2	2	8	20	16
3	3	7	20	21
4	4	6	20	24
5	5	5	20	25

Chapter 9 Section 1 Question 3 Page 482

a) The question asks you to investigate the dimensions of various rectangles with an area of 12 square units.

b) Answers will vary. A sample answer is shown.

Let the space between two pins on the geoboard be 1 unit and use an elastic band to make different rectangles with an area of 12 square units. Start with a width of 1 unit and a length of 12 units. Then, increase the width by 1 unit and decrease the length to maintain an area of 12 square units.

Rectangle	Width (units)	Length (units)	Area (square units)	Perimeter (units)
1	1	12	12	26
2	2	6	12	16
3	3	4	12	14

Chapter 9 Section 1 Question 4 Page 483

a)

Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
1	1	16	34	16
2	2	8	20	16
3	4	4	16	16

b) The greater the perimeter, the higher the cost of the shed, since a greater length of wall is needed.

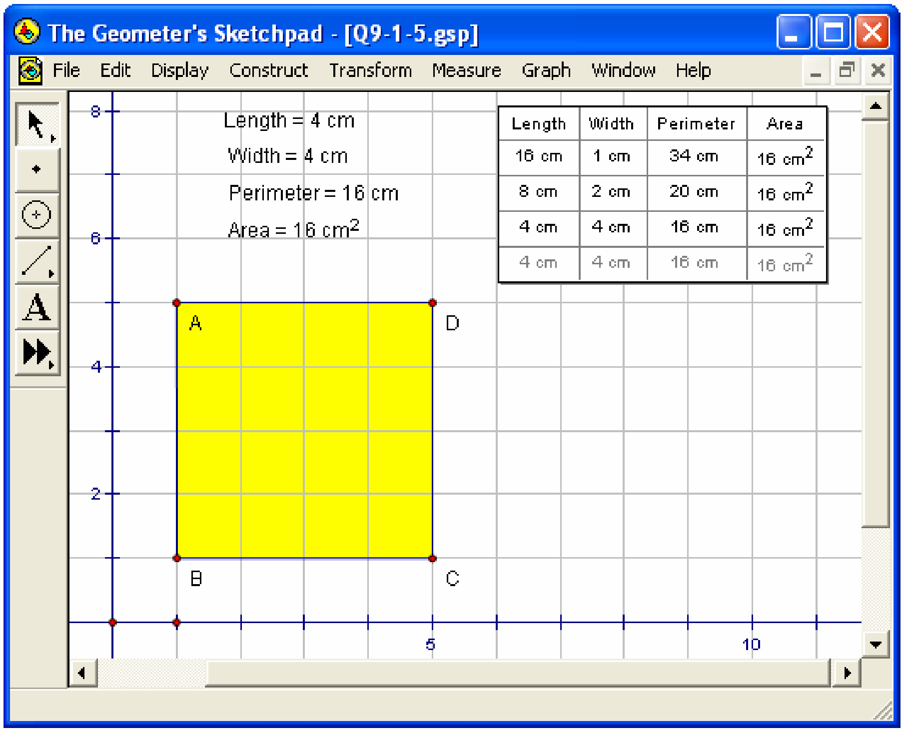
c) Rectangle 3 (a square) with dimensions 4 m by 4 m will be the most economical.

d) Answers will vary. A sample answer is shown.

You must consider the type and quality of the material used to construct the shed, and build it with attention to protecting what will be stored in it.

Chapter 9 Section 1 Question 5 Page 483

A rectangle with dimensions 4 m by 4 m encloses the greatest area for the same amount of fencing. Sketches may vary. A sample sketch is shown. Click here to load the sketch.



Chapter 9 Section 1 Question 6 Page 483

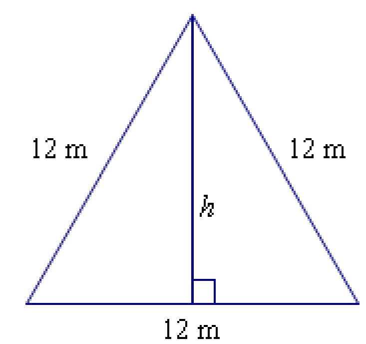
Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
1	1	15	32	15
2	2	14	32	28
3	3	13	32	39
4	4	12	32	48
5	5	11	32	55
6	6	10	32	60
7	7	9	32	63
8	8	8	32	64

The maximum area that Colin can enclose is 64 m^2, using a square 8 m by 8 m. Click here to load the spreadsheet.

Chapter 9 Section 1 Question 7 Page 483

a) Regular polygons enclose the greatest area.

b) For a triangle, the greatest area is enclosed using an equilateral triangle with side length 12 m.



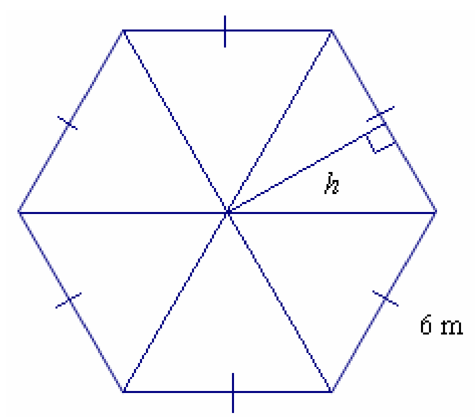
$$12^2 = 6^2 + h^2$$
$$144 = 36 + h^2$$
$$108 = h^2$$
$$\sqrt{108} = h$$
$$10.39 \doteq h$$

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 12 \times 10.39 \\ &= 62.35 \end{aligned}$$

The area of the triangle is about 62.35 m^2.

For a rectangle, the greatest area is enclosed by a square with side length 9 m. The area is 9×9, or 81 m^2.

For a hexagon, the greatest area is enclosed by a regular side length of 6 m.



$$6^2 = 3^2 + h^2$$
$$36 = 9 + h^2$$
$$27 = h^2$$
$$\sqrt{27} = h$$
$$5.20 \doteq h$$

$$\begin{aligned} A_{\text{triangle}} &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 6 \times 5.20 \\ &= 15.6 \end{aligned}$$

$$\begin{aligned} A_{\text{hexagon}} &= 6A_{\text{triangle}} \\ &= 6 \times 15.6 \\ &= 93.6 \end{aligned}$$

The area of the hexagon is about 93.6 m^2.

For a circle with a circumference of 36 m, the radius is $\frac{36}{2\pi}$, or approximately 5.73 m.

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \times 5.73^2 \\ &\doteq 103.15 \end{aligned}$$

The area of the circle is about 103.15 m^2.

c) The shape of the enclosure affects its area. Different shapes result in different areas. The greatest area can be achieved by using a circle.

Chapter 9 Section 2 Perimeter and Area Relationships of a Rectangle

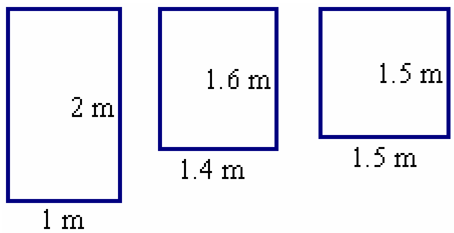
Chapter 9 Section 2 Question 1 Page 487

The maximum area occurs when a square shape is used.

a) 5 m × 5 m **b)** 9 m × 9 m **c)** 12.5 m × 12.5 m **d)** 20.75 m × 20.75 m

Chapter 9 Section 2 Question 2 Page 488

a) Answers will vary. Sample answers are shown.



b) The maximum area occurs when a square shape is used, 1.5 m by 1.5 m.

Chapter 9 Section 2 Question 3 Page 488

a) The maximum area occurs when a square shape is used, 20.5 m by 20.5 m.

b) The same area cannot be enclosed using 2 m long barriers. It is not possible to create a dimension of 20.5 m using 2 m barriers.

c)

$$\begin{aligned} A_{\text{usingrope}} &= 20.5 \times 20.5 \\ &= 420.25 \end{aligned}$$

$$\begin{aligned} A_{\text{usingbarriers}} &= 20 \times 20 \\ &= 400 \end{aligned}$$

If rope is used, you can enclose 420.25 – 400, or 20.25 m^2 more area.

Chapter 9 Section 2 Question 4 Page 488

Answers will vary. A spreadsheet solution is shown. Let the length represent the side formed by the barn. The maximum area occurs with two widths of 4 m and one length of 8 m of fencing. Click here to load the spreadsheet.

Rectangle	Width (m)	Length (m)	Sum of Lengths of Three Sides (m)	Area (m^2)
1	1	14	16	14
2	2	12	16	24
3	3	10	16	30
4	4	8	16	32
5	5	6	16	30
6	6	4	16	24
7	7	2	16	14

Chapter 9 Section 2 Question 5 Page 488

a) Use 5 pieces on a side to form sides that are 2.8 × 5, or 14 m long.

$$\begin{aligned} A &= 14^2 \\ &= 196 \end{aligned}$$

The maximum area that can be enclosed is 196 m^2.

b) Use 10 pieces on a side to form sides that are 2.8 × 10, or 28 m long.

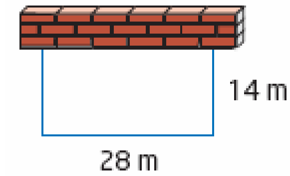
$$\begin{aligned} A &= 28^2 \\ &= 784 \end{aligned}$$

The maximum area that can be enclosed is 784 m^2.

Chapter 9 Section 2 Question 6 Page 488

From question 4, the maximum area occurs when one length is formed by the wall, and the length is twice the width.

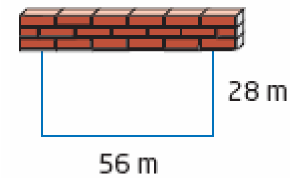
a) Since you need a length that is twice the width, use 10 pieces for the length, and 5 pieces for each width, for dimensions of 28 m by 14 m.



$$\begin{aligned} A &= 14 \times 28 \\ &= 392 \end{aligned}$$

The existing border provides 392 – 196, or 196 m^2 of additional area.

b) Since you need a length that is twice the width, use 20 pieces for the length, and 10 pieces for each width, for dimensions of 56 m by 28 m.



$$\begin{aligned} A &= 28 \times 56 \\ &= 1568 \end{aligned}$$

The existing border provides 1568 – 784, or 784 m^2 of additional area.

Chapter 9 Section 2 Question 7 Page 488

Answers will vary. A spreadsheet investigation is shown. Click here to load the spreadsheet.

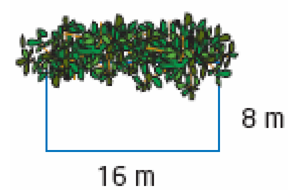
Rectangle	Width (m)	Length (m)	Sum of Lengths of Two Sides (m)	Area (m^2)
1	1	39	40	39
2	2	38	40	76
3	3	37	40	111
4	4	36	40	144
5	5	35	40	175
6	6	34	40	204
7	7	33	40	231
8	8	32	40	256
9	9	31	40	279
10	10	30	40	300
11	11	29	40	319
12	12	28	40	336
13	13	27	40	351
14	14	26	40	364
15	15	25	40	375
16	16	24	40	384
17	17	23	40	391
18	18	22	40	396
19	19	21	40	399
20	20	20	40	400

The maximum area of 400 m^2 occurs when a square area 20 m by 20 m is used.

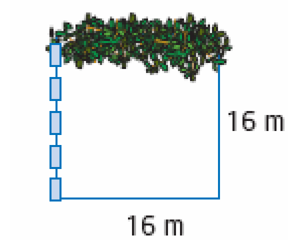
Chapter 9 Section 2 Question 8 Page 489

When 4 sides are required, the maximum area occurs when a square of side length 8 m is used, resulting in an area of 8^2, or 64 m^2.

When one side is a hedge, the maximum area occurs when the hedge is used as a length, and the length is twice the width, for dimensions of 16 m by 8 m, and an area of 16×8, or 128 m^2.



When a hedge and a fence are used, the maximum area occurs when a square is used, for dimensions of 16 m by 16 m, and an area of 16^2, or 256 m^2. A spreadsheet investigation is shown. Click here to load the spreadsheet.



Rectangle	Width (m)	Length (m)	Sum of Lengths of Two Sides (m)	Area (m^2)
1	1	31	32	31
2	2	30	32	60
3	3	29	32	87
4	4	28	32	112
5	5	27	32	135
6	6	26	32	156
7	7	25	32	175
8	8	24	32	192
9	9	23	32	207
10	10	22	32	220
11	11	21	32	231
12	12	20	32	240
13	13	19	32	247
14	14	18	32	252
15	15	17	32	255
16	16	16	32	256

Chapter 9 Section 2 Question 9 Page 489

a)

Rectangle	Width (m)	Length (m)	Area (m^2)	Fence Used (m)
1	1	72	72	74
2	2	36	72	40
3	3	24	72	30
4	4	18	72	26
5	5	14.4	72	24.4
6	6	12	72	24

b) The minimum length of fence occurs when the building is used as one length, and the length is twice the width, for dimensions of 12 m by 6 m.

c) The minimum length of fence is 24 m.

Chapter 9 Section 2 Question 10 Page 489

Answers will vary.

Chapter 9 Section 2 Question 11 Page 489

Answers will vary. Sample answers are shown.

a) A minimum perimeter for a given area is important to know if cost of materials for enclosing the area is a factor, such as fencing in a pasture for livestock.

