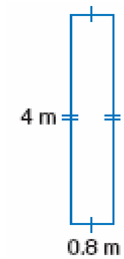
Chapter 8 Measurement Relationships

Chapter 8 Get Ready

Chapter 8 Get Ready Question 1 Page 414

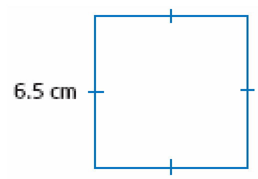
a) $P = 2(4 + 0.8)$
$= 2(4.8)$
$= 9.6$

The perimeter is 9.6 m.



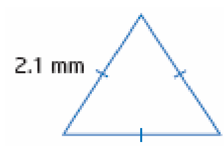
b) $P = 4(6.5)$
$= 26$

The perimeter is 26 cm.



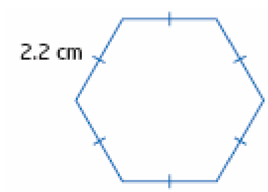
c) $P = 2.1 + 2.1 + 2.1$
$= 6.3$

The perimeter is 6.3 mm.



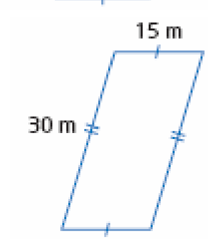
d) $P = 6(2.2)$
$= 13.2$

The perimeter is 13.2 cm.



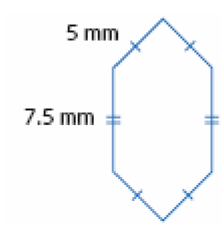
e) $P = 2(15 + 30)$
$= 2(45)$
$= 90$

The perimeter is 90 m.



f) $P = 2(7.5) + 4(5)$
$= 15 + 20$
$= 35$

The perimeter is 35 mm.

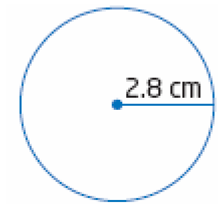


Chapter 8 Get Ready Question 2 Page 414

a) $C = 2\pi(2.8)$

$\doteq 17.6$

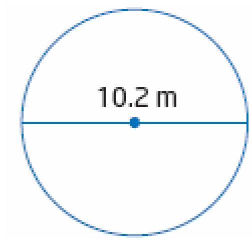
The circumference is approximately 17.6 cm.



b) $C = \pi(10.2)$

$\doteq 32.0$

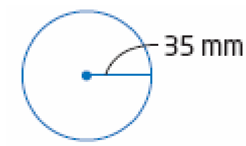
The circumference is approximately 32.0 m.



c) $C = 2\pi(35)$

$\doteq 219.9$

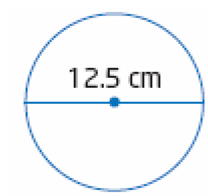
The circumference is approximately 219.9 mm.



d) a) $C = \pi(12.5)$

$\doteq 39.3$

The circumference is approximately 39.3 cm.

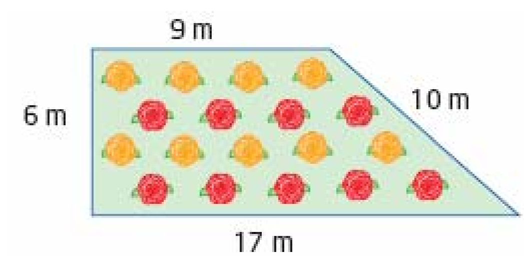


Chapter 8 Get Ready Question 3 Page 414

$P = 9 + 10 + 17 + 6$

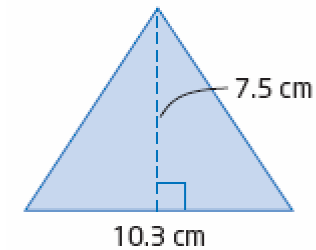
$= 42$

The perimeter is 42 m.



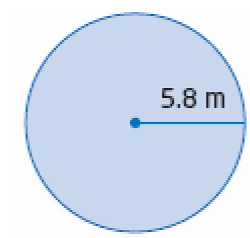
Chapter 8 Get Ready **Question 4** **Page 415**

a)
$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(10.3)(7.5) \\ &\doteq 38.6 \end{aligned}$$



The area is approximately 38.6 cm^2.

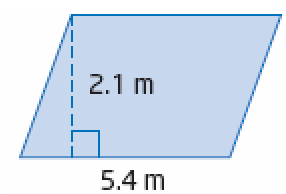
b)
$$\begin{aligned} A &= \pi r^2 \\ &= \pi(5.8)^2 \\ &\doteq 105.7 \end{aligned}$$



The area is approximately 105.7 m^2.

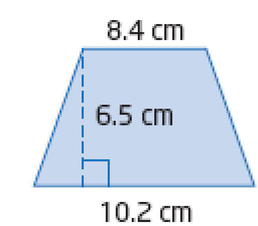
Chapter 8 Get Ready **Question 5** **Page 415**

a)
$$\begin{aligned} A &= bh \\ &= 5.4 \times 2.1 \\ &= 11.34 \end{aligned}$$



The area is 11.34 m^2.

b)
$$\begin{aligned} A &= \frac{1}{2}h(a+b) \\ &= \frac{1}{2}(6.5)(10.2+8.4) \\ &= 60.45 \end{aligned}$$

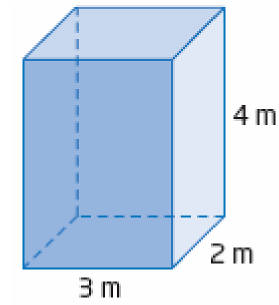


The area is 60.45 cm^2.

Chapter 8 Get Ready Question 6 Page 416

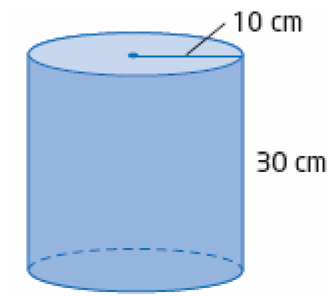
a) $SA = 2lw + 2wh + 2lh$
$= 2(3 \times 2) + 2(2 \times 4) + 2(3 \times 4)$
$= 12 + 16 + 24$
$= 52$

The surface area is 52 m^2.



b) $SA = 2\pi r^2 2\pi rh$
$= 2\pi(10)^2 + 2\pi(10)(30)$
$\doteq 2513$

The surface area is approximately 2513 cm^2.



Chapter 8 Get Ready Question 7 Page 416

a) $V = lwh$
$= 3 \times 2 \times 4$
$= 24$

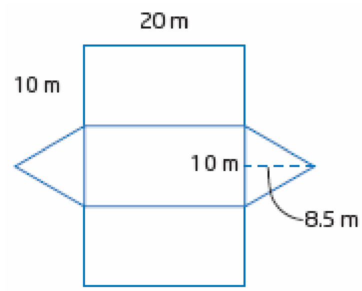
The volume is 24 m^3.

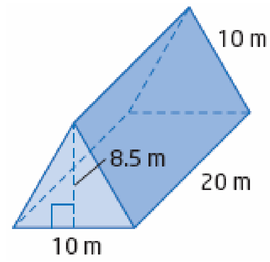
b) $V = \pi r^2 h$
$= \pi(10)^2(30)$
$\doteq 9425$

The volume is approximately 9425 cm^3.

Chapter 8 Get Ready **Question 8** **Page 416**

a)





b) $SA = 3A_{\text{face}} + 2A_{\text{base}}$

$$= 3(20 \times 10) + 2\left(\frac{1}{2} \times 10 \times 8.5\right)$$
$$= 600 + 85$$
$$= 685$$

The surface area is 685 m^2.