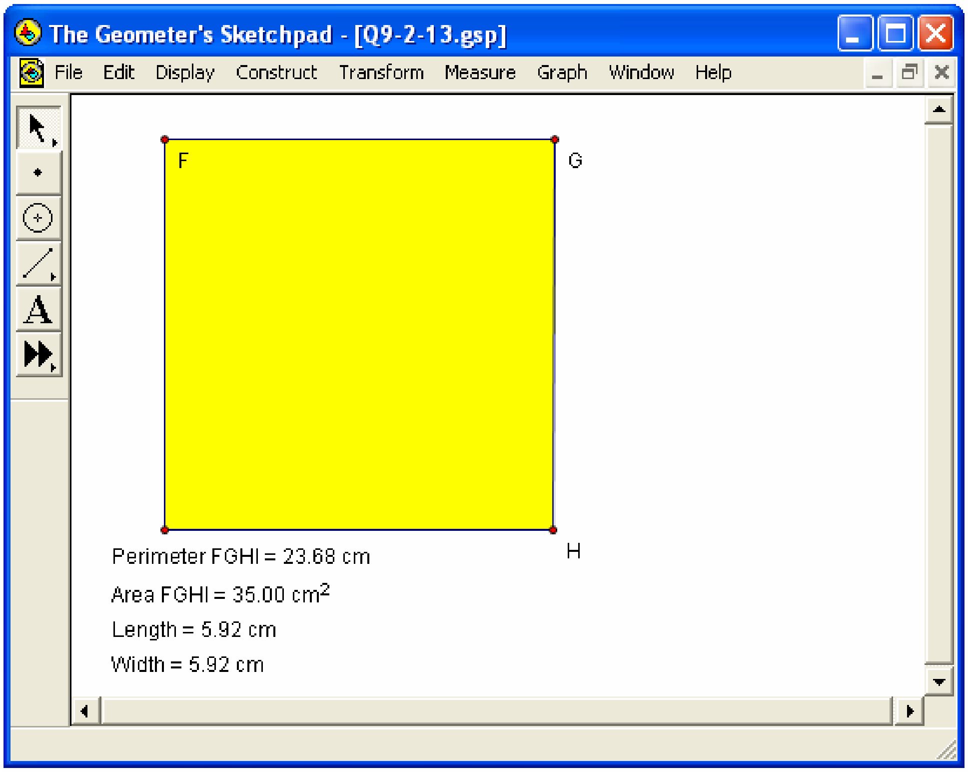
b) The maximum area for a given perimeter is important to know if space available should be maximized, such as a storage shed.

Chapter 9 Section 2 Question 12 Page 490

Solutions for the Achievement Checks are shown in the Teacher's Resource.

Chapter 9 Section 2 Question 13 Page 490

The maximum area occurs when a square is used of side length of approximately 5.92 m. Investigations may vary. A solution using dynamic geometry software is shown. Click here to load the sketch.



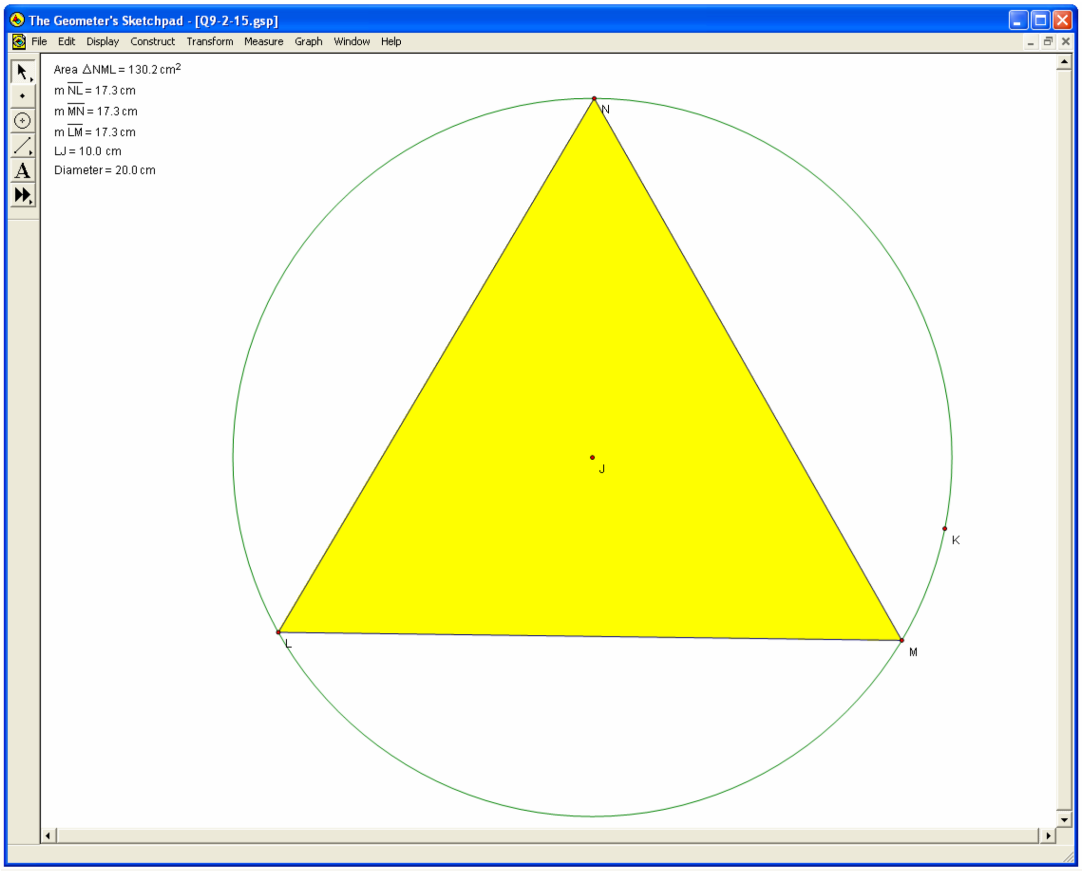
Chapter 9 Section 2 Question 14 Page 490

Answers will vary. A spreadsheet investigation is shown. The minimum perimeter is 20 m using dimensions of 5 m by 10 m. Click here to load the spreadsheet.

Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
1	1	50.0	52.0	50
2	2	25.0	29.0	50
3	3	16.7	22.7	50
4	4	12.5	20.5	50
5	5	10.0	20.0	50

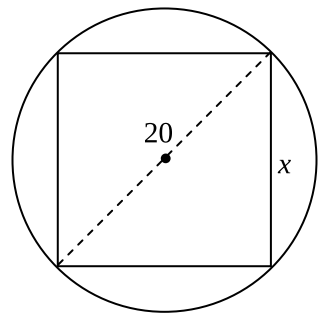
Chapter 9 Section 2 Question 15 Page 490

Answers will vary. An investigation using dynamic geometry software is shown. The maximum area occurs when an equilateral triangle of side length 17.3 cm is used. Click here to load the sketch.



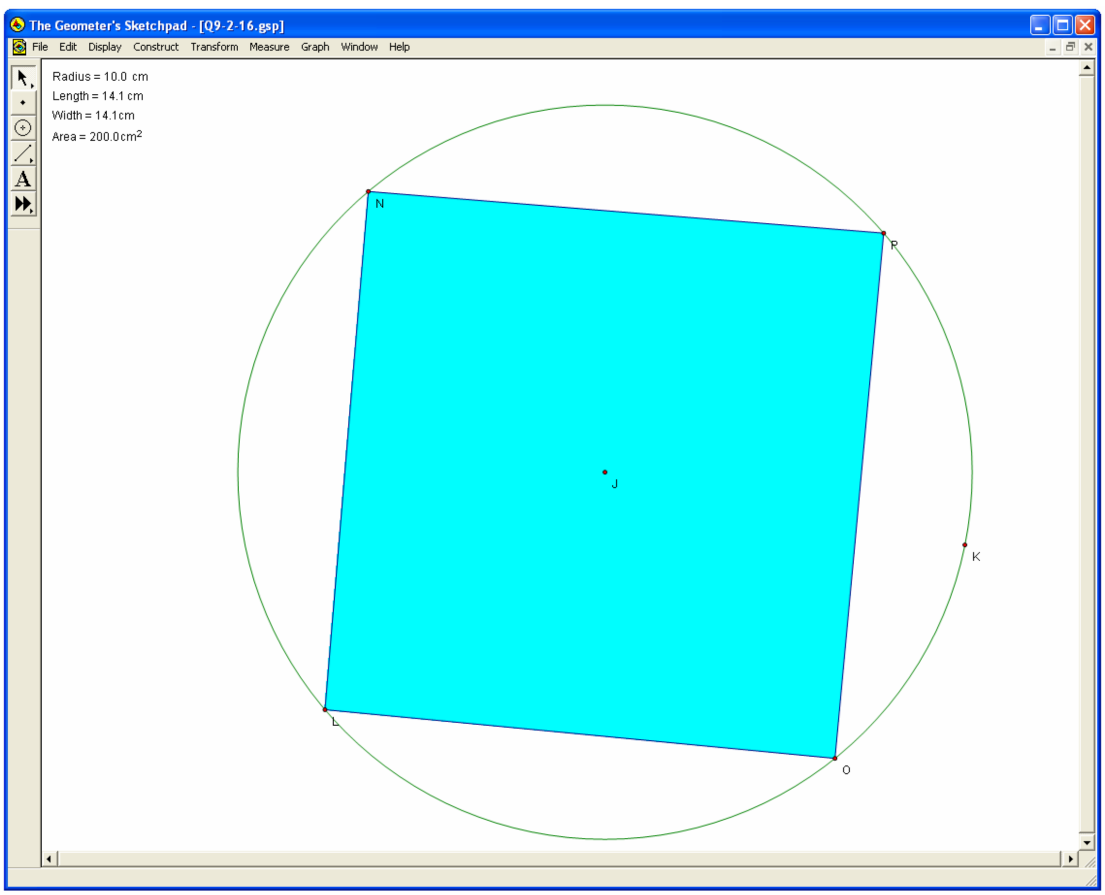
Chapter 9 Section 2 Question 16 Page 490

Methods will vary. The maximum area occurs when a square is used of side length of approximately 14.1 cm.



$$x^2 + x^2 = 20^2$$
$$2x^2 = 400$$
$$\frac{2x^2}{2} = \frac{400}{2}$$
$$x^2 = 200$$
$$x = \sqrt{200}$$
$$x \doteq 14.1$$

Investigation with dynamic geometry software confirms the result. Click here to load the sketch.



Chapter 9 Section 2 Question 17 Page 490

Ranjeet is correct. If the string is used to enclose a circle, the circle will have a greater area than the square.

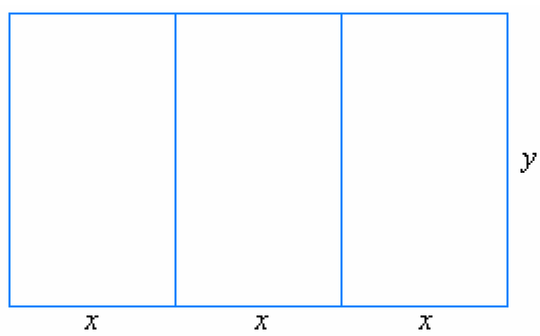
$$C = 2\pi r$$
$$24 = 2\pi r$$
$$\frac{24}{2\pi} = \frac{2\pi r}{2\pi}$$
$$3.82 \doteq r$$

$$A = \pi \times 3.82^2$$
$$\doteq 45.8$$

Chapter 9 Section 2 Question 18 Page 490

Consider the layout of the three adjoining fields shown. The total length of fence is $6x + 4y = 500$. Investigations may vary. A spreadsheet investigation is shown. Click here to load the spreadsheet.

The maximum area occurs when $x = 41.7$ m and $y = 62.45$ m.

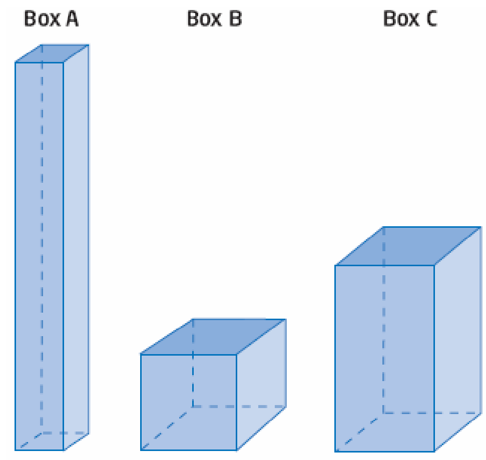


x (m)	y (m)	Fencing (m)	Area (m^2)
40.00	65.00	500.00	2600.00
40.10	64.85	500.00	2600.49
40.20	64.70	500.00	2600.94
40.30	64.55	500.00	2601.37
40.40	64.40	500.00	2601.76
40.50	64.25	500.00	2602.13
40.60	64.10	500.00	2602.46
40.70	63.95	500.00	2602.77
40.80	63.80	500.00	2603.04
40.90	63.65	500.00	2603.29
41.00	63.50	500.00	2603.50
41.10	63.35	500.00	2603.69
41.20	63.20	500.00	2603.84
41.30	63.05	500.00	2603.97
41.40	62.90	500.00	2604.06
41.50	62.75	500.00	2604.13
41.60	62.60	500.00	2604.16
41.70	62.45	500.00	2604.17
41.80	62.30	500.00	2604.14
41.90	62.15	500.00	2604.09
42.00	62.00	500.00	2604.00
42.10	61.85	500.00	2603.89
42.20	61.70	500.00	2603.74
42.30	61.55	500.00	2603.57
42.40	61.40	500.00	2603.36

Chapter 9 Section 3 Minimize the Surface Area of a Square-Based Prism

Chapter 9 Section 3 Question 1 Page 495

From least to greatest surface area the prisms are ranked B, C, and A. The cubic shape has the least surface area. The thinnest shape has the greatest surface area.