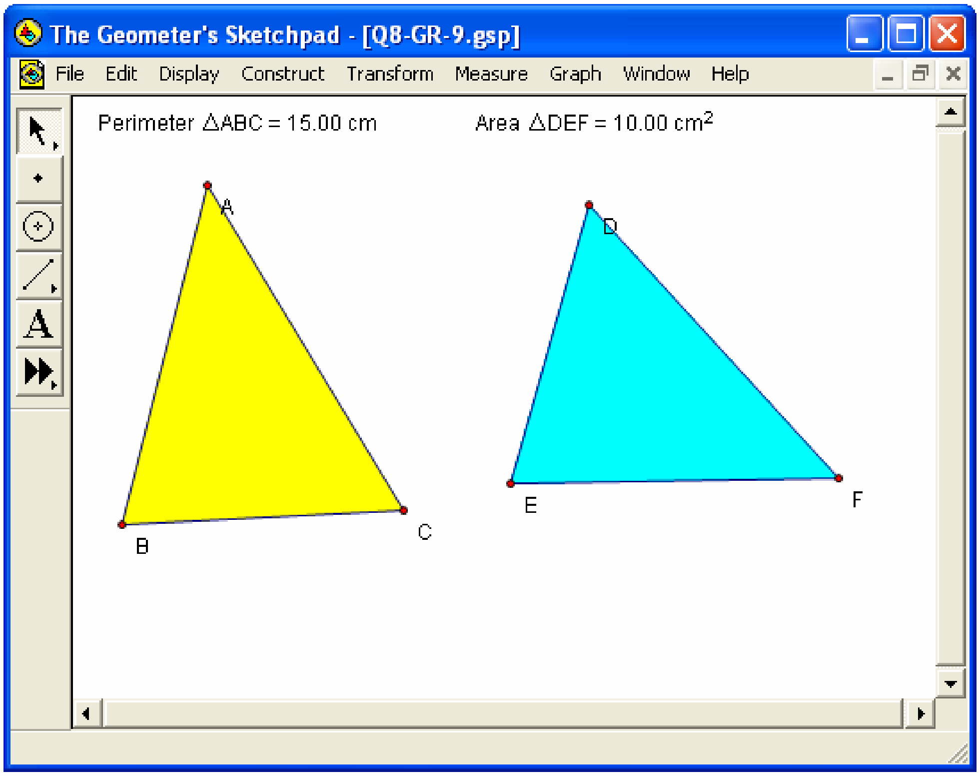
c) $V = A_{\text{base}} \times h$

$$= \frac{1}{2} \times 10 \times 8.5 \times 20$$
$$= 850$$

The volume is 850 m^3.

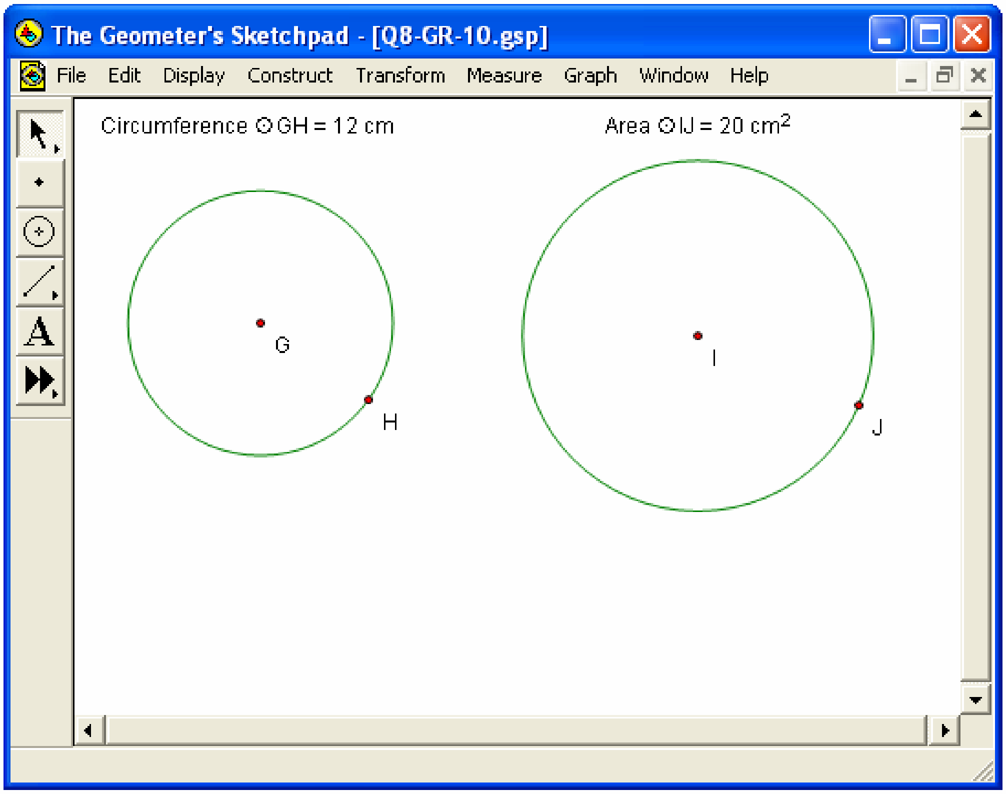
Chapter 8 Get Ready Question 9 Page 417

Answers will vary. A sample sketch is shown. Click here to load the sketch.



Chapter 8 Get Ready Question 10 Page 417

Answers will vary. A sample sketch is shown. Click here to load the sketch.

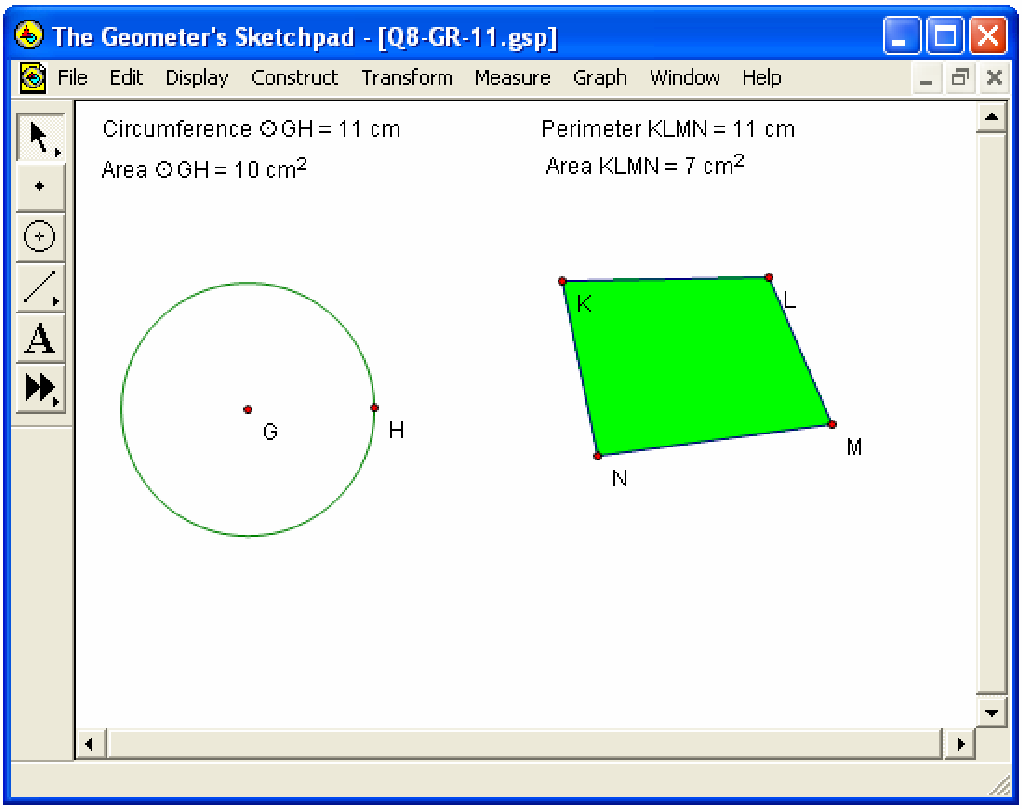


Chapter 8 Get Ready Question 11 Page 417

a) Answers will vary. A sample sketch is shown. Click here to load the sketch.

b) Answers will vary.

c) The quadrilateral does not have the same area as a circle with the same perimeter.

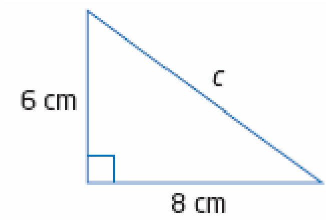


Chapter 8 Section 1: Apply the Pythagorean Theorem

Chapter 8 Section 1 Question 1 Page 423

a)

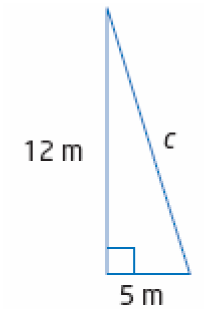
$$c^2 = 6^2 + 8^2$$
$$c^2 = 36 + 64$$
$$c^2 = 100$$
$$\sqrt{c^2} = \sqrt{100}$$
$$c = 10$$



The length of the hypotenuse is 10 cm.

b)

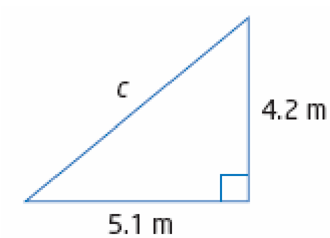
$$c^2 = 12^2 + 5^2$$
$$c^2 = 144 + 25$$
$$c^2 = 169$$
$$\sqrt{c^2} = \sqrt{169}$$
$$c = 13$$



The length of the hypotenuse is 13 m.

c)

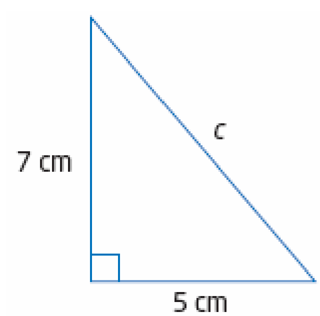
$$c^2 = 4.2^2 + 5.1^2$$
$$c^2 = 17.64 + 26.01$$
$$c^2 = 43.65$$
$$\sqrt{c^2} = \sqrt{43.65}$$
$$c \doteq 6.6$$



The length of the hypotenuse is approximately 6.6 m.

d)

$$c^2 = 7^2 + 5^2$$
$$c^2 = 49 + 25$$
$$c^2 = 74$$
$$\sqrt{c^2} = \sqrt{74}$$
$$c \doteq 8.6$$

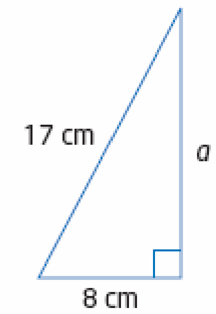


The length of the hypotenuse is approximately 8.6 cm.