Chapter 9 Section 3 Question 2 Page 495

a)

$$\begin{aligned} V &= s^3 \\ 512 &= s^3 \\ \sqrt[3]{512} &= \sqrt[3]{s^3} \\ \sqrt[3]{512} &= s \\ 8 &= s \end{aligned}$$

The square-based prism with the least surface area is a cube with a side length of 8 cm.

b)

$$\begin{aligned} V &= s^3 \\ 1000 &= s^3 \\ \sqrt[3]{1000} &= \sqrt[3]{s^3} \\ \sqrt[3]{1000} &= s \\ 10 &= s \end{aligned}$$

The square-based prism with the least surface area is a cube with a side length of 10 cm.

c)

$$\begin{aligned} V &= s^3 \\ 750 &= s^3 \\ \sqrt[3]{750} &= \sqrt[3]{s^3} \\ \sqrt[3]{750} &= s \\ 9.1 &\doteq s \end{aligned}$$

The square-based prism with the least surface area is a cube with a side length of 9.1 cm.

d)

$$\begin{aligned} V &= s^3 \\ 1200 &= s^3 \\ \sqrt[3]{1200} &= \sqrt[3]{s^3} \\ \sqrt[3]{1200} &= s \\ 10.6 &\doteq s \end{aligned}$$

The square-based prism with the least surface area is a cube with a side length of 10.6 cm.

Chapter 9 Section 3 Question 3 Page 495

a)

$$\begin{aligned} SA &= 6s^2 \\ &= 6 \times 8^2 \\ &= 384 \end{aligned}$$

The surface area of the prism is 384 cm^2.

b)

$$\begin{aligned} SA &= 6s^2 \\ &= 6 \times 10^2 \\ &= 600 \end{aligned}$$

The surface area of the prism is 600 cm^2.

c)

$$\begin{aligned} SA &= 6s^2 \\ &= 6 \times 9.1^2 \\ &\doteq 497 \end{aligned}$$

The surface area of the prism is about 497 cm^2.

d)

$$\begin{aligned} SA &= 6s^2 \\ &= 6 \times 10.6^2 \\ &\doteq 674 \end{aligned}$$

The surface area of the prism is about 674 cm^2.

Chapter 9 Section 3 Question 4 Page 495

$$\begin{aligned} V &= s^3 \\ 3200 &= s^3 \\ \sqrt[3]{3200} &= s \\ 14.7 &\doteq s \end{aligned}$$

The square-based prism with the least surface area is a cube with a side length of about 14.7 cm.

Chapter 9 Section 3 Question 5 Page 496

a)

$$V = s^3$$
$$4000 = s^3$$
$$\sqrt[3]{4000} = s$$
$$15.9 \doteq s$$

The box with the least surface area is a cube with a side length of about 15.9 cm.

b) Answers will vary. Sample answers are shown.

A square-based prism is difficult to pick up with one hand to pour the laundry soap. Manufacturers may also want a large front on the box to display the company logo and brand name.

Chapter 9 Section 3 Question 6 Page 496

a)

$$V = s^3$$
$$750 = s^3$$
$$\sqrt[3]{750} = s$$
$$9.09 \doteq s$$

The box with the least surface area is a cube with a side length of 9.09 cm.

b)

$$SA = 6s^2$$
$$= 6 \times 9.09^2$$
$$\doteq 495.8$$

The minimum area of cardboard required is about 495.8 cm^2.

Chapter 9 Section 3 Question 7 Page 496

2.5 L = 2500 cm^3

$$V = s^3$$
$$2500 = s^3$$
$$\sqrt[3]{2500} = s$$
$$13.6 \doteq s$$

$$SA = 6s^2$$
$$= 6 \times 13.6^2$$
$$\doteq 1110$$

The minimum area of cardboard required is about 1110 cm^2.

Chapter 9 Section 3 Question 8 Page 496

a) A spreadsheet solution is shown. The prism has a base length of 17.1 cm and a height of 8.5 cm, for a volume of 2500 cm^3, and a minimum surface area. Click here to load the spreadsheet.

Base (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
16.0	9.8	2500.0	881.0
16.1	9.6	2500.0	880.3
16.2	9.5	2500.0	879.7
16.3	9.4	2500.0	879.2
16.4	9.3	2500.0	878.7
16.5	9.2	2500.0	878.3
16.6	9.1	2500.0	878.0
16.7	9.0	2500.0	877.7
16.8	8.9	2500.0	877.5
16.9	8.8	2500.0	877.3
17.0	8.7	2500.0	877.2
17.1	8.5	2500.0	877.2
17.2	8.5	2500.0	877.2
17.3	8.4	2500.0	877.3
17.4	8.3	2500.0	877.5

b) The dimensions are different from the box in question 7.

c) The lidless box requires less material.

Chapter 9 Section 3 Question 9 Page 496

a) 200mL = 200 cm^3

$$V = s^3$$
$$200 = s^3$$
$$\sqrt[3]{200} = s$$
$$5.8 \doteq s$$

The box with a minimum surface area is a cube with a side length of 5.8 cm.

b) Answers will vary. A sample answer is shown.

Cubical boxes are harder to hold, and the cube would be very small.

c) Answers will vary.

Chapter 9 Section 3 Question 10 Page 497

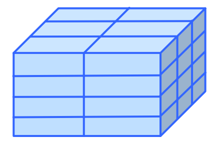
Answers will vary.

Chapter 9 Section 3 Question 11 Page 497

You cannot make a cube with an integral side length using all 100 cubes. Find dimensions that are as close to a cube as possible, such as $5 \times 5 \times 4$.

Chapter 9 Section 3 Question 12 Page 497

a) Pack the boxes as shown.



b) This is the closest that 24 boxes can be stacked to form a cube, which provides the minimum surface area.

c) Answers will vary. A sample answer is shown.

Packing 24 boxes per carton is not the most economical use of cardboard. A cube can be created to package 6 tissue boxes: length 1 box (1 × 24 cm), width 2 boxes (2 × 12 cm), and height 3 boxes (3 × 8 cm).

Chapter 9 Section 3 Question 13 Page 497

A spreadsheet solution is shown. The warehouse should be built with a base length of 12.6 m and a height of 6.3 m for a volume of 1000 m^3 and a surface area of 476.22 m^2. Click here to load the spreadsheet.

Base (m)	Height (m)	Volume (m^3)	Surface Area (m^2)
12.0	6.9	1000.0	477.33
12.1	6.8	1000.0	476.99
12.2	6.7	1000.0	476.71
12.3	6.6	1000.0	476.49
12.4	6.5	1000.0	476.34
12.5	6.4	1000.0	476.25
12.6	6.3	1000.0	476.22
12.7	6.2	1000.0	476.25
12.8	6.1	1000.0	476.34

Chapter 9 Section 3 Question 14 Page 497

$$\begin{aligned} V &= s^3 \\ 216\,000 &= s^3 \\ \sqrt[3]{216\,000} &= s \\ 60 &= s \end{aligned}$$

$$\begin{aligned} SA &= 6s^2 \\ &= 6 \times 60^2 \\ &= 21\,600 \end{aligned}$$

The least amount of cardboard required is 21 600 × 1.10, or 23 760 cm^2.

Chapter 9 Section 3 **Question 15** **Page 497**

$$
\begin{aligned}
V &= s^3 \\
2700 &= s^3 \\
\sqrt[3]{2700} &= s \\
13.92 &\doteq s
\end{aligned}
$$

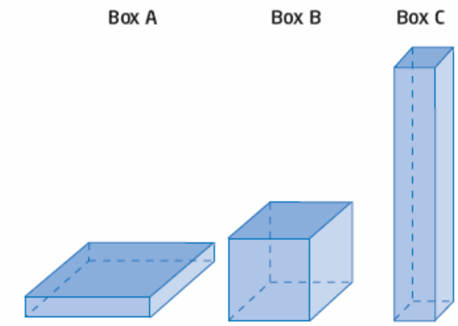
$$
\begin{aligned}
SA &= 6s^2 \\
&= 6 \times 13.92^2 \\
&\doteq 1162.6
\end{aligned}
$$

The length of a side is 13.92 cm. Each flap has a base and a height of $\frac{1}{3} \times 13.92$, or 4.64 cm. The area of flaps needed is $4 \times \frac{1}{2} \times 4.64^2$, or about 43.1 cm^2. The total area of cardboard required for a box is 1162.6 + 43.1, or 1205.7 cm^2.

Chapter 9 Section 4 Maximize the Volume of a Square-Based Prism

Chapter 9 Section 4 Question 1 Page 501

The prisms in order of volume from greatest to least are B, C, and A.



Chapter 9 Section 4 Question 2 Page 502

a)

$$SA = 6s^2$$
$$150 = 6s^2$$
$$\frac{150}{6} = \frac{6s^2}{6}$$
$$25 = s^2$$
$$\sqrt{25} = s$$
$$5 = s$$

The square-based prism with the maximum volume is a cube with a side length of 5 cm.

b)

$$SA = 6s^2$$
$$2400 = 6s^2$$
$$\frac{2400}{6} = \frac{6s^2}{6}$$
$$400 = s^2$$
$$\sqrt{400} = s$$
$$20 = s$$

The square-based prism with the maximum volume is a cube with a side length of 20 m.

c)

$$SA = 6s^2$$
$$750 = 6s^2$$
$$\frac{750}{6} = \frac{6s^2}{6}$$
$$125 = s^2$$
$$\sqrt{125} = s$$
$$11.2 \doteq s$$

The square-based prism with the maximum volume is a cube with a side length of about 11.2 cm.

d)

$$\begin{aligned} SA &= 6s^2 \\ 1200 &= 6s^2 \\ \frac{1200}{6} &= \frac{6s^2}{6} \\ 200 &= s^2 \\ \sqrt{200} &= s \\ 14.1 &\doteq s \end{aligned}$$

The square-based prism with the maximum volume is a cube with a side length of about 14.1 m.

Chapter 9 Section 4 Question 3 Page 502

a)

$$\begin{aligned} V &= s^3 \\ &= 5^3 \\ &= 125 \end{aligned}$$

The volume is 125 cm^3.

b)

$$\begin{aligned} V &= s^3 \\ &= 20^3 \\ &= 8000 \end{aligned}$$

The volume is 8000 m^3.

c)

$$\begin{aligned} V &= s^3 \\ &= 11.2^3 \\ &= 1405 \end{aligned}$$

The volume is about 1405 cm^3.

d)

$$\begin{aligned} V &= s^3 \\ &= 14.1^3 \\ &\doteq 2803 \end{aligned}$$

The volume is about 2803 m^3.

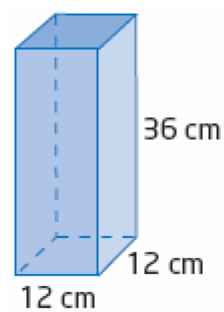
Chapter 9 Section 4 Question 4 Page 502

A spreadsheet solution is shown. The maximum volume occurs with a cube of side length 10.8 cm, for a volume of 1260.1 cm^3. Click here to load the spreadsheet.

Base (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
10.0	12.5	1250.0	700.0
10.1	12.3	1252.3	700.0
10.2	12.1	1254.4	700.0
10.3	11.8	1256.1	700.0
10.4	11.6	1257.6	700.0
10.5	11.4	1258.7	700.0
10.6	11.2	1259.5	700.0
10.7	11.0	1260.0	700.0
10.8	10.8	1260.1	700.0
10.9	10.6	1260.0	700.0
11.0	10.4	1259.5	700.0

Chapter 9 Section 4 Question 5 Page 502

a)
$$\begin{aligned} SA &= 4A_{\text{side}} + 2A_{\text{bottom}} \\ &= 4(12 \times 36) + 2(12 \times 12) \\ &= 1728 + 288 \\ &= 2016 \end{aligned}$$



$$\begin{aligned} V &= lwh \\ &= 12 \times 12 \times 36 \\ &= 5184 \end{aligned}$$

The surface area is 2016 cm^2, and the volume is 5184 cm^3.

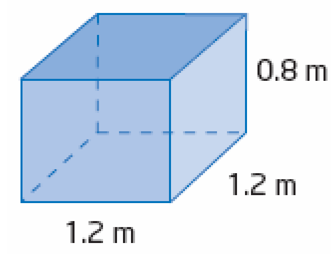
b)
$$\begin{aligned} SA &= 6s^2 \\ 2016 &= 6s^2 \\ \frac{2016}{6} &= \frac{6s^2}{6} \\ 336 &= s^2 \\ \sqrt{336} &= s \\ 18.3 &\doteq s \end{aligned}$$

The box with maximum volume is a cube with a side length of 18.3 cm.

c)
$$\begin{aligned} V &= s^3 \\ &= 18.3^3 \\ &\doteq 6128 \end{aligned}$$

The volume of the box in part b) is about 6128 cm^3, which is greater than the volume of the box in part a).

Chapter 9 Section 4 Question 6 Page 502



a) $SA = 4A_{side} + 2A_{bottom}$

$= 4(1.2 \times 0.8) + 2(1.2 \times 1.2)$

$= 3.84 + 2.88$

$= 6.72$

$V = lwh$

$= 1.2 \times 1.2 \times 0.8$

$= 1.152$

The surface area is 6.72 m^2, and the volume is 1.152 m^3.

b) $SA = 6s^2$

$6.72 = 6s^2$

$\frac{6.72}{6} = \frac{6s^2}{6}$

$1.12 = s^2$

$\sqrt{1.12} = s$

$1.1 \doteq s$

The box with maximum volume is a cube with a side length of 1.1 m.

c) $V = s^3$

$= 1.1^3$

$= 1.331$

The volume of the box in part b) is 1.331 m^3, which is greater than the volume of the box in part a).

Chapter 9 Section 4 Question 7 Page 502

a) $SA = 6s^2$

$12 = 6s^2$

$\frac{12}{6} = \frac{6s^2}{6}$

$2 = s^2$

$\sqrt{2} = s$

$1.4 \doteq s$

The box with maximum volume is a cube with a side length of 1.4 m.

b) $V = s^3$

$= 1.4^3$

$\doteq 3$

The volume of the box is about 3 m^3.

Chapter 9 Section 4 Question 8 Page 503

a)

$$\begin{aligned} SA &= 6s^2 \\ 2500 &= 6s^2 \\ \frac{2500}{6} &= \frac{6s^2}{6} \\ \frac{1250}{3} &= s^2 \\ \sqrt{\frac{1250}{3}} &= s \\ 20.4 &\doteq s \end{aligned}$$

The box with maximum volume is a cube with a side length of 20.4 cm.

b) $V = s^3$

$= 20.4^3$

$\doteq 8490$

The volume of the box is about 8490 cm^3.

c) $\text{Empty Space} = V_{\text{box}} - V_{\text{drive}}$

$= 8490 - 14 \times 20 \times 2.5$

$= 7790$

The volume of empty space is 7790 cm^3.

d) Answers will vary. A sample answer is shown.

Assume that there is no empty space in the box. The DVD would fit into the cube with enough room around the edges for the shredded paper. The shredded paper is tightly packed.

Chapter 9 Section 4 Question 9 Page 503

Solutions for the Achievement Checks are shown in the Teacher's Resource.

Chapter 9 Section 4 **Question 10** **Page 503**

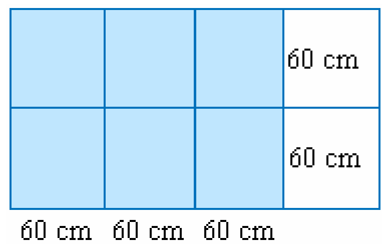
a) Dylan has 120 × 240, or 28 800 cm^2 of plywood available.

$$SA = 6s^2$$
$$28\,800 = 6s^2$$
$$\frac{28\,800}{6} = \frac{6s^2}{6}$$
$$4800 = s^2$$
$$\sqrt{4800} = s$$
$$69.3 \doteq s$$

Ideally, Dylan needs a cube with a side length 69.3 cm.

b) Diagrams will vary. A sample answer is shown.

Dylan needs 6 pieces of wood measuring 69.3 cm by 69.3 cm. These cannot be cut from a piece of wood measuring 120 cm by 240 cm. Dylan's closest option is to cut 6 pieces measuring 60 cm by 60 cm, as shown.

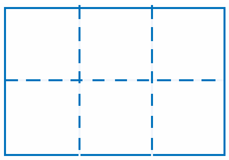


c) Answers will vary. A sample answer is shown.

Assume that Dylan does not want to cut some of the wasted wood, and glue it onto his pieces to make bigger pieces. Assume that the saw cuts are negligible.

Chapter 9 Section 4 Question 11 Page 503

a)



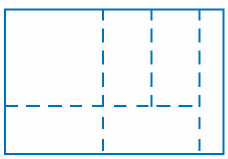
b) $V = s^3$

$= 10^3$

$= 1000$

The volume of the box is 1000 cm^3.

c)



d) Assume that the height is half the base length. From the diagram, the base length will be $\frac{2}{3} \times 20$, or 13.3 cm, and the height will be 6.7 cm.

$V = lwh$

$= 13.3 \times 13.3 \times 6.7$

$\doteq 1185.2$

The volume of the box is about 1185.2 cm^3.

e) Answers will vary. A sample answer is shown.

Assume that the cuts waste a negligible amount of glass.

Chapter 9 Section 5 Maximize the Volume of a Cylinder

Chapter 9 Section 5 Question 1 Page 508

a)

$$SA = 6\pi r^2$$
$$1200 = 6\pi r^2$$
$$\frac{\overset{200}{\cancel{1200}}}{\underset{1}{\cancel{6}}\pi} = \frac{\overset{1}{\cancel{6\pi}}r^2}{\underset{1}{\cancel{6\pi}}}$$
$$\frac{200}{\pi} = r^2$$
$$\sqrt{\frac{200}{\pi}} = r$$
$$7.98 \doteq r$$

$$h = 2 \times 7.98$$
$$= 15.96$$

The radius of the cylinder is 7.98 cm, and the height is 15.96 cm.

b)

$$SA = 6\pi r^2$$
$$10 = 6\pi r^2$$
$$\frac{\overset{5}{\cancel{10}}}{\underset{3}{\cancel{6}}\pi} = \frac{\overset{1}{\cancel{6\pi}}r^2}{\underset{1}{\cancel{6\pi}}}$$
$$\frac{5}{3\pi} = r^2$$
$$\sqrt{\frac{5}{3\pi}} = r$$
$$0.73 \doteq r$$

$$h = 2 \times 0.73$$
$$= 1.46$$

The radius of the cylinder is 0.73 m, and the height is 1.46 m.

c)

$$SA = 6\pi r^2$$
$$125 = 6\pi r^2$$
$$\frac{125}{6\pi} = \frac{6\pi r^2}{6\pi}$$
$$\frac{125}{6\pi} = r^2$$
$$\sqrt{\frac{125}{6\pi}} = r$$
$$2.58 \doteq r$$

$$h = 2 \times 2.58$$
$$= 5.16$$

The radius of the cylinder is 2.58 cm, and the height is 5.16 cm.

d)

$$SA = 6\pi r^2$$
$$6400 = 6\pi r^2$$
$$\frac{\overset{3200}{\cancel{6400}}}{\underset{3}{\cancel{6}}\,\pi} = \frac{\overset{1}{\cancel{6\pi}}\, r^2}{\underset{1}{\cancel{6\pi}}}$$
$$\frac{3200}{3\pi} = r^2$$
$$\sqrt{\frac{3200}{3\pi}} = r$$
$$18.43 \doteq r$$

$$h = 2 \times 18.43$$
$$= 36.86$$

The radius of the cylinder is 18.43 mm, and the height is 36.86 mm.

Chapter 9 Section 5 Question 2 Page 508

a) $V = \pi r^2 h$

$= \pi \times 7.98^2 \times 15.96$

$\doteq 3193$

The volume of the cylinder is about 3193 cm^3.

b) $V = \pi r^2 h$

$= \pi \times 0.73^2 \times 1.46$

$\doteq 2$

The volume of the cylinder is about 2 m^3.

c) $V = \pi r^2 h$

$= \pi \times 2.58^2 \times 5.16$

$\doteq 108$

The volume of the cylinder is about 108 cm^3.

d) $V = \pi r^2 h$

$= \pi \times 18.43^2 \times 36.86$

$\doteq 39\,333$

The volume of the cylinder is about 39 333 cm^3.

Chapter 9 Section 5 Question 3 Page 508

$$SA = 6\pi r^2$$
$$8 = 6\pi r^2$$
$$\frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{6}}\pi} = \frac{\overset{1}{\cancel{6\pi}} r^2}{\underset{1}{\cancel{6\pi}}}$$
$$\frac{4}{3\pi} = r^2$$
$$\sqrt{\frac{4}{3\pi}} = r$$
$$0.65 \doteq r$$

$$h = 2 \times 0.65$$
$$= 1.3$$

$$V = \pi r^2 h$$
$$= \pi \times 0.65^2 \times 1.3$$
$$\doteq 2$$

The volume of fuel that the tank can hold is about 2 m^3.

Chapter 9 Section 5 Question 4 Page 508

a)

$$SA = 6\pi r^2$$
$$72 = 6\pi r^2$$
$$\frac{\overset{12}{\cancel{72}}}{\underset{1}{\cancel{6}}\pi} = \frac{\overset{1}{\cancel{6\pi}}\, r^2}{\underset{1}{\cancel{6\pi}}}$$
$$\frac{12}{\pi} = r^2$$
$$\sqrt{\frac{12}{\pi}} = r$$
$$2.0 \doteq r$$

$$h = 2 \times 2.0$$
$$= 4.0$$

The radius of the cylinder is 2.0 m, and the height is 4.0 m.

b) $V = \pi r^2 h$

$$= \pi \times 2.0^2 \times 4.0$$
$$\doteq 50.265$$

The volume is about 50.265 m^3, or 50 265 L.

c) Answers will vary. A sample answer is shown.

Assume that no metal will be wasted in the building process, and that no metal is being overlapped.

Chapter 9 Section 5 Question 5 Page 509

a) The height of the optimal cylinder is 12 cm.

b) The cylinder will hold $\frac{12}{0.2}$, or 60 CDs.

c) Answers will vary. A sample answer is shown.

Assume that only the dimensions of the CDs need to be considered, that no extra space is left for the container's closing mechanism, and that the plastic container has negligible thickness.

Chapter 9 Section 5 Question 6 Page 509

a) Answers will vary. A sample answer is shown.

Adjust the surface area formula for the new cylinder, isolate the height and run a few trials using a spreadsheet to find the maximum volume.

b)

$$SA = \pi r^2 + 2\pi rh$$

$$SA - \pi r^2 = 2\pi rh$$

$$\frac{SA - \pi r^2}{2\pi r} = \frac{2\pi rh}{2\pi r}$$

$$\frac{SA - \pi r^2}{2\pi r} = h$$

The radius and height are both 7.3 cm, for a volume of 1213.9 cm^3. Click here to load the spreadsheet.

Radius (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
7.0	7.9	1211.2	500.0
7.1	7.7	1212.8	500.0
7.2	7.5	1213.7	500.0
7.3	7.3	1213.9	500.0
7.4	7.1	1213.5	500.0
7.5	6.9	1212.3	500.0
7.6	6.7	1210.5	500.0
7.7	6.5	1207.9	500.0
7.8	6.3	1204.6	500.0
7.9	6.1	1200.5	500.0

Chapter 9 Section 5 Question 7 Page 509

a) Answers will vary. A possible answer is that a cylinder will have the greatest volume.

b)

$$
\begin{aligned}
SA_{\text{cylinder}} &= 6\pi r^2 \\
2400 &= 6\pi r^2 \\
\frac{2400}{6\pi} &= \frac{6\pi r^2}{6\pi} \\
\frac{400}{\pi} &= r^2 \\
\sqrt{\frac{400}{\pi}} &= r \\
11.28 &\doteq r
\end{aligned}
$$

$$
\begin{aligned}
h &= 2 \times 11.28 \\
&= 22.56
\end{aligned}
$$

$$
\begin{aligned}
V_{\text{cylinder}} &= \pi r^2 h \\
&= \pi \times 11.28^2 \times 22.56 \\
&\doteq 9018
\end{aligned}
$$

$$
\begin{aligned}
SA_{\text{prism}} &= 6s^2 \\
2400 &= 6s^2 \\
\frac{2400}{6} &= \frac{6s^2}{6} \\
400 &= s \\
\sqrt{400} &= s \\
20 &= s
\end{aligned}
$$

$$
\begin{aligned}
V_{\text{prism}} &= s^3 \\
&= 20^3 \\
&= 8000
\end{aligned}
$$

The cylinder has a volume of about 9018 cm^2, while the square-based prism has a volume of 8000 cm^2.

Chapter 9 Section 5 Question 8 Page 509

a) Answers will vary. A possible answer is that the sphere will produce the greatest volume.

b)

$$SA_{\text{sphere}} = 4\pi r^2$$
$$2000 = 4\pi r^2$$
$$\frac{2000}{4\pi} = \frac{4\pi r^2}{4\pi}$$
$$\frac{500}{\pi} = r^2$$
$$\sqrt{\frac{500}{\pi}} = r$$
$$12.62 \doteq r$$

The sphere has a radius of 12.62 cm.

$$SA_{\text{cylinder}} = 6\pi r^2$$
$$2000 = 6\pi r^2$$
$$\frac{2000}{6\pi} = \frac{6\pi r^2}{6\pi}$$
$$\frac{1000}{3\pi} = r^2$$
$$\sqrt{\frac{1000}{3\pi}} = r$$
$$10.30 \doteq r$$

$$h = 2 \times 10.30$$
$$= 20.60$$

The cylinder has a radius of 10.30 cm and a height of 20.60 cm.

$$SA_{\text{cube}} = 6s^2$$
$$2000 = 6s^2$$
$$\frac{2000}{6} = \frac{6s^2}{6}$$
$$\frac{1000}{3} = s^2$$
$$\sqrt{\frac{1000}{3}} = s$$
$$18.26 \doteq s$$

The square-based prism has a side length of 18.26 cm.

c) $V_{\text{sphere}} = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 12.62^3$

$\doteq 8419.1$

The sphere has a volume of about 8419.1 cm^3.

$V_{\text{cylinder}} = \pi r^2 h$

$= \pi \times 10.30^2 \times 20.60$

$\doteq 6865.8$

The cylinder has a volume of about 6865.8 cm^3.

$V_{\text{cube}} = s^3$

$= 18.26^3$

$\doteq 6088.4$

The square-based prism has a volume of about 6088.4 cm^3.

d) The sphere has the greatest volume. This will always be the case.

e) For a given surface area, volume of a sphere > volume of a cylinder > volume of a square-based prism.

Chapter 9 Section 5 **Question 9** **Page 509**

You have 2 m^2 of metal to work with.

a) For a cylinder with a top and a bottom, the maximum volume occurs for a radius of 0.33 cm and a height of 0.63 cm. Click here to load the spreadsheet.

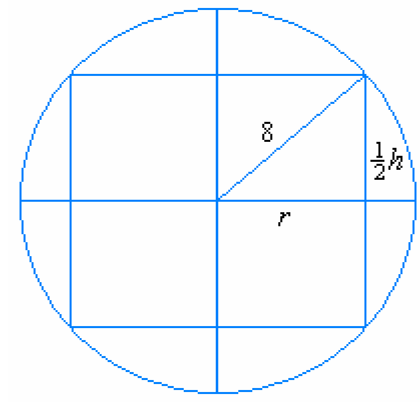
Radius (m)	Height (m)	Volume (m^3)	Surface Area (m^2)
0.30	0.76	0.21518	2
0.31	0.72	0.21641	2
0.32	0.67	0.21706	2
0.33	0.63	0.21710	2
0.34	0.60	0.21652	2
0.35	0.56	0.21530	2
0.36	0.52	0.21343	2
0.37	0.49	0.21087	2
0.38	0.46	0.20761	2
0.39	0.43	0.20364	2

b) For a cylinder with no top, the maximum volume occurs with a radius and height of 0.46 m. Click here to load the spreadsheet.