Chapter 8 Section 1 Question 2 Page 423

a)

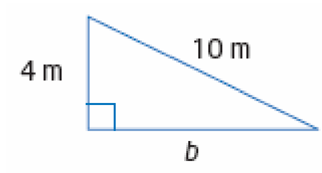
$$17^2 = a^2 + 8^2$$
$$289 = a^2 + 64$$
$$289 - 64 = a^2 + 64 - 64$$
$$225 = a^2$$
$$\sqrt{225} = \sqrt{a^2}$$
$$15 = a$$



The length of side *a* is 15 cm.

b)

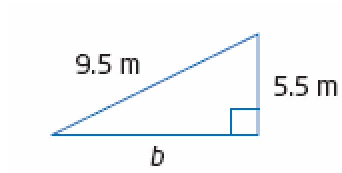
$$10^2 = b^2 + 4^2$$
$$100 = b^2 + 16$$
$$100 - 16 = b^2 + 16 - 16$$
$$84 = b^2$$
$$\sqrt{84} = \sqrt{b^2}$$
$$9.2 \doteq b$$



The length of side *b* is approximately 9.2 m.

c)

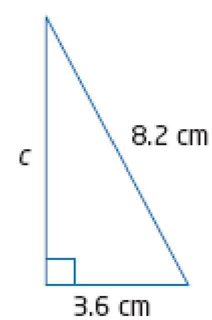
$$9.5^2 = b^2 + 5.5^2$$
$$90.25 = b^2 + 30.25$$
$$90.25 - 30.25 = b^2 + 30.25 - 30.25$$
$$60 = b^2$$
$$\sqrt{60} = \sqrt{b^2}$$
$$7.7 \doteq b$$



The length of side *b* is approximately 7.7 m.

d)

$$8.2^2 = c^2 + 3.6^2$$
$$67.24 = c^2 + 12.96$$
$$67.24 - 12.96 = c^2 + 12.96 - 12.96$$
$$54.28 = c^2$$
$$\sqrt{54.28} = \sqrt{c^2}$$
$$7.4 \doteq c$$

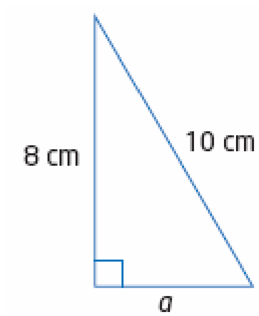


The length of side *c* is approximately 7.4 cm.

Chapter 8 Section 1 **Question 3** **Page 423**

a)

$$10^2 = a^2 + 8^2$$
$$100 = a^2 + 64$$
$$100 - 64 = a^2 + 64 - 64$$
$$36 = a^2$$
$$\sqrt{36} = \sqrt{a^2}$$
$$6 = a$$

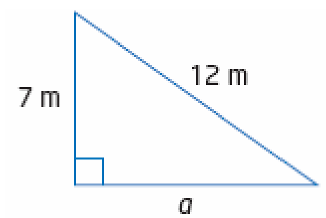


$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(6)(8)$$
$$= 24$$

The area of the right triangle is 24 cm^2.

b)

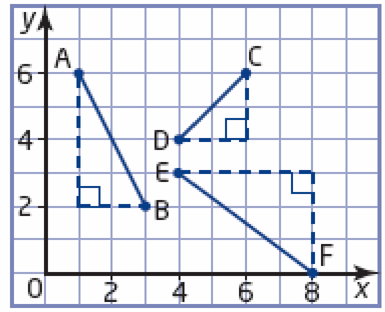
$$12^2 = a^2 + 7^2$$
$$144 = a^2 + 49$$
$$144 - 49 = a^2 + 49 - 49$$
$$95 = a^2$$
$$\sqrt{95} = \sqrt{a^2}$$
$$9.75 \doteq a$$



$$A = \frac{1}{2}(9.75)(7)$$
$$\doteq 34.1$$

The area of the right triangle is approximately 34.1 m^2.

Chapter 8 Section 1 Question 4 Page 424



a)

$$\text{AB}^2 = 4^2 + 2^2$$
$$\text{AB}^2 = 16 + 4$$
$$\text{AB}^2 = 20$$
$$\sqrt{\text{AB}^2} = \sqrt{20}$$
$$\text{AB} \doteq 4.5$$

The length of line segment AB is approximately 4.5 units.

b)

$$\text{CD}^2 = 2^2 + 2^2$$
$$\text{CD}^2 = 4 + 4$$
$$\text{CD}^2 = 8$$
$$\sqrt{\text{CD}^2} = \sqrt{8}$$
$$\text{CD} \doteq 2.8$$

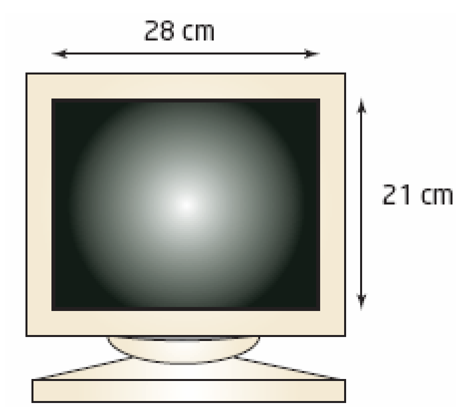
The length of line segment CD is approximately 2.8 units.

c)

$$\text{EF}^2 = 4^2 + 3^2$$
$$\text{EF}^2 = 16 + 9$$
$$\text{EF}^2 = 25$$
$$\sqrt{\text{EF}^2} = \sqrt{25}$$
$$\text{EF} = 5$$

The length of line segment EF is 5 units.

Chapter 8 Section 1 Question 5 Page 424

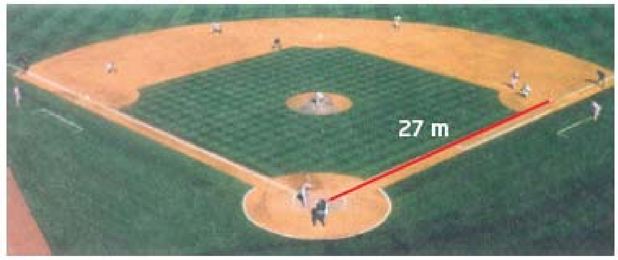


$$d^2 = 28^2 + 21^2$$
$$d^2 = 784 + 441$$
$$d^2 = 1225$$
$$\sqrt{d^2} = \sqrt{1225}$$
$$d = 35$$

The length of the diagonal is 35 cm.

Chapter 8 Section 1 Question 6 Page 424

$$d^2 = 27^2 + 27^2$$
$$d^2 = 729 + 729$$
$$d^2 = 1458$$
$$\sqrt{d^2} = \sqrt{1225}$$
$$d \doteq 38$$



The second-base player must throw the ball approximately 38 m to reach home plate.

Chapter 8 Section 1 Question 7 Page 424

$$42^2 = s^2 + s^2$$
$$1764 = 2s^2$$
$$\frac{1764}{2} = \frac{2s^2}{2}$$
$$882 = s^2$$
$$\sqrt{882} = \sqrt{s^2}$$
$$29.7 \doteq s$$

$$P = 4s$$
$$= 4(29.7)$$
$$\doteq 119$$

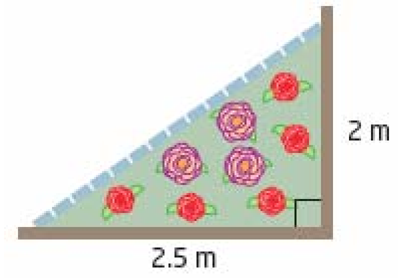
The perimeter of the courtyard is approximately 119 m.