Radius (m)	Height (m)	Volume (m^3)	Surface Area (m^2)
0.38	0.65	0.2938	2
0.39	0.62	0.2968	2
0.40	0.60	0.2995	2
0.41	0.57	0.3017	2
0.42	0.55	0.3036	2
0.43	0.53	0.3051	2
0.44	0.50	0.3062	2
0.45	0.48	0.3069	2
0.46	0.46	0.3071	2
0.47	0.44	0.3069	2
0.48	0.18	0.1326	2
0.49	0.16	0.1204	2
0.50	0.14	0.1073	2

Chapter 9 Section 5 Question 10 Page 509

Methods may vary. A solution using a spreadsheet, and another using dynamic geometry software are shown. The volume is maximized at 1238.22 cm^3 for a radius of 6.53 cm and a height of 9.24 cm.



$$r^2 + \left(\frac{1}{2}h\right)^2 = 8^2$$

$$r^2 + \frac{1}{4}h^2 = 64$$

$$\frac{1}{4}h^2 = 64 - r^2$$

$$4 \times \frac{1}{4}h^2 = 4 \times \left(64 - r^2\right)$$

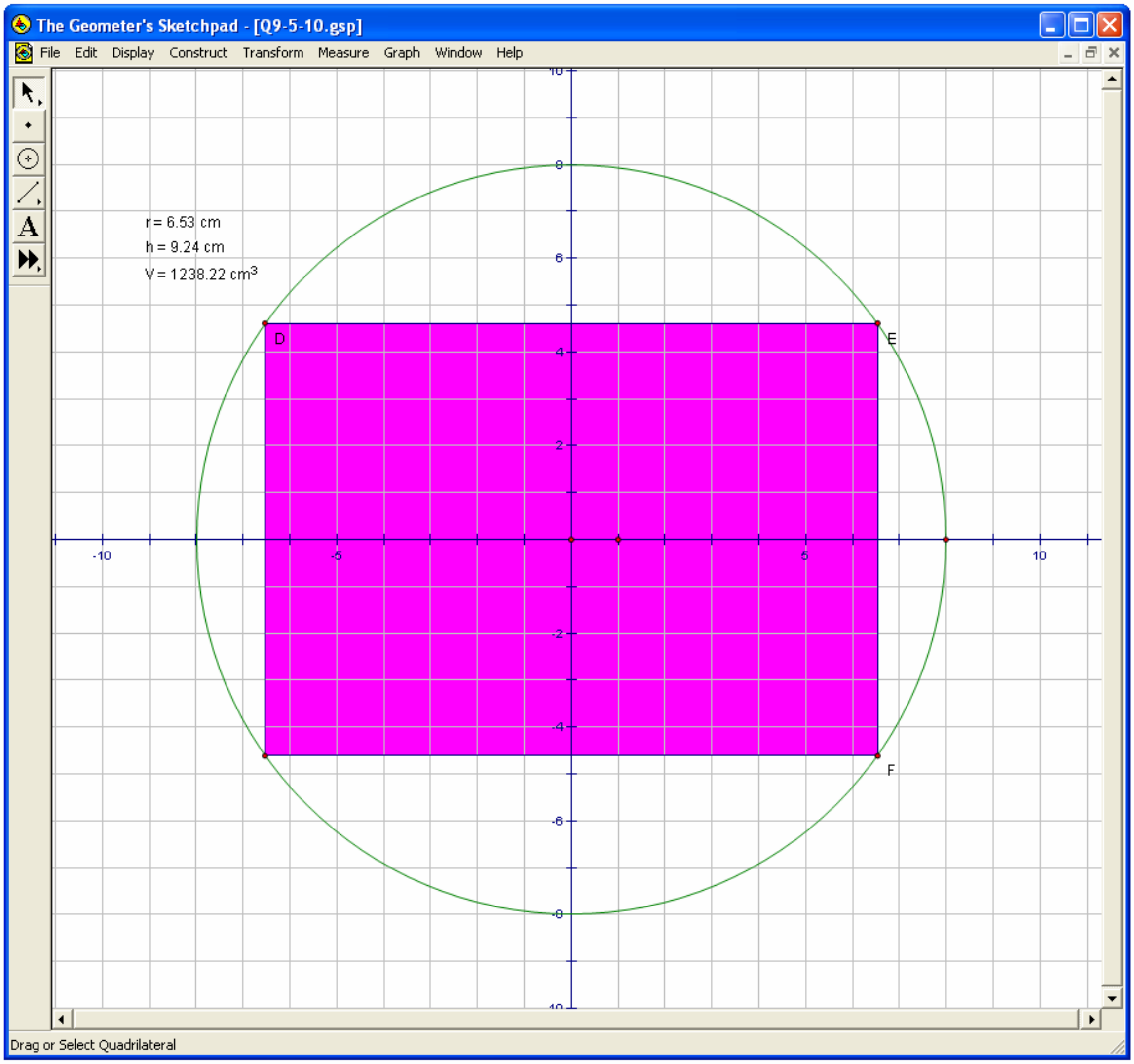
$$h^2 = 256 - 4r^2$$

$$h = \sqrt{256 - 4r^2}$$

Use this formula for *h*, as well as the formulas for volume and surface area, in a spreadsheet. Click here to load the spreadsheet.

Radius (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
6.50	9.33	1238.04	646.40
6.51	9.30	1238.14	646.66
6.52	9.27	1238.20	646.92
6.53	9.24	1238.22	647.16
6.54	9.21	1238.21	647.40
6.55	9.19	1238.16	647.63
6.56	9.16	1238.08	647.85
6.57	9.13	1237.97	648.07

Click here to load the sketch.



Chapter 9 Section 6 Minimize the Surface Area of a Cylinder

Chapter 9 Section 6 Question 1 Page 513

a)

$$
\begin{aligned}
V &= 2\pi r^3 \\
1200 &= 2\pi r^3 \\
\frac{\overset{600}{\cancel{1200}}}{\underset{1}{\cancel{2}}\pi} &= \frac{\overset{1}{\cancel{2\pi}} r^3}{\underset{1}{\cancel{2\pi}}} \\
\frac{600}{\pi} &= r^3 \\
\sqrt[3]{\frac{600}{\pi}} &= r \\
5.8 &\doteq r
\end{aligned}
$$

$$
\begin{aligned}
h &= 2 \times 5.8 \\
&= 11.6
\end{aligned}
$$

The radius of the cylinder with minimum surface area is 5.8 cm, and the height is 11.6 cm.

b)

$$
\begin{aligned}
V &= 2\pi r^3 \\
1 &= 2\pi r^3 \\
\frac{1}{2\pi} &= \frac{2\pi r^3}{2\pi} \\
\frac{1}{2\pi} &= r^3 \\
\sqrt[3]{\frac{1}{2\pi}} &= r \\
0.5 &= r
\end{aligned}
$$

$$
\begin{aligned}
h &= 2 \times 0.5 \\
&= 1.0
\end{aligned}
$$

The radius of the cylinder with minimum surface area is 0.5 m, and the height is 1.0 m.

c)

$$V = 2\pi r^3$$
$$225 = 2\pi r^3$$
$$\frac{225}{2\pi} = \frac{2\pi r^3}{2\pi}$$
$$\frac{225}{2\pi} = r^3$$
$$\sqrt[3]{\frac{225}{2\pi}} = r$$
$$3.3 \doteq r$$

$$h = 2 \times 3.3$$
$$= 6.6$$

The radius of the cylinder with minimum surface area is 3.3 cm, and the height is 6.6 cm.

d)

$$V = 2\pi r^3$$
$$4 = 2\pi r^3$$
$$\frac{\overset{2}{\cancel{4}}}{\underset{1}{\cancel{2}}\pi} = \frac{\overset{1}{\cancel{2\pi}}r^3}{\underset{1}{\cancel{2\pi}}}$$
$$\frac{2}{\pi} = r^3$$
$$\sqrt[3]{\frac{2}{\pi}} = r$$
$$0.9 \doteq r$$

$$h = 2 \times 0.9$$
$$= 1.8$$

The radius of the cylinder with minimum surface area is 0.9 m, and the height is 1.8 m.

Chapter 9 Section 6 Question 2 Page 513

a) $SA = 2\pi r^2 + 2\pi rh$

$= 2\pi \times 5.8^2 + 2\pi \times 5.8 \times 11.6$

$\doteq 634$

The surface area of the cylinder is about 634 cm^2.

b) $SA = 2\pi r^2 + 2\pi rh$

$= 2\pi \times 0.5^2 + 2\pi \times 0.5 \times 1.0$

$\doteq 5$

The surface area of the cylinder is about 5 m^2.

c) $SA = 2\pi r^2 + 2\pi rh$

$= 2\pi \times 3.3^2 + 2\pi \times 3.3 \times 6.6$

$\doteq 205$

The surface area of the cylinder is about 205 cm^2.

d) $SA = 2\pi r^2 + 2\pi rh$

$= 2\pi \times 0.9^2 + 2\pi \times 0.9 \times 1.8$

$\doteq 15$

The surface area of the cylinder is about 15 m^2.

Chapter 9 Section 6 Question 3 Page 513

$$V = 2\pi r^3$$

$$540 = 2\pi r^3$$

$$\frac{\overset{270}{\cancel{540}}}{\underset{1}{\cancel{2}}\pi} = \frac{\overset{1}{\cancel{2\pi}} r^3}{\underset{1}{\cancel{2\pi}}}$$

$$\frac{270}{\pi} = r^3$$

$$\sqrt[3]{\frac{270}{\pi}} = r$$

$$4.4 \doteq r$$

$$h = 2 \times 4.4$$

$$= 8.8$$

The radius of the cylinder with minimum surface area is 4.4 cm, and the height is 8.8 cm.

Chapter 9 Section 6 Question 4 Page 514

a)

$$
\begin{aligned}
V &= 2\pi r^3 \\
5000 &= 2\pi r^3 \\
\frac{\overset{2500}{\cancel{5000}}}{\underset{1}{\cancel{2}}\pi} &= \frac{\overset{1}{\cancel{2\pi}}\, r^3}{\underset{1}{\cancel{2\pi}}} \\
\frac{2500}{\pi} &= r^3 \\
\sqrt[3]{\frac{2500}{\pi}} &= r \\
9.3 &\doteq r
\end{aligned}
$$

$$
\begin{aligned}
h &= 2 \times 9.3 \\
&= 18.6
\end{aligned}
$$

The radius of the cylinder with minimum surface area is 9.3 cm, and the height is 18.6 cm.

b) Answers will vary. A sample answer is shown.

Assume that no extra material will be needed to enclose the volume.

Chapter 9 Section 6 Question 5 Page 514

$$
\begin{aligned}
V &= 2\pi r^3 \\
12\,000 &= 2\pi r^3 \\
\frac{\overset{6000}{\cancel{12\,000}}}{\underset{1}{\cancel{2}}\pi} &= \frac{\overset{1}{\cancel{2\pi}}\, r^3}{\underset{1}{\cancel{2\pi}}} \\
\frac{6000}{\pi} &= r^3 \\
\sqrt[3]{\frac{6000}{\pi}} &= r \\
12.4 &\doteq r
\end{aligned}
$$

$$
\begin{aligned}
h &= 2 \times 12.4 \\
&= 24.8
\end{aligned}
$$

The radius of the cylinder with minimum surface area is 12.4 cm, and the height is 24.8 cm.

Chapter 9 Section 6 Question 6 Page 514

a)

$$V = 2\pi r^3$$
$$375 = 2\pi r^3$$
$$\frac{375}{2\pi} = \frac{2\pi r^3}{2\pi}$$
$$\frac{375}{2\pi} = r^3$$
$$\sqrt[3]{\frac{375}{2\pi}} = r$$
$$3.9 \doteq r$$

$$h = 2 \times 3.9$$
$$= 7.8$$

The radius of the cylinder with minimum surface area is 3.9 cm, and the height is 7.8 cm.

b) $SA = 2\pi r^2 + 2\pi rh$

$$= 2\pi \times 3.9^2 + 2\pi \times 3.9 \times 7.8$$
$$\doteq 287$$

The cost of the aluminum required is 12 × 0.001 × 287, or $3.44.

Chapter 9 Section 6 Question 7 Page 514

Answers will vary. A sample answer is shown.

It is not always practical to use cylinders with the optimum volume. They may be harder to use, to handle, to carry, or to store.

Chapter 9 Section 6 Question 8 Page 514

$$
\begin{aligned}
V_{\text{cylinder}} &= 2\pi r^3 \\
500 &= 2\pi r^3 \\
\frac{\overset{250}{\cancel{500}}}{\underset{1}{\cancel{2}}\,\pi} &= \frac{\overset{1}{\cancel{2\pi}}\, r^3}{\underset{1}{\cancel{2\pi}}} \\
\frac{250}{\pi} &= r^3 \\
\sqrt[3]{\frac{250}{\pi}} &= r \\
4.30 &\doteq r
\end{aligned}
$$

$$
\begin{aligned}
V_{\text{cube}} &= s^3 \\
500 &= s^3 \\
\sqrt[3]{500} &= s \\
7.94 &\doteq s
\end{aligned}
$$

$$
\begin{aligned}
SA_{\text{cube}} &= 6s^2 \\
&= 6 \times 7.94^2 \\
&\doteq 378
\end{aligned}
$$

$$
\begin{aligned}
h &= 2 \times 4.30 \\
&= 8.60
\end{aligned}
$$

$$
\begin{aligned}
SA_{\text{cylinder}} &= 2\pi r^2 + 2\pi rh \\
&= 2\pi \times 4.30^2 + 2\pi \times 4.30 \times 8.60 \\
&\doteq 349
\end{aligned}
$$

A cylinder will have a surface area of about 349 cm^2, and a cube will have a surface area of about 378 cm^2. A cylinder is more cost efficient.

Chapter 9 Section 6 Question 9 Page 514

The cafeteria does not appear to designed to minimize heat loss The cylindrical shape is taller than its diameter. However, there is a large glass area which would encourage solar heating.

Chapter 9 Section 6 Question 10 Page 515

Solutions for the Achievement Checks are shown in the Teacher's Resource.

Chapter 9 Section 6 **Question 11** **Page 515**

a)

Radius (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
7.0	9.7	1500.0	582.5
7.1	9.5	1500.0	580.9
7.2	9.2	1500.0	579.5
7.3	9.0	1500.0	578.4
7.4	8.7	1500.0	577.4
7.5	8.5	1500.0	576.7
7.6	8.3	1500.0	576.2
7.7	8.1	1500.0	575.9
7.8	7.8	1500.0	575.7
7.9	7.7	1500.0	575.8
8.0	7.5	1500.0	576.1

The minimum surface area for the open cylinder occurs with a radius of 7.8 cm and a height of 7.8 cm. Click here to load the spreadsheet.

b) The minimum surface area is about 576 cm^2.

c) Answers will vary. A sample answer is shown.

Assume that the only cardboard needed is used to enclose the required volume so there is no wastage.

Chapter 9 Section 6 Question 12 Page 515

a) Answers will vary. A possible answer is that the sphere will have the minimum surface area for a given volume.

b)

$$\begin{aligned} V_{\text{cube}} &= s^3 \\ 1000 &= s^3 \\ \sqrt[3]{1000} &= s \\ 10 &= s \end{aligned}$$

$$\begin{aligned} SA_{\text{cube}} &= 6s^2 \\ &= 6 \times 10^2 \\ &= 600 \end{aligned}$$

The surface area of a cube with a volume of 1000 cm^3 is 600 cm^2.

$$\begin{aligned} V_{\text{cylinder}} &= 2\pi r^3 \\ 1000 &= 2\pi r^3 \\ \frac{1000}{2\pi} &= \frac{2\pi r^3}{2\pi} \\ \frac{500}{\pi} &= r^3 \\ \sqrt[3]{\frac{500}{\pi}} &= r \\ 5.42 &\doteq r \end{aligned}$$

$$\begin{aligned} h &= 2 \times 5.42 \\ &= 10.84 \end{aligned}$$

$$\begin{aligned} SA_{\text{cylinder}} &= 2\pi r^2 + 2\pi rh \\ &= 2\pi \times 5.42^2 + 2\pi \times 5.42 \times 10.84 \\ &\doteq 553.7 \end{aligned}$$

The minimum surface area of a cylinder with a volume of 1000 cm^3 is about 553.7 cm^2.

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

$$1000 = \frac{4}{3}\pi r^3$$

$$3 \times 1000 = 3 \times \frac{4}{3}\pi r^3$$

$$3000 = 4\pi r^3$$

$$\frac{3000}{4\pi} = \frac{4\pi r^3}{4\pi}$$

$$\frac{750}{\pi} = r^3$$

$$\sqrt[3]{\frac{750}{\pi}} = r$$

$$6.20 \doteq r$$

$$SA_{\text{sphere}} = 4\pi r^2$$

$$= 4\pi \times 6.20^2$$

$$\doteq 483.1$$

The surface area of a sphere with a volume of 1000 cm^3 is about 483.1 cm^2.

The sphere has the least surface area.

Chapter 9 Section 6 Question 13 Page 515

To enclose a maximum volume, use a sphere.

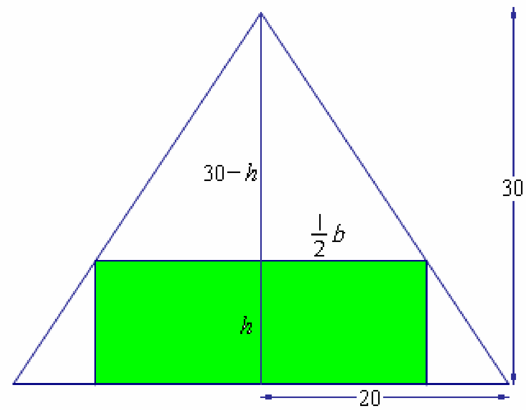
$$
\begin{aligned}
SA &= 4\pi r^2 \\
3584 &= 4\pi r^2 \\
\frac{3584}{4\pi} &= \frac{4\pi r^2}{4\pi} \\
\frac{896}{\pi} &= r^2 \\
\sqrt{\frac{896}{\pi}} &= r \\
16.89 &\doteq r
\end{aligned}
$$

$$
\begin{aligned}
V &= \frac{4}{3}\pi r^3 \\
&= \frac{4}{3}\pi \times 16.89^3 \\
&\doteq 20\,183
\end{aligned}
$$

The greatest volume that can be enclosed is about 20 183 cm^3.

Chapter 9 Section 6 **Question 14** **Page 515**

Consider a square-based prism of base length b and height h inscribed in a cone of radius 20 cm and height 30 cm, as shown. Using similar triangles,



$$\frac{30}{20} = \frac{30-h}{0.5b}$$

$$1.5 = \frac{30-h}{0.5b}$$

$$0.5b \times 1.5 = 0.5b \times \frac{30-h}{0.5b}$$

$$0.75b = 30 - h$$

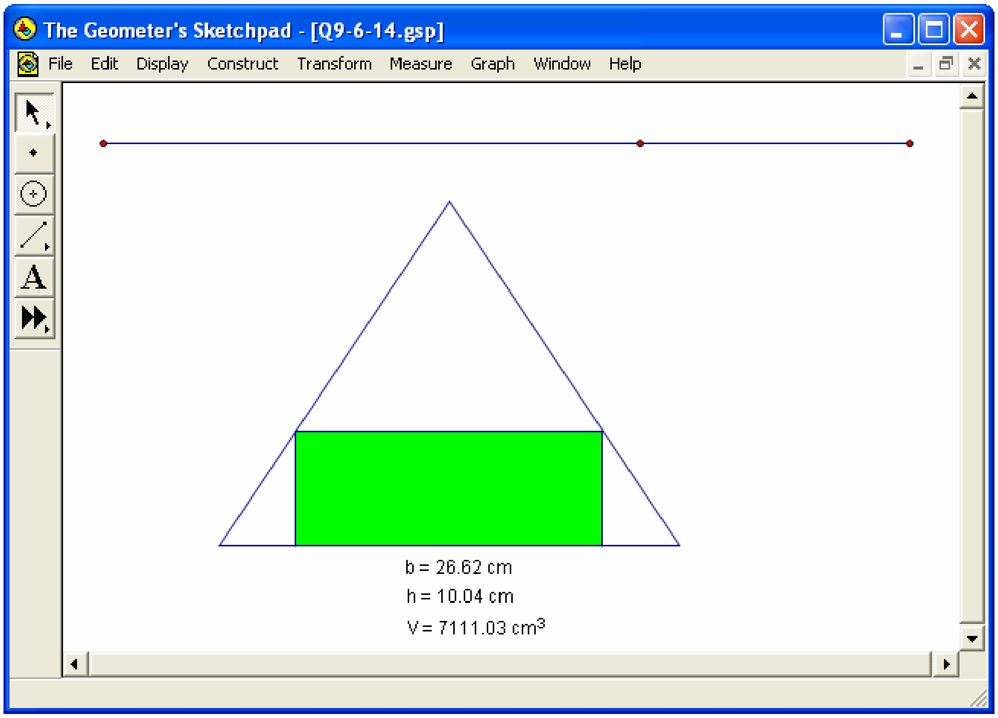
$$h = 30 - 0.75b$$

Use this relation to investigate the volume of the inscribed square-based prism. A sample spreadsheet is shown. Click here to load the spreadsheet.

Base (cm)	Height (cm)	Volume (cm^3)
26.55	10.09	7110.70
26.56	10.08	7110.77
26.57	10.07	7110.83
26.58	10.07	7110.89
26.59	10.06	7110.94
26.60	10.05	7110.98
26.61	10.04	7111.01
26.62	10.04	7111.05
26.63	10.03	7111.07
26.64	10.02	7111.09
26.65	10.01	7111.10
26.66	10.01	7111.11
26.67	10.00	7111.11
26.68	9.99	7111.11
26.69	9.98	7111.09
26.70	9.98	7111.08

The maximum volume of 7111.11 cm^3 occurs with a base length of 26.67 cm and a height of 10 cm.

Alternatively, you can use dynamic geometry software to investigate the inscribed square-based prism. A sample sketch is shown, resulting in a similar answer. Click here to load the sketch.



Chapter 9 Section 6 Question 15 Page 515

Use a spreadsheet to investigate the surface area with a constant volume. Solve the volume formula for a cone for *h*. Calculate the slant height from the Pythagorean Theorem. The minimum surface area of 225.4 cm^2 occurs with a radius of 4.24 cm and a height of 11.95 cm. Click here to load the spreadsheet.

$$V = \frac{1}{3}\pi r^2 h$$

$$3 \times V = 3 \times \frac{1}{3}\pi r^2 h$$

$$3V = \pi r^2 h$$

$$\frac{3V}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$$

$$\frac{3V}{\pi r^2} = h$$

$$s^2 = r^2 + h^2$$

$$s = \sqrt{r^2 + h^2}$$

Radius (cm)	Height (cm)	Volume (cm^3)	Slant Height (cm)	Surface Area (cm^2)
4.20	12.18	225.00	12.88	225.4183
4.21	12.12	225.00	12.83	225.4081
4.22	12.07	225.00	12.78	225.4014
4.23	12.01	225.00	12.73	225.3980
4.24	11.95	225.00	12.68	225.3979
4.25	11.90	225.00	12.63	225.4012
4.26	11.84	225.00	12.58	225.4078
4.27	11.78	225.00	12.53	225.4178
4.28	11.73	225.00	12.49	225.4311

Chapter 9 Section 6 Question 16 Page 515

Use a spreadsheet to investigate the volume with a constant surface area. Solve the formula for the surface area of a cone to determine the formula for the slant height. Use the Pythagorean theorem to calculate the height of the cone. The maximum volume of 977.205 cm^3 occurs with a radius of 6.91 cm and a height of 19.54 cm. Click here to load the spreadsheet.

$$SA = \pi r^2 + \pi rs$$
$$SA - \pi r^2 = \pi rs$$
$$\frac{SA - \pi r^2}{\pi r} = \frac{\pi rs}{\pi r}$$
$$\frac{SA - \pi r^2}{\pi r} = s$$

$$s^2 = r^2 + h^2$$
$$s^2 - r^2 = h^2$$
$$h = \sqrt{s^2 - r^2}$$

Radius (cm)	Slant Height (cm)	Height (cm)	Surface Area (cm^2)	Volume (cm^3)
6.85	21.03	19.88	600.00	977.059
6.86	20.98	19.83	600.00	977.104
6.87	20.93	19.77	600.00	977.140
6.88	20.88	19.71	600.00	977.169
6.89	20.83	19.66	600.00	977.189
6.90	20.78	19.60	600.00	977.201
6.91	20.73	19.54	600.00	977.205
6.92	20.68	19.49	600.00	977.201
6.93	20.63	19.43	600.00	977.188
6.94	20.58	19.37	600.00	977.168

Chapter 9 Review

Chapter 9 Review Question 1 Page 516

a)

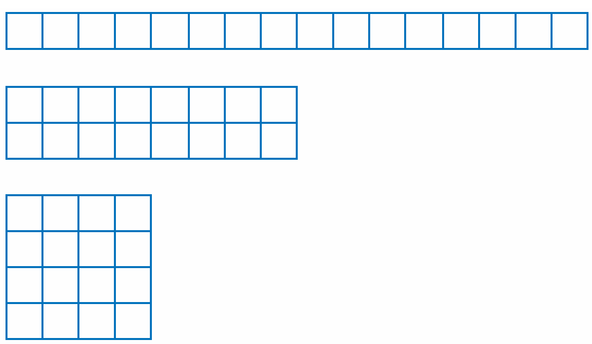
Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
1	1	19	40	19
2	2	18	40	36
3	3	17	40	51
4	4	16	40	64
5	5	15	40	75
6	6	14	40	84
7	7	13	40	91
8	8	12	40	96
9	9	11	40	99
10	10	10	40	100

b) There are 10 possible rectangles, assuming that side lengths are integers.

c) Choose a 10 m by 10 m rectangle in order to maximize the play area of the sandbox.

Chapter 9 Review Question 2 Page 516

a)



b)

Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
1	1	16	34	16
2	2	8	20	16
3	4	4	16	16

c) The 4 m by 4 m garden is the most economical. For the same enclosed area, it has the least perimeter. Fewer edging bricks will be required.

Chapter 9 Review **Question 3** **Page 516**

A square shape has the minimum perimeter. Make the whiteboard 1 m by 1 m.

Chapter 9 Review **Question 4** **Page 516**

a) The maximum area occurs with a square of side length 30 cm, for an area of 30×30, or 900 m^2.

b) The maximum area occurs with one length equal to twice the width. Use two widths of 30 m each, and one length of 60 m, for an area of 30×60, or 1800 m^2.

Chapter 9 Review **Question 5** **Page 516**

a) The most economical rink is a square with a side length of $\sqrt{1800}$, or about 42.4 m.

b) Answers will vary. A sample answer is shown.