Chapter 8 Section 1 Question 8 Page 424

$$125^2 = h^2 + 50^2$$
$$15\,625 = h^2 + 2500$$
$$15\,625 - 2500 = h^2 + 2500 - 2500$$
$$13\,125 = h^2$$
$$\sqrt{13\,125} = \sqrt{h^2}$$
$$114.56 \doteq h$$

The height of the kite above the tree is 114.56 – 10, or 104.56 m.

Chapter 8 Section 1 Question 9 Page 424

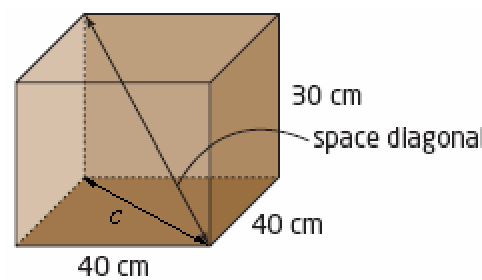


$$c^2 = 2^2 + 2.5^2$$
$$c^2 = 4 + 6.25$$
$$c^2 = 10.25$$
$$\sqrt{c^2} = \sqrt{10.25}$$
$$c \doteq 3.2$$

The third side measures approximately 3.2 m.

Emily will need $\frac{3.2}{0.3}$, or about 11 border stones.

Chapter 8 Section 1 Question 10 Page 425

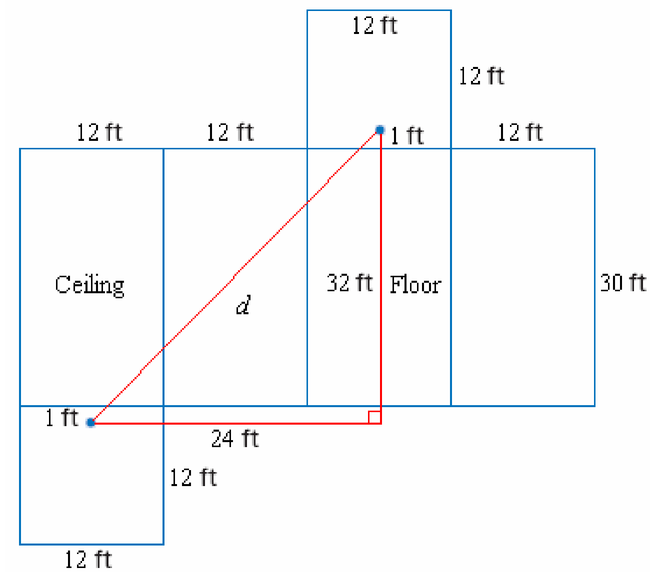


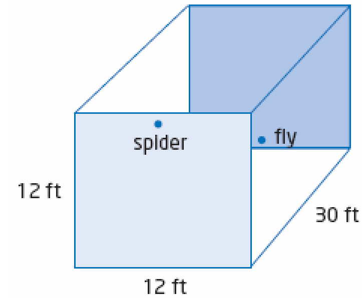
$$c^2 = 40^2 + 40^2$$
$$c^2 = 1600 + 1600$$
$$c^2 = 3200$$
$$\sqrt{c^2} = \sqrt{3200}$$
$$c \doteq 56.6$$

$$d^2 = 56.6^2 + 30^2$$
$$d^2 = 3203.56 + 900$$
$$d^2 = 4103.56$$
$$\sqrt{d^2} = \sqrt{4103.56}$$
$$d \doteq 64$$

The length of the space diagonal is approximately 64 cm.

Chapter 8 Section 1 **Question 11** **Page 425**



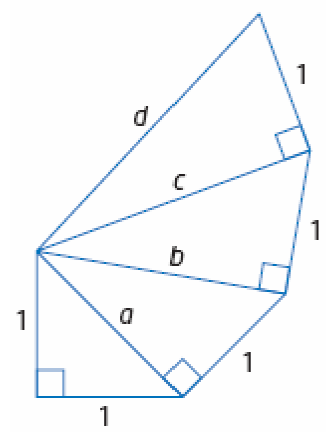


Refer to the net shown.

$$
\begin{aligned}
d^2 &= 24^2 + 32^2 \\
d^2 &= 576 + 1024 \\
d^2 &= 1600 \\
\sqrt{d^2} &= \sqrt{1600} \\
d &= 40
\end{aligned}
$$

The spider must crawl a distance of 40 ft to reach the fly.

Chapter 8 Section 1 **Question 12** **Page 425**



a)

$a^2 = 1^2 + 1^2$
$a^2 = 1 + 1$
$a^2 = 2$
$a = \sqrt{2}$

$b^2 = 1^2 + \left(\sqrt{2}\right)^2$
$b^2 = 1 + 2$
$b^2 = 3$
$b = \sqrt{3}$

$c^2 = 1^2 + \left(\sqrt{3}\right)^2$
$c^2 = 1 + 3$
$c^2 = 4$
$c = \sqrt{4}$

$d^2 = 1^2 + 2^2$
$d^2 = 1 + 5$
$d^2 = 5$
$d = \sqrt{5}$

b) $A = \frac{1}{2} \times 1 \times 1 + \frac{1}{2} \times 1 \times \sqrt{2} + \frac{1}{2} \times 1 \times \sqrt{3} + \frac{1}{2} \times 1 \times \sqrt{4}$

$= \frac{1}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} + \frac{\sqrt{4}}{2}$

c) As you add right triangles to the spiral pattern, the area will increase by $\frac{\sqrt{\text{Number of Triangles}}}{2}$.

Chapter 8 Section 1 **Question 13** **Page 425**

a) This name is appropriate because this set of three whole numbers satisfies the Pythagorean theorem.

b) Multiples of a Pythagorean triple are also Pythagorean triples. One example is shown.

$$2(3,4,5) = (6,8.10)$$
$$\begin{aligned} 6^2 + 8^2 &= 36 + 64 \\ &= 100 \\ &= 10^2 \end{aligned}$$

c) Triples of the form ($m^2 - n^2$, $2mn$, $m^2 + n^2$) are Pythagorean triples, with some restrictions on the values of m and n. Examples are shown. Click here to load the spreadsheet.

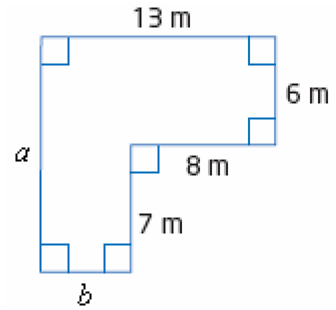
m	n	$m^2 - n^2$	$2mn$	$m^2 + n^2$	$(m^2 - n^2)^2 + (2mn)^2$	$(m^2 + n^2)^2$
2	1	3	4	5	25	25
3	1	8	6	10	100	100
3	2	5	12	13	169	169
4	1	15	8	17	289	289
4	2	12	16	20	400	400
4	3	7	24	25	625	625
5	1	24	10	26	676	676
5	2	21	20	29	841	841
5	3	16	30	34	1156	1156
5	4	9	40	41	1681	1681

d) The restrictions on the values of m and n are $m > n > 0$.

Chapter 8 Section 2 Perimeter and Area of Composite Figures

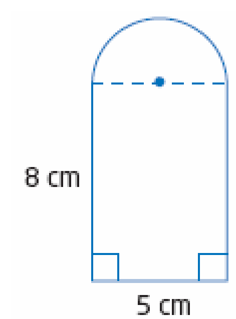
Chapter 8 Section 2 Question 1 Page 432

a)
$$\begin{aligned} a &= 6 + 7 \\ &= 13 \\ b &= 13 - 8 \\ &= 5 \\ P &= 13 + 6 + 8 + 7 + 5 + 13 \\ &= 52 \end{aligned}$$



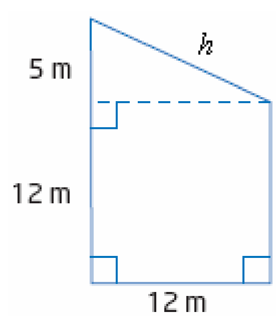
The perimeter of the figure is 52 m.

b)
$$\begin{aligned} P_{\text{semicircle}} &= \frac{1}{2}\pi(5) \\ &\doteq 8 \\ P &= 8 + 8 + 8 + 5 \\ &= 29 \end{aligned}$$



The perimeter of the figure is about 29 cm.

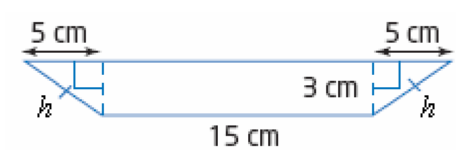
c)
$$\begin{aligned} h^2 &= 5^2 + 12^2 \\ h^2 &= 25 + 144 \\ h^2 &= 169 \\ h &= \sqrt{169} \\ h &= 13 \end{aligned}$$



$$\begin{aligned} P &= 12 + 12 + 12 + 5 + 13 \\ &= 54 \end{aligned}$$

The perimeter of the figure is 54 m.

d)
$$\begin{aligned} h^2 &= 5^2 + 3^2 \\ h^2 &= 25 + 9 \\ h^2 &= 34 \\ h &= \sqrt{34} \\ h &\doteq 6 \end{aligned}$$



$$\begin{aligned} P &= 15 + 6 + 5 + 15 + 5 + 6 \\ &= 52 \end{aligned}$$

The perimeter of the figure is about 52 cm.

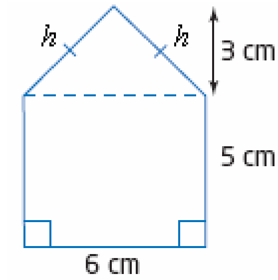
e) $h^2 = 3^2 + 3^2$

$h^2 = 9 + 9$

$h^2 = 18$

$h = \sqrt{18}$

$h \doteq 4$



$P = 6 + 5 + 4 + 4 + 5$

$= 24$

The perimeter of the figure is about 24 cm.

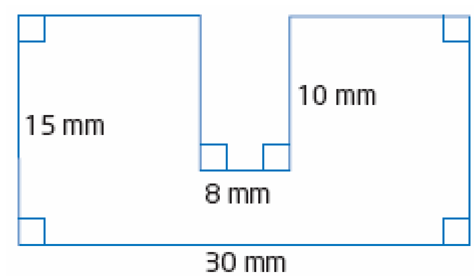
Chapter 8 Section 2 Question 2 Page 432

a) $A = A_{\text{rectangle}} - A_{\text{cutout}}$

$= 30 \times 15 - 8 \times 10$

$= 450 - 80$

$= 370$



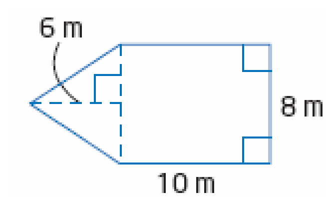
The area of the figure is 370 mm^2.

b) $A = A_{\text{rectangle}} + A_{\text{triangle}}$

$= 10 \times 8 + \frac{1}{2} \times 8 \times 6$

$= 80 + 24$

$= 104$

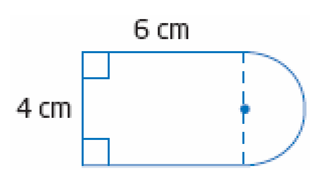


The area of the figure is 104 m^2.

c) $A = A_{\text{rectangle}} + A_{\text{semicircle}}$

$= 4 \times 6 + \frac{1}{2} \times \pi \times 2^2$

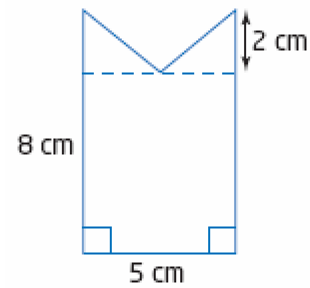
$\doteq 30$



The area of the figure is approximately 30 cm^2.

d) $A = A_{\text{rectangle}} - A_{\text{triangle}}$

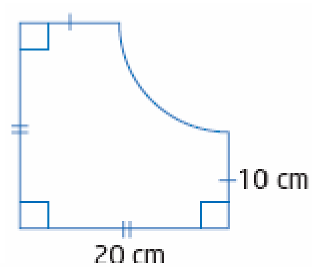
$$= 5 \times 10 - \frac{1}{2} \times 5 \times 2$$
$$= 50 - 5$$
$$= 45$$



The area of the figure is 45 cm^2.

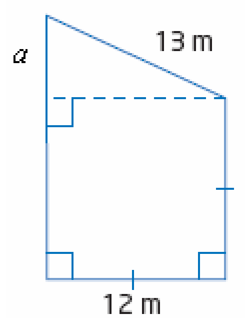
e) $A = A_{\text{square}} - A_{\text{quartercircle}}$

$$= 20 \times 20 - \frac{1}{4} \times \pi \times 10^2$$
$$\doteq 321$$



The area of the figure is approximately 321 cm^2.

f)



$$13^2 = a^2 + 12^2$$
$$169 = a^2 + 144$$
$$169 - 144 = a^2 + 144 - 144$$
$$25 = a^2$$
$$\sqrt{25} = a$$
$$5 = a$$

$A = A_{\text{square}} + A_{\text{triangle}}$

$$= 12 \times 12 + \frac{1}{2} \times 12 \times 5$$
$$= 144 + 30$$
$$= 174$$

The area of the figure is 174 cm^2.

Chapter 8 Section 2 Question 3 Page 433

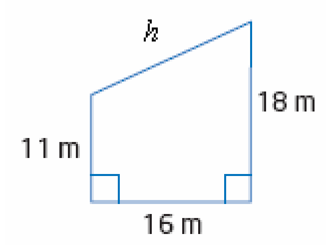
a) $h^2 = 7^2 + 16^2$

$h^2 = 49 + 256$

$h^2 = 305$

$h = \sqrt{305}$

$h \doteq 17$



$P = 17 + 18 + 16 + 11$

$= 62$

The length of fencing needed is about 62 m.

b) $A = \frac{1}{2}(16)(11 + 18)$

$= 232$

The area of the yard is 232 m^2.

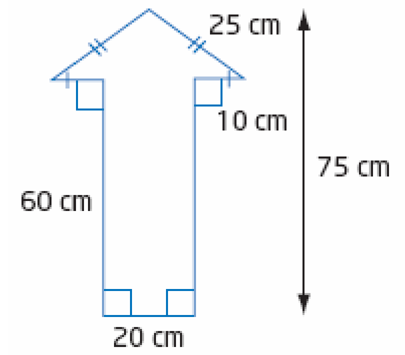
c) To find the perimeter:

Step 1: Use the Pythagorean theorem to determine the length of the unknown side.

Step 2: Add the dimensions of the outer boundary to determine the perimeter.

To find the area: Use the formula for the area of a trapezoid.

Chapter 8 Section 2 Question 4 Page 433



a) $A = A_{\text{rectangle}} + A_{\text{triangle}}$

$$= 20 \times 60 + \frac{1}{2} \times 40 \times 15$$
$$= 1200 + 300$$
$$= 1500$$

The area of one side of one arrow is 1500 cm^2.

b) There are 12 sides to be painted.

$$12 \times 1500 = 18\,000$$

$$18\,000 \text{ cm}^2 = \frac{18\,000}{100 \times 100} \text{ m}^2$$
$$= 1.8 \text{ m}^2$$

Since one can of paint covers 2 m^2, only one can will need to be purchased.

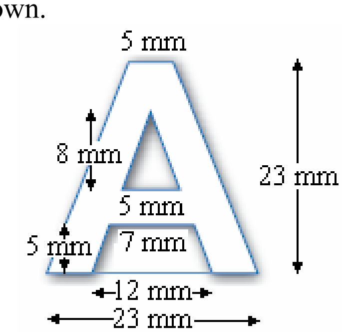
c) $\text{Cost} = \$3.95 + 0.08 \times \$3.95 + 0.07 \times \$3.95$

$$= \$4.54$$

The cost of paint will be \$4.54.

Chapter 8 Section 2 Question 5 Page 433

Measurements may vary. Sample measurements are shown.



$A = A_{\text{big trapezoid}} - A_{\text{small trapezoid}} - A_{\text{triangle}}$

$$= \frac{1}{2} \times 23 \times (23 + 5) - \frac{1}{2} \times 5 \times (12 + 7) - \frac{1}{2} \times 5 \times 8$$
$$= 254.5$$