A square ice rink may not be best as skaters may want longer straight runs to gain speed.

Chapter 9 Review **Question 6** **Page 516**

Side Length of Square Base (cm)	Area of Square Base (cm^2)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
9.45	89.30	9.80	875	548.9754
9.46	89.49	9.78	875	548.9621
9.47	89.68	9.76	875	548.9500
9.48	89.87	9.74	875	548.9391
9.49	90.06	9.72	875	548.9295
9.50	90.25	9.70	875	548.9211
9.51	90.44	9.67	875	548.9138
9.52	90.63	9.65	875	548.9079
9.53	90.82	9.63	875	548.9031
9.54	91.01	9.61	875	548.8995
9.55	91.20	9.59	875	548.8971
9.56	91.39	9.57	875	548.8960
9.57	91.58	9.55	875	548.8960
9.58	91.78	9.53	875	548.8973
9.59	91.97	9.51	875	548.8997

A cube measuring about 9.6 cm on a side requires the least amount of material. Click here to load the spreadsheet.

Chapter 9 Review **Question 7** **Page 516**

a) 1 L = 1000 cm^3

$$V = s^3$$
$$1000 = s^3$$
$$\sqrt[3]{1000} = s$$
$$10 = s$$

The box that requires the minimum amount of material is a cube with a side length of 10 cm.

b) Answers will vary. A sample answer is shown.

The surface area of a cylinder that contains the same volume will be less than the surface area of the box. The manufacturer could save on packaging costs.

A cube-shaped box is harder to pick up than a more rectangular box.

Chapter 9 Review **Question 8** **Page 517**

3 L = 3000 cm^3

$$V = s^3$$
$$3000 = s^3$$
$$\sqrt[3]{3000} = s$$
$$14.4 \doteq s$$

$$SA = 6s^2$$
$$= 6 \times 14.4^2$$
$$\doteq 1244$$

An area of about 1244 cm^2 of cardboard is required to make the box.

Chapter 9 Review **Question 9** **Page 517**

Side Length of Square Base (m)	Area of Square Base (m^2)	Surface Area (m^2)	Height (m)	Volume (m^3)
0.50	0.25	2	0.75	0.18750
0.51	0.26	2	0.73	0.18867
0.52	0.27	2	0.70	0.18970
0.53	0.28	2	0.68	0.19056
0.54	0.29	2	0.66	0.19127
0.55	0.30	2	0.63	0.19181
0.56	0.31	2	0.61	0.19219
0.57	0.32	2	0.59	0.19240
0.58	0.34	2	0.57	0.19244
0.59	0.35	2	0.55	0.19231
0.60	0.36	2	0.53	0.19200

The maximum volume occurs when a cube of side length approximately 0.58 m is used. Click here to load the spreadsheet.

Chapter 9 Review **Question 10** **Page 517**

$$SA = 6s^2$$
$$1200 = 6s^2$$
$$\frac{1200}{6} = \frac{6s^2}{6}$$
$$200 = s^2$$
$$\sqrt{200} = s$$
$$14.1 \doteq s$$

The maximum volume occurs when using a cube with a side length of approximately 14.1 cm.

Chapter 9 Review **Question 11** **Page 517**

It is not possible to cut six 14.1 cm by 14.1 cm pieces from a 60 cm by 20 cm piece of cardboard. Only four such pieces fit into these dimensions.

Chapter 9 Review Question 12 Page 517

Radius (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
6.100	12.686	1482.918	720.000
6.110	12.645	1483.005	720.000
6.120	12.604	1483.081	720.000
6.130	12.564	1483.145	720.000
6.140	12.523	1483.198	720.000
6.150	12.483	1483.239	720.000
6.160	12.443	1483.269	720.000
6.170	12.402	1483.287	720.000
6.180	12.362	1483.293	720.000
6.190	12.322	1483.288	720.000
6.200	12.283	1483.271	720.000
6.210	12.243	1483.242	720.000

The maximum volume of 1483.29 cm^3 occurs with a radius of 6.18 cm and a height of 12.36 cm.
Click here to load the spreadsheet.

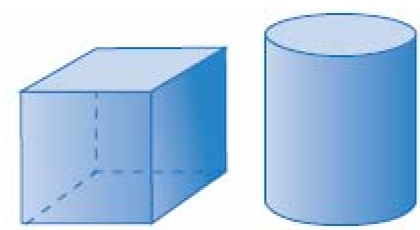
Chapter 9 Review Question 13 Page 517

Since there is no lid, you must change the formula for height from

$$h = \frac{SA - 2\pi r^2}{2\pi r} \text{ to } h = \frac{SA - \pi r^2}{2\pi r}.$$

Chapter 9 Review Question 14 Page 517

Answers will vary. A sample answer is shown.



A cylinder will have a greater volume using the same amount of cardboard, but the square-based prism may be easier for customers to store.

Chapter 9 Review Question 15 Page 517

a)

Radius (cm)	Base Area (cm^2)	Volume (cm^3)	Height (cm)	Surface Area (cm^2)
3.90	47.78	400	8.37	300.6955
3.91	48.03	400	8.33	300.6615
3.92	48.27	400	8.29	300.6316
3.93	48.52	400	8.24	300.6055
3.94	48.77	400	8.20	300.5833
3.95	49.02	400	8.16	300.5650
3.96	49.27	400	8.12	300.5506
3.97	49.51	400	8.08	300.5400
3.98	49.76	400	8.04	300.5332
3.99	50.01	400	8.00	300.5302
4.00	50.27	400	7.96	300.5310
4.01	50.52	400	7.92	300.5355
4.02	50.77	400	7.88	300.5438

The minimum surface area is 300.53 cm^2 when the radius is 3.99 cm, and the height is 8.00 cm. Click here to load the spreadsheet.

b) Answers will vary. A sample answer is shown.

Assume there is no waste material while making the pop can.

Chapter 9 Review Question 16 Page 517

a) The minimum surface area occurs when the height equals the diameter of 12.2 cm. The number of CDs that the container will hold is $\frac{12.2}{0.2}$, or 61.

b) Answers will vary. A sample answer is shown.

Assume that no extra space is allowed inside the container.

c)
$$\begin{aligned} SA &= 2\pi r^2 + 2\pi rh \\ &= 2\pi \times 6.1^2 + 2\pi \times 6.1 \times 12.2 \\ &\doteq 701.4 \end{aligned}$$

The amount of material required is about 701.4 cm^2.

Chapter 9 Chapter Test

Chapter 9 Chapter Test Question 1 Page 518

The field should be a square with a side length of 100 m. Answer B.

Chapter 9 Chapter Test Question 2 Page 518

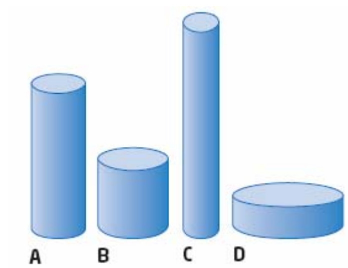
8 L = 8000 cm^3

$$
\begin{aligned}
V &= s^3 \\
8000 &= s^3 \\
\sqrt[3]{8000} &= s \\
20 &= s
\end{aligned}
$$

The box should be a cube with side length 20 cm. Answer D.

Chapter 9 Chapter Test Question 3 Page 518

The surface area is a minimum when the diameter equals the height. Answer B.



Chapter 9 Chapter Test Question 4 Page 518

$$
\begin{aligned}
SA &= 6s^2 \\
600 &= 6s^2 \\
\frac{600}{6} &= \frac{6s^2}{6} \\
100 &= s^2 \\
\sqrt{100} &= s \\
10 &= s
\end{aligned}
$$

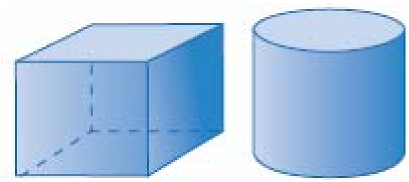
The volume is a maximum when a cube with a side length of 10 cm is used. Answer A.

Chapter 9 Chapter Test Question 5 Page 518

The area is a maximum when a square shape of side length 50 cm is used.

Chapter 9 Chapter Test Question 6 Page 518

Their volumes of the containers are equal, since they have the same base area and the same height. The cylinder requires less material to make.



Chapter 9 Chapter Test Question 7 Page 518

a) 5 L = 5000 cm^3

$$\begin{aligned} V &= s^3 \\ 5000 &= s^3 \\ \sqrt[3]{5000} &= s \\ 17.1 &\doteq s \end{aligned}$$

The minimum surface area occurs when a cube of side length approximately 17.1 cm is used.

b) Answers will vary. A sample answer is shown.

Assume that no material is overlapped, and that no extra material is required for sealing purposes.

Chapter 9 Chapter Test Question 8 Page 518

a)

$$SA = 6s^2$$
$$8.64 = 6s^2$$
$$\frac{8.64}{6} = \frac{6s^2}{6}$$
$$1.44 = s^2$$
$$\sqrt{1.44} = s$$
$$1.2 = s$$

The maximum volume occurs when a cube of side length 1.2 m is used.

b)

$$V = s^3$$
$$= 1.2^3$$
$$= 1.728$$

The volume of the box is 1.728 m^3.

c) The material available for each of the smaller boxes is $\frac{8.64}{3}$, or 2.88 m^2.

$$SA = 6s^2$$
$$2.88 = 6s^2$$
$$\frac{2.88}{6} = \frac{6s^2}{6}$$
$$0.48 = s^2$$
$$\sqrt{0.48} = s$$
$$0.69 \doteq s$$

Each small box is a cube with a side length of approximately 0.69 cm.

d)

$$V_{small} = s^3$$
$$= 0.69^3$$
$$\doteq 0.33$$

The total volume of the three small bins is 3×0.33, or 0.99 m^3. This is less than the volume of the original large bin.

Chapter 9 Chapter Test Question 9 Page 519

Radius (m)	Height (m)	Volume (m^3)	Surface Area (m^2)
18.50	18.60	20000.00	3237.3722
18.51	18.58	20000.00	3237.3668
18.52	18.56	20000.00	3237.3633
18.53	18.54	20000.00	3237.3617
18.54	18.52	20000.00	3237.3620
18.55	18.50	20000.00	3237.3641
18.56	18.48	20000.00	3237.3681
18.57	18.46	20000.00	3237.3741
18.58	18.44	20000.00	3237.3818

The minimum surface area occurs with a radius of 18.53 m and a height of 18.54 m. Click here to load the spreadsheet.

Chapter 9 Chapter Test Question 10 Page 519

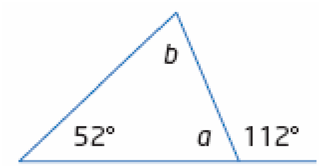
Base (m)	Height (m)	Volume (m^3)	Surface Area (m^2)
0.8	1.1	0.672	4.0
0.9	0.9	0.718	4.0
1.0	0.8	0.750	4.0
1.1	0.6	0.767	4.0
1.2	0.5	0.768	4.0
1.3	0.4	0.751	4.0
1.4	0.4	0.714	4.0
1.5	0.3	0.656	4.0

The maximum volume occurs with a base length of 1.2 m and a height of 0.5 m. Click here to load the spreadsheet.

Chapters 7 to 9 Review **Question 1 Page 520**

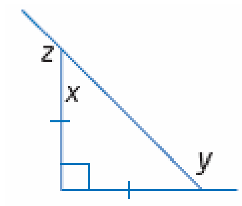
a)

$$\begin{aligned} a &= 180° - 112° \\ &= 68° \end{aligned}$$



$$\begin{aligned} b + 52° &= 112° \\ b &= 112° - 52° \\ b &= 60° \end{aligned}$$

b)



$$\begin{aligned} 2x + 90° &= 180° \\ 2x &= 180° - 90° \\ 2x &= 90° \\ \frac{2x}{2} &= \frac{90°}{2} \\ x &= 45° \end{aligned}$$

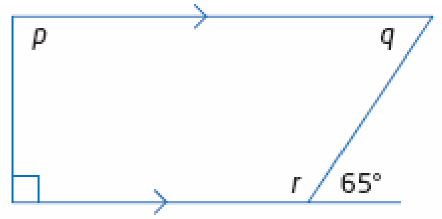
$$\begin{aligned} z &= 90° + 45° \\ &= 135° \end{aligned}$$

$$\begin{aligned} y &= 90° + 45° \\ &= 135° \end{aligned}$$

Chapters 7 to 9 Review **Question 2** **Page 520**

a)

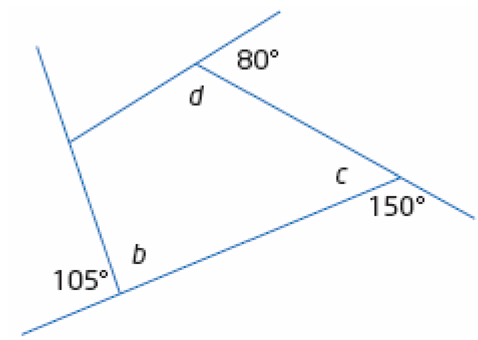
$$\begin{aligned} r &= 180° - 65° \\ &= 115° \end{aligned}$$



$$\begin{aligned} q + 115° &= 180° \\ q &= 180° - 115° \\ q &= 65° \end{aligned}$$

$$\begin{aligned} p + 65° + 115° + 90° &= 360° \\ p + 270° &= 360° \\ p &= 360° - 270° \\ p &= 90° \end{aligned}$$

b) $$\begin{aligned} b &= 180° - 105° \\ &= 75° \end{aligned}$$

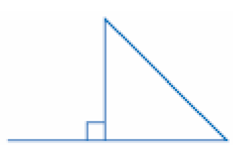


$$\begin{aligned} c &= 180° - 150° \\ &= 30° \end{aligned}$$

$$\begin{aligned} d &= 180° - 80° \\ &= 100° \end{aligned}$$

Chapters 7 to 9 Review **Question 3** **Page 520**

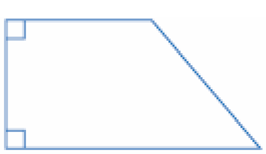
a)



b) Each exterior angle and its adjacent interior angle have a sum of 180°. Thus an exterior right angle has an adjacent interior right angle. This cannot occur in a triangle because two right interior angles have a sum of 180°, leaving no room for the triangle's third angle.

c)

d)



Chapters 7 to 9 Review **Question 4** **Page 520**

a)

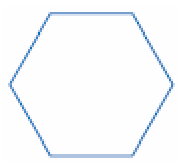
$$\begin{aligned}180(n-2)&=144n\\180n-360&=144n\\180n-360+360-144n&=144n+360-144n\\36n&=360\\\frac{36n}{36}&=\frac{360}{36}\\n&=10\end{aligned}$$

The polygon has 10 sides.

b) The sum of the exterior angles is 360º for all polygons.