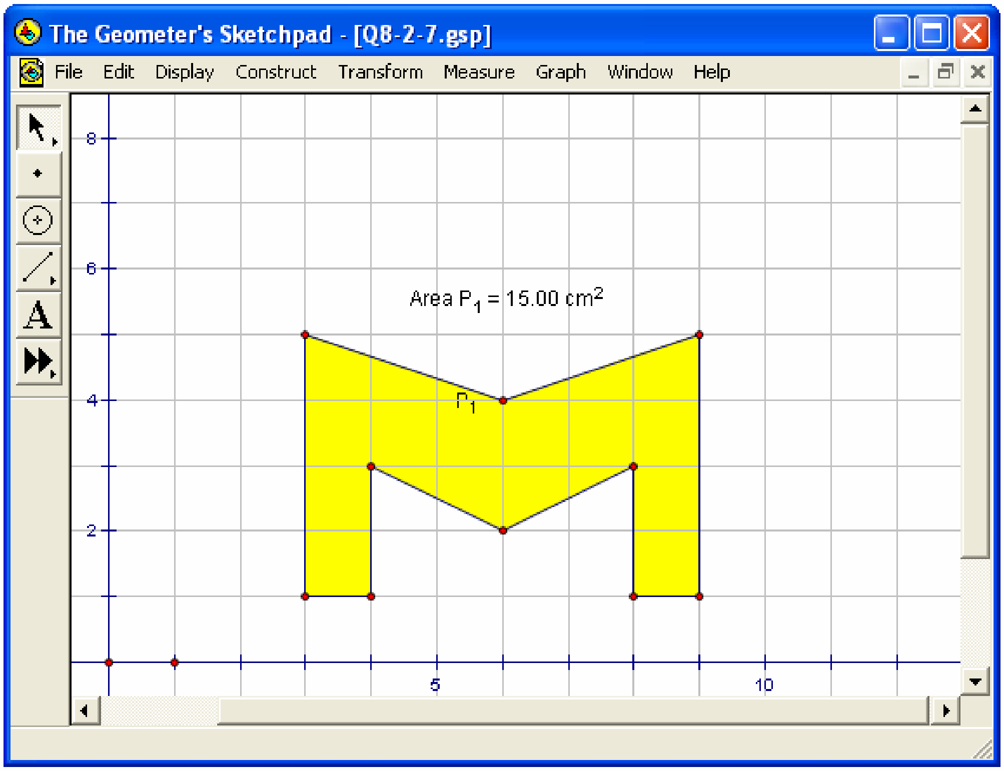
The area is 300 mm^2, to the nearest hundred square millimetres.

Chapter 8 Section 2 Question 6 Page 433

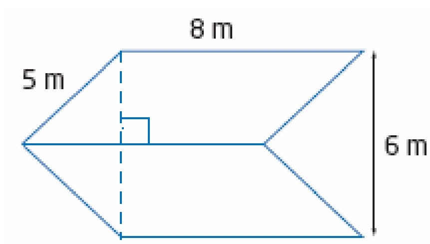
Answers will vary. See the solution for question 7 for a sample logo.

Chapter 8 Section 2 Question 7 Page 433

Answers will vary. A sample sketch is shown. Click here to load the sketch.



Chapter 8 Section 2 Question 8 Page 433



a) $P = 5+8+5+5+8+5$
$= 36$

The perimeter is 36 m. The plants are ro be placed every 20 cm, or 0.2 m.

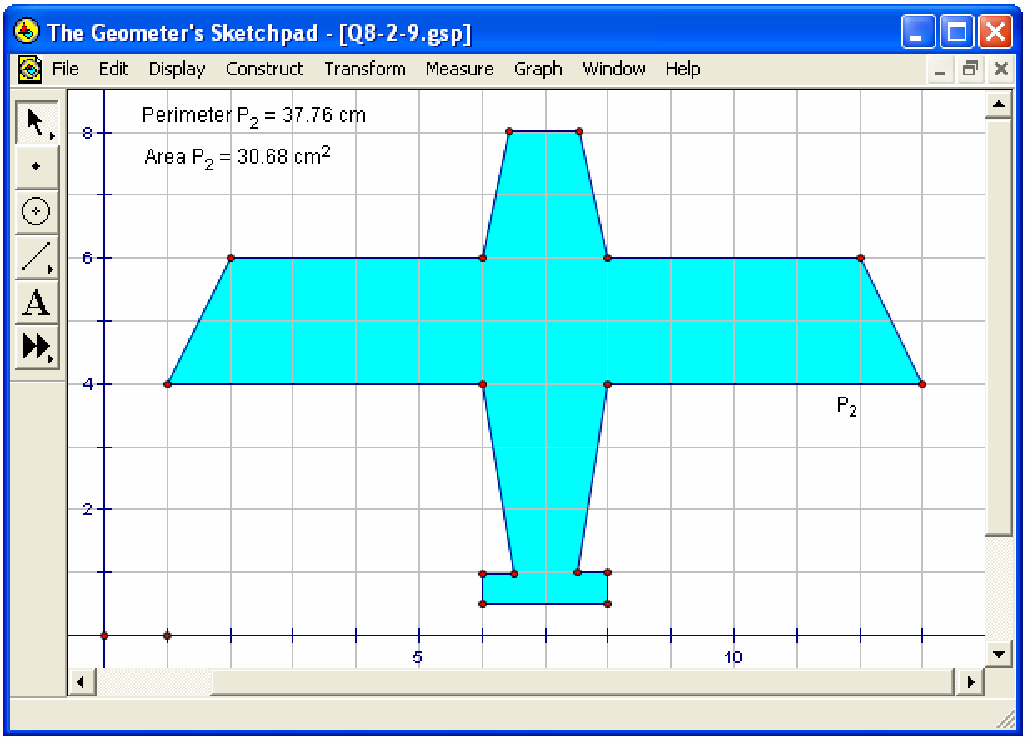
Emily will need $\frac{36}{0.2}$, or 180 plants.

b) $A = 2A_{\text{parallelogram}}$
$= 2(8 \times 3)$
$= 48$

The area of the garden is 48 m^2.

Chapter 8 Section 2 Question 9 Page 433

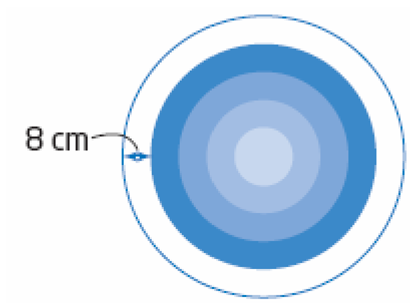
Answers will vary. A sample sketch is shown. Click here to load the sketch.



Chapter 8 Section 2 Question 10 Page 434

a)

$$\begin{aligned} A_{\text{outer ring}} &= A_{\text{target}} - A_{\text{up to first inner ring}} \\ &= \pi \times 40^2 - \pi \times 32^2 \\ &\doteq 1810 \end{aligned}$$



The area of the outer ring is approximately 1810 cm^2.

b)

$$\begin{aligned} \frac{A_{\text{outer ring}}}{A_{\text{target}}} &= \frac{1810}{\pi \times 40^2} \\ &\doteq 0.36 \end{aligned}$$

The area of the outer ring is about 36% of the area of the total area.

Chapter 8 Section 2 Question 11 Page 434

a) $s^2 = 5$

$s = \sqrt{5}$

$s \doteq 2.2$

The length of one side of the patio is approximately 2.2 m.

b) The perimeter of the patio is 4×2.2, or 9 m to the nearest metre.

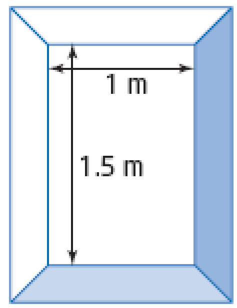
Chapter 8 Section 2 Question 12 Page 434

$A_{frame} = A_{outside} - A_{picture}$

$= 1.7 \times 1.2 - 1.5 \times 1$

$= 0.54$

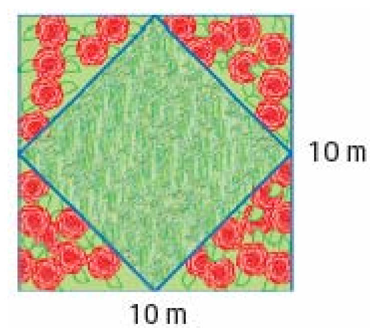
The area of the frame is 0.54 m^2, or 5400 cm^2.



Chapter 8 Section 2 Question 13 Page 434

Solutions for the Achievement Checks are shown in the Teacher's Resource.

Chapter 8 Section 2 Question 14 Page 435



a) $s^2 = 5^2 + 5^2$

$s^2 = 25 + 25$

$s^2 = 50$

The area of the lawn is 50 m^2.

b) The four flower beds make up the same area as the lawn. The area of the lawn is four times the area of one flower bed.

c) When a square is inscribed within a square, four congruent triangles are always formed. However, the answer to part b) is only true when the vertices of the inscribed square are at the midpoints of the outer square.

Chapter 8 Section 2 Question 15 Page 435

Doubling the radius of a circle results in four times the area.

Consider a circle with a radius *r*, and another with radius 2*r*.

$$A_r = \pi r^2$$

$$\begin{aligned} A_{2r} &= \pi(2r)^2 \\ &= \pi(4r^2) \\ &= 4\pi r^2 \\ &= 4A_r \end{aligned}$$

The area of the second circle is four times the area of the first.

Chapter 8 Section 2 Question 16 Page 435

a) You must add the previous two terms to obtain the next term: 34, 55, 89, and 144.

b) The areas are: 1, 2, 6, 15, 40, 104, ...

$$\begin{aligned} &1 \times 1 = 1 \\ &1 \times 2 = 2 \\ &2 \times 3 = 6 \\ &3 \times 5 = 15 \\ &5 \times 8 = 40 \\ &8 \times 13 = 104 \end{aligned}$$

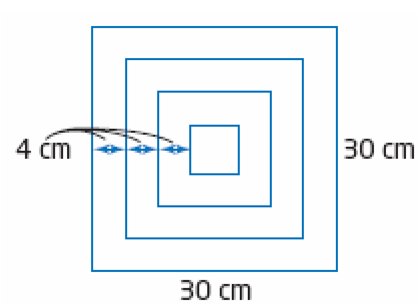
c) Answers will vary.

d) Answers will vary.

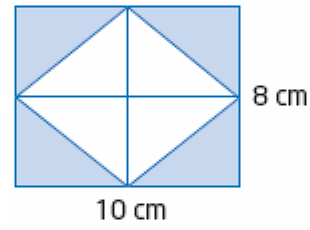
Chapter 8 Section 2 Question 17 Page 435

$$\begin{aligned} \frac{P_{\text{smallest square}}}{P_{\text{largest square}}} &= \frac{4 \times 6}{4 \times 30} \\ &= \frac{1}{5} \end{aligned}$$

The ratio of the perimeter of the smallest square to the perimeter of the largest square is 1:5.



Chapter 8 Section 2 Question 18 Page 435



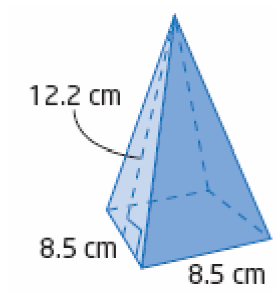
The figure can be divided into 8 congruent triangles. The area of the shaded region is one-half the area of the rectangle.

$$\begin{aligned} A_{\text{shaded region}} &= \frac{1}{2} \times A_{\text{rectangle}} \\ &= \frac{1}{2} \times 10 \times 8 \\ &= 40 \end{aligned}$$

The area of the shaded region is 40 cm^2.

Chapter 8 Section 3 Surface Area and Volume of Prisms and Pyramids

Chapter 8 Section 3 Question 1 Page 441



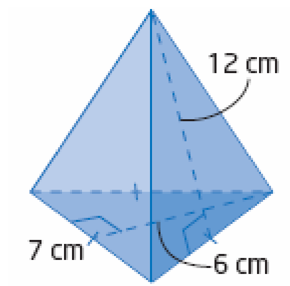
a) $SA = A_{\text{base}} + 4A_{\text{triangle}}$

$= 8.5 \times 8.5 + 4\left(\frac{1}{2} \times 8.5 \times 12.2\right)$

$= 72.25 + 207.4$

$= 279.65$

The surface area is 279.65 cm^2.



b) $SA = A_{\text{base}} + 3A_{\text{triangle}}$

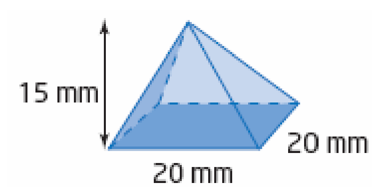
$= \frac{1}{2} \times 7 \times 6 + 3\left(\frac{1}{2} \times 7 \times 12\right)$

$= 21 + 126$

$= 147$

The surface area is 147 cm^2.

Chapter 8 Section 3 Question 2 Page 441

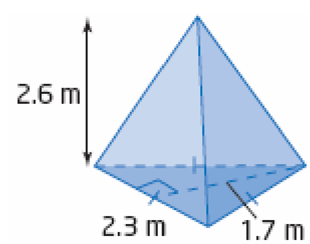


a) $V = \frac{1}{3} A_{\text{base}} \times h$

$= \frac{1}{3} \times 20^2 \times 15$

$= 2000$

The volume is 2000 mm^3.



b) $V = \frac{1}{3} A_{\text{base}} \times h$

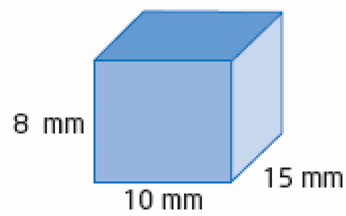
$= \frac{1}{3} \times \left(\frac{1}{2} \times 2.3 \times 1.7\right) \times 2.6$

$\doteq 2$

The volume is approximately 2 m^3.

Chapter 8 Section 3 Question 3 Page 441

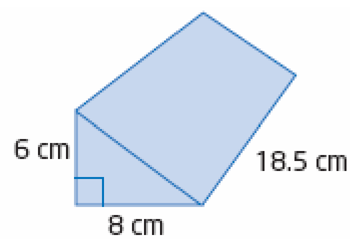
a) $SA = 2A_{bottom} + 2A_{sides} + 2A_{front}$
$= 2(10 \times 15) + 2(8 \times 15) + 2(10 \times 8)$
$= 300 + 240 + 160$
$= 700$



The surface area is 700 mm^2.

b)

$c^2 = 6^2 + 8^2$
$c^2 = 36 + 64$
$c^2 = 100$
$c = \sqrt{100}$
$c = 10$

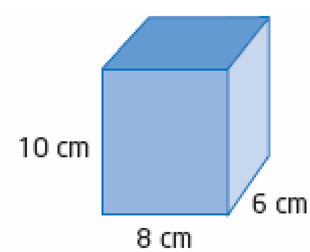


$SA = 2A_{base} + A_{left\ side} + A_{bottom} + A_{right\ side}$
$= 2\left(\frac{1}{2} \times 8 \times 6\right) + 6 \times 18.5 + 8 \times 18.5 + 10 \times 18.5$
$= 48 + 111 + 148 + 185$
$= 492$

The surface area is 492 cm^2.

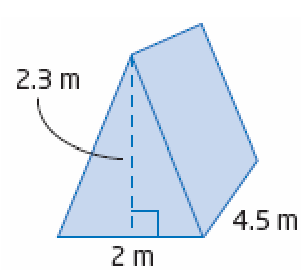
Chapter 8 Section 3 Question 4 Page 441

a) $V = A_{base} \times h$
$= (10 \times 8) \times 6$
$= 480$



The volume is 480 cm^2.

b) $V = A_{base} \times h$
$= \left(\frac{1}{2} \times 2 \times 2.3\right) \times 4.5$
$= 10.35$



The volume is 10.35 m^2.

Chapter 8 Section 3 Question 5 Page 441

a)
$$\begin{aligned} SA &= 2A_{\text{bottom}} + 2A_{\text{sides}} + 2A_{\text{front}} \\ &= 2(3 \times 2) + 2(2 \times 4) + 2(3 \times 4) \\ &= 12 + 16 + 24 \\ &= 52 \end{aligned}$$

The surface area is 52 m^2.

b)
$$\begin{aligned} V &= A_{\text{base}} \times h \\ &= (3 \times 2) \times 4 \\ &= 24 \end{aligned}$$

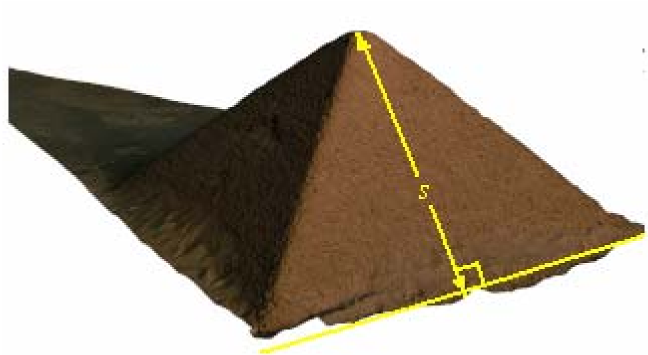
The volume is 24 m^2.

Chapter 8 Section 3 Question 6 Page 441

$$\begin{aligned} SA &= A_{\text{base}} \times h \\ 3000 &= (20 \times 5) \times h \\ 3000 &= 100h \\ \frac{3000}{100} &= \frac{100h}{100} \\ 30 &= h \end{aligned}$$

The height of the cereal box is 30 cm.

Chapter 8 Section 3 **Question 7** **Page 442**



a) $V = \frac{1}{3}A_{\text{base}} \times h$

$= \frac{1}{3} \times 220^2 \times 105$

$= 1\,694\,000$

The volume is 1 694 000 m^3.

b)

$s^2 = 110^2 + 105^2$

$s^2 = 12\,100 = 11\,025$

$s^2 = 23\,125$

$s = \sqrt{23\,125}$

$s \doteq 152.1$

$SA = A_{\text{base}} + 4A_{\text{triangle}}$

$= 220 \times 220 + 4\left(\frac{1}{2} \times 220 \times 152.1\right)$

$= 48\,400 + 66\,924$

$= 115\,324$

The surface area is about 115 324 m^2.