Chapters 7 to 9 Review **Question 5** **Page 520**

a)

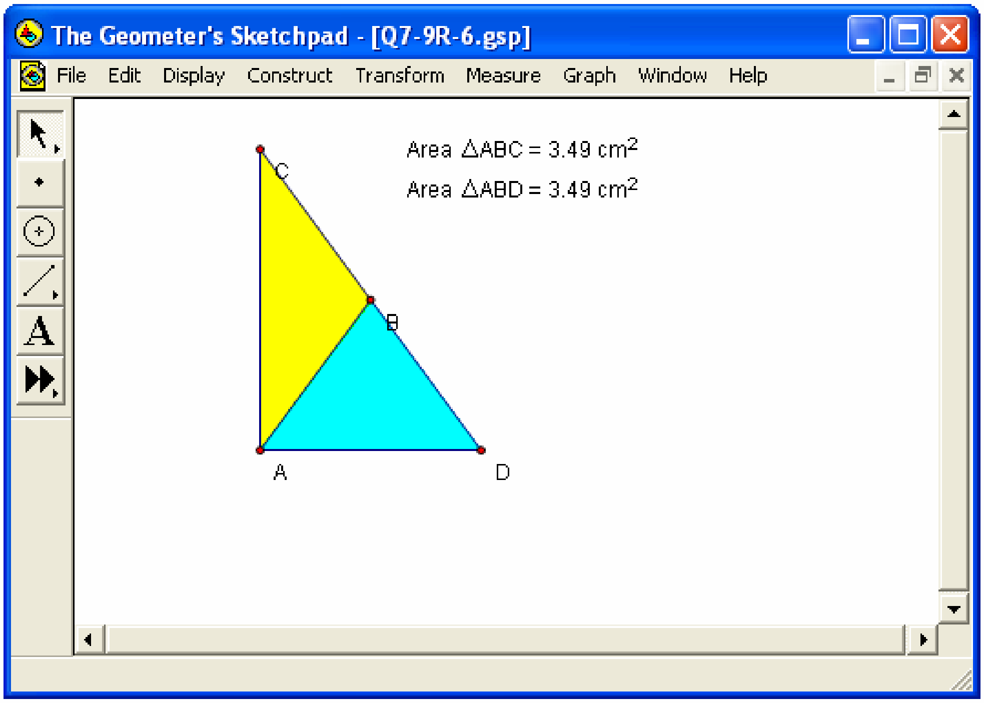


b) Answers will vary. A sample answer is shown.

You can use dynamic geometry software to rotate a line segment about one of its endpoints five times through an angle of 60º. Then, join the endpoints of the line segments formed.

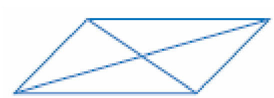
Chapters 7 to 9 Review **Question 6** **Page 520**

Adam is correct. The median from the hypotenuse divides the area of a right triangle into two equal parts. You can verify this conjecture using dynamic geometry software. A sample sketch is shown. Click here to load the sketch.



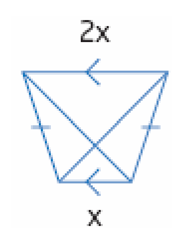
Chapters 7 to 9 Review **Question 7** **Page 520**

a) It is false that the diagonals of a parallelogram are equal in length. A counter-example is shown.



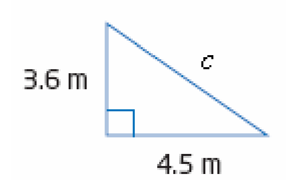
b) It is true that the line segment joining the midpoints of two sides of a triangle is always parallel to the third side. You can use dynamic geometry software to show that interior angles add to 180º, making the line segments parallel.

c) It is false that the diagonals of a trapezoid are never equal in length. A counter-example is shown.



Chapters 7 to 9 Review **Question 8** **Page 520**

a)



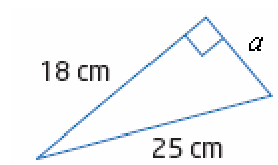
$$c^2 = 3.6^2 + 4.5^2$$
$$c^2 = 12.96 + 20.25$$
$$c^2 = 33.21$$
$$c = \sqrt{33.21}$$
$$c \doteq 5.8$$

$$P = 5.8 + 3.6 + 4.5$$
$$= 13.9$$

$$A = \frac{1}{2} \times 4.5 \times 3.6$$
$$= 8.1$$

The perimeter is 13.9 m, and the area is 8.1 m^2.

b)



$$25^2 = a^2 + 18^2$$
$$625 = a^2 + 324$$
$$625 - 324 = a^2$$
$$301 = a^2$$
$$\sqrt{301} = a$$
$$a \doteq 17.3$$

$$P = 17.3 + 18 + 25$$
$$= 60.3$$

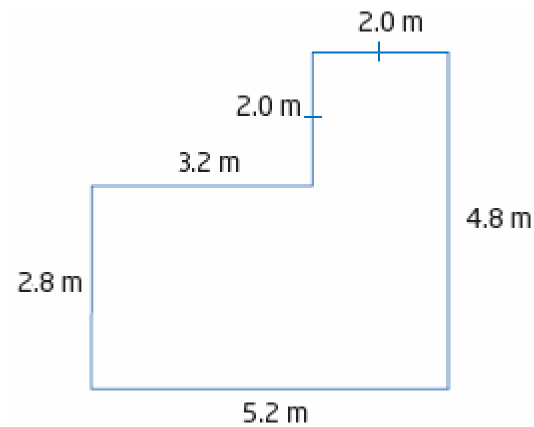
$$A = \frac{1}{2} \times 18 \times 17.3$$
$$= 155.7$$

The perimeter is 60.3 cm, and the area is 155.7 cm^2.

Chapters 7 to 9 Review **Question 9** **Page 520**

$$
\begin{aligned}
P &= 5.2 + 4.8 + 2.0 + 2.0 + 3.2 + 2.8 \\
&= 20.0
\end{aligned}
$$

$$
\begin{aligned}
A &= A_{\text{rectangle}} - A_{\text{cutout}} \\
&= 5.2 \times 4.8 - 3.2 \times 2.0 \\
&= 24.96 - 6.4 \\
&= 18.56
\end{aligned}
$$

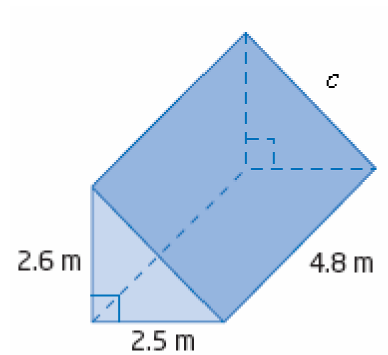


The perimeter is 20.0 m, and the area is 18.56 m^2.

Chapters 7 to 9 Review **Question 10** **Page 521**

a)

$$
\begin{aligned}
c^2 &= 2.6^2 + 2.5^2 \\
c^2 &= 6.76 + 6.25 \\
c^2 &= 13.01 \\
c &= \sqrt{13.01} \\
c &\doteq 3.6
\end{aligned}
$$



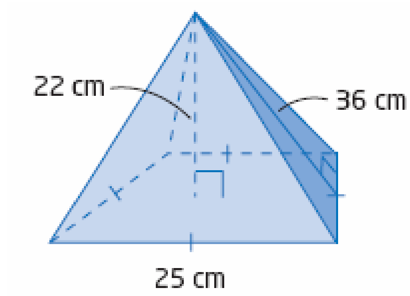
$$
\begin{aligned}
SA &= 2A_{\text{base}} + A_{\text{left side}} + A_{\text{bottom}} + A_{\text{right side}} \\
&= 2 \times \left(\frac{1}{2} \times 2.5 \times 2.6 \right) + 2.6 \times 4.8 + 2.5 \times 4.8 + 3.6 \times 4.8 \\
&= 6.5 + 12.48 + 12 + 17.28 \\
&\doteq 48.3
\end{aligned}
$$

$$
\begin{aligned}
V &= A_{\text{base}} \times h \\
&= \left(\frac{1}{2} \times 2.5 \times 2.6 \right) \times 4.8 \\
&= 15.6
\end{aligned}
$$

The surface area is approximately 48.3 m^2, and the volume is 15.6 m^3.

b)

$$SA = A_{\text{base}} + 4A_{\text{triangle}}$$
$$= 25 \times 25 + 4\left(\frac{1}{2} \times 25 \times 36\right)$$
$$= 625 + 1800$$
$$= 2425$$



$$V = \frac{1}{3} A_{\text{base}} \times h$$
$$= \frac{1}{3} \times 25^2 \times 22$$
$$\doteq 4583.3$$

The surface area is 2425 cm^2, and the volume is approximately 4583.3 cm^3.

Chapters 7 to 9 Review **Question 11** **Page 521**

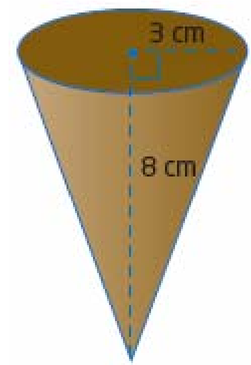
325 mL = 325 cm^3

$$V = \pi r^2 h$$
$$325 = \pi \times 3.6^2 \times h$$
$$325 = 12.96\pi h$$
$$\frac{325}{12.96\pi} = \frac{12.96\pi h}{12.96\pi}$$
$$\frac{325}{12.96\pi} = h$$
$$8.0 \doteq h$$

The height of the can is 8.0 cm.

Chapters 7 to 9 Review   Question 12   Page 521

a)



$$s^2 = 8^2 + 3^2$$
$$s^2 = 64 + 9$$
$$s^2 = 73$$
$$s = \sqrt{73}$$
$$s \doteq 8.5$$

$$\begin{aligned} SA &= \pi rs + \pi r^2 \\ &= \pi \times 3 \times 8.5 + \pi \times 3^2 \\ &\doteq 108 \end{aligned}$$

The area of paper required is about 108 cm^2.

b) $$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3} \times \pi \times 3^2 \times 8 \\ &\doteq 75 \end{aligned}$$

The volume of the cone is approximately 75 cm^3.

Chapters 7 to 9 Review   Question 13   Page 521

a) $$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \times 20^3 \\ &\doteq 33\,510 \end{aligned}$$

The volume of the golf ball is approximately 33 510 mm^3.

b) $$\begin{aligned} SA &= 4\pi r^2 \\ &= 4\pi \times 20^2 \\ &\doteq 5027 \end{aligned}$$

The surface area of the golf ball is approximately 5027 mm^2.

c) The entire surface of a golf ball is covered with small indentations (commonly known as dimples). Due to the presence of dimples, the actual surface area of the golf ball is greater and the volume of the golf ball is less than that calculated in parts a) and b).

Chapters 7 to 9 Review **Question 14** **Page 521**

a) Allie should make a square garden, using 13 pieces, or 6.5 m, on a side.

b) The area of the garden is 6.5^2, or 42.25 m^2.

c) The perimeter of the garden is 4×6.5, or 26 m.

Chapters 7 to 9 Review **Question 15** **Page 521**

$$\begin{aligned} V &= s^3 \\ 10\,000 &= s^3 \\ \sqrt[3]{10\,000} &= s \\ 21.5 &\doteq s \end{aligned}$$

$$\begin{aligned} SA &= 6s^2 \\ &= 6 \times 21.5^2 \\ &\doteq 2774 \end{aligned}$$

The area of cardboard required is about 2774 cm^2.

Chapters 7 to 9 Review **Question 16** **Page 521**

a)

$$\begin{aligned} SA &= 6s^2 \\ 150 &= 6s^2 \\ \frac{150}{6} &= \frac{6s^2}{6} \\ 25 &= s^2 \\ \sqrt{25} &= s \\ 5 &= s \end{aligned}$$

The maximum volume occurs with a cube of side length 5 cm.

b)

Radius (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
2.5	7.0	138.4	150.0
2.6	6.6	139.8	150.0
2.7	6.1	140.7	150.0
2.8	5.6	141.0	150.0
2.9	5.3	140.9	150.0
3.0	5.0	140.2	150.0

The maximum volume of 141 cm^2 occurs with a radius of 2.8 cm and a height of 5.6 cm.
Click here to load the spreadsheet.

Radius (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
3.880	8.140	385.000	293.043
3.890	8.099	385.000	293.021
3.900	8.057	385.000	293.003
3.910	8.016	385.000	292.989
3.920	7.975	385.000	292.979
3.930	7.935	385.000	292.972
3.940	7.894	385.000	292.969
3.950	7.854	385.000	292.970
3.960	7.815	385.000	292.975
3.970	7.776	385.000	292.983

The minimum surface area of about 293 cm^2 occurs with a radius of 3.9 cm and a height of 7.9 cm. Click here to load the spreadsheet.