Chapter 8 Section 3 Question 8 Page 442

$$\begin{aligned}
V &= \frac{1}{3}A_{base} \times h \\
2\,211\,096 &= \frac{1}{3} \times 215^2 \times h \\
2\,211\,096 &= \frac{46\,225h}{3} \\
3 \times 2\,211\,096 &= 3 \times \frac{46\,225h}{3} \\
6\,633\,288 &= 46\,225h \\
\frac{6\,633\,288}{46\,225} &= \frac{46\,225h}{46\,225} \\
143.5 &\doteq h
\end{aligned}$$

The height of the pyramid is approximately 143.5 m.

Chapter 8 Section 3 Question 9 Page 442

$$\begin{aligned}
V &= A_{base} \times h \\
&= 40 \times 26 \\
&= 1040
\end{aligned}$$

The volume is 1040 cm^3, or 1.04 L. It will hold 1 L of milk.

Chapter 8 Section 3 Question 10 Page 442

a)

$$\begin{aligned}
V &= A_{base} \times h \\
3000 &= 100h \\
\frac{3000}{100} &= \frac{100h}{100} \\
30 &= h
\end{aligned}$$

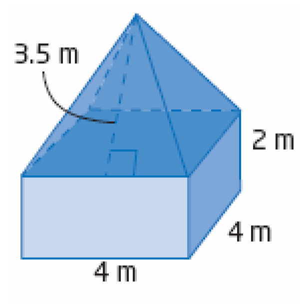
The height of the prism is 30 cm.

b) Assume that there are no irregularities (bumps/dimples) on the surface, the top of the juice container is flat, and the container is completely full.

Chapter 8 Section 3 Question 11 Page 442

a)

$$3.5^2 = 2^2 + h^2$$
$$12.25 = 4 + h^2$$
$$8.25 = h^2$$
$$\sqrt{8.25} = h$$
$$2.9 \doteq h$$



$$\begin{aligned} V &= V_{\text{prism}} + V_{\text{pyramid}} \\ &= 4 \times 4 \times 2 + \frac{1}{3} \times 4^2 \times 2.9 \\ &\doteq 47 \end{aligned}$$

The volume of the shed is about 47 m^3.

b) $$\begin{aligned} SA &= 4A_{\text{rectangle}} + 4A_{\text{triangle}} \\ &= 4(2 \times 4) + 4\left(\frac{1}{2} \times 4 \times 3.5\right) \\ &= 32 + 28 \\ &= 60 \end{aligned}$$

The surface area is 60 m^2. Adam will need $\frac{60}{4}$, or 15 cans of paint.

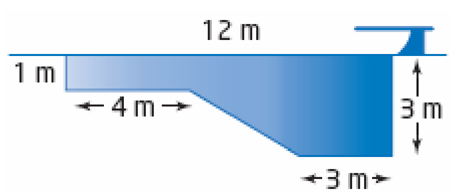
c) $$\begin{aligned} \text{Cost} &= 15 \times \$16.95 \times 1.15 \\ &= \$292.39 \end{aligned}$$

The total cost is $292.39.

Chapter 8 Section 3 Question 12 Page 442

a) Answers will vary. A possible estimate is about 80 m^3, or 80 000 L.

b) $$\begin{aligned} V &= \left(A_{\text{rectangle}} - A_{\text{trapezoid}}\right) \times \text{width} \\ &= \left(12 \times 3 - \frac{1}{2} \times 2 \times (4 + 9)\right) \times 4 \\ &= (36 - 13) \times 4 \\ &= 92 \end{aligned}$$



The volume of the pool is 92 m^3, or 92 000 L.

c) At 100 L/min, it will take $\frac{92\ 000}{100}$, or 920 min (15 h 20 min) to fill the pool.

Chapter 8 Section 3 Question 13 Page 443

a) Predictions may vary. A sample prediction is that doubling the height doubles the volume.

b)

$$\begin{aligned} V &= A_{\text{base}} \times h \\ &= \left(\frac{1}{2} \times 6 \times 8\right) \times 10 \\ &= 240 \end{aligned}$$

$$\begin{aligned} V &= A_{\text{base}} \times h \\ &= \left(\frac{1}{2} \times 6 \times 8\right) \times 20 \\ &= 480 \end{aligned}$$

Doubling the height from 10 cm to 20 cm doubles the volume from 240 cm^3 to 480 cm^3.

c) Answers will vary. The sample prediction was accurate.

d) This is true in general. Doubling the height doubles the volume of the prism.

Chapter 8 Section 3 Question 14 Page 443

Solutions for the Achievement Checks are shown in the Teacher's Resource.

Chapter 8 Section 3 Question 15 Page 443

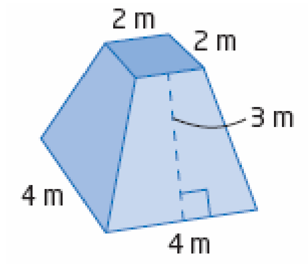
The height of the pyramid is three times the height of the prism.

$$\begin{aligned} V_{\text{pyramid}} &= \frac{1}{3} A_{\text{base}} \times h \\ &= \frac{1}{3} lwh \end{aligned}$$

$$\begin{aligned} V_{\text{prism}} &= A_{\text{base}} \times h \\ &= lwh \end{aligned}$$

If the two volumes are equal, then the height of the pyramid must be three times the height of the prism because *w* and *l* are the same for both.

Chapter 8 Section 3 Question 16 Page 443



a) $SA = A_{\text{bottom}} + A_{\text{top}} + 4 \times A_{\text{trapezoid}}$

$$= 4 \times 4 + 2 \times 2 + 4\left(\frac{1}{2} \times 3(4 + 2)\right)$$
$$= 16 + 4 + 36$$
$$= 56$$

The surface area of the frustum is 56 m^2.

b) The area to be painted it 56 – 16, or 40 m^2. The cost is 40 × \$49.50, or \$1980.00.

Chapter 8 Section 3 Question 17 Page 443

a) $SA = 2(2l \times 2w + 2w \times 2h + 2l \times 2h)$

$$= 2(4lw + 4wh + 4lh)$$
$$= 2(4(lw + wh + lh))$$
$$= 8(lw + wh + lh)$$

The surface area quadruples if each dimension is doubled.

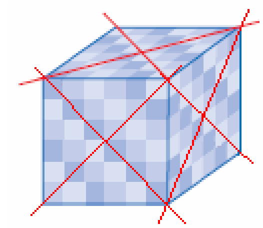
b) $V = 2l \times 2w \times 2h$

$$= 8lwh$$

The volume increases by 8 times if each dimension is doubled.

Chapter 8 Section 3 Question 18 Page 443

All cubes along diagonals will be cut.



Consider the 6 x 6 x 6 cube as made up of a 4 x 4 x 4 cube, and a 6 x 6 x 6 shell around it.

Consider the top face of the 4 x 4 x 4 cube. When this face is cut, all cubes marked x will be cut.

x	o	o	x
o	x	x	o
o	x	x	o
x	o	o	x

When the next cuts are made from the right side, the cubes marked o in the top and bottom rows will be cut. When the final cuts are made from the front side, the cubes marked o on the left and right sides will be cut. Hence, all cubes in the 4 x 4 x 4 cube will be cut.

Now consider the 6 x 6 x 6 shell. When the top face cuts are made, all cubes marked x will be cut.

x	o	o	o	o	x
o	x	o	o	x	o
o	o	x	x	o	o
o	o	x	x	o	o
o	x	o	o	x	o
x	o	o	o	o	x

When the next cuts are made from the right side, the cubes marked o in the top and bottom rows will be cut. When the final cuts are made from the front side, the cubes marked o on the left and right sides will be cut. This leaves 8 cubes uncut, as shown.

x	x	x	x	x	x
x	x	**o**	**o**	x	x
x	**o**	x	x	**o**	x
x	**o**	x	x	**o**	x
x	x	**o**	**o**	x	x
x	x	x	x	x	x

This pattern will occur on all six faces, leaving 6×8, or 48 cubes uncut.

Chapter 8 Section 4 Surface Area of a Cone

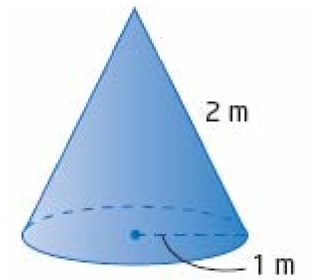
Chapter 8 Section 4 Question 1 Page 447

a) $SA = \pi rs + \pi r^2$

$= \pi \times 1 \times 2 + \pi \times 1^2$

$\doteq 9$

The surface area is approximately 9 m^2.

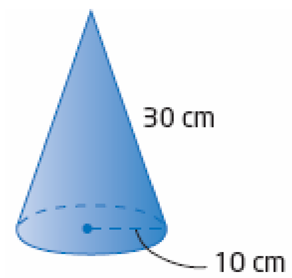


b) $SA = \pi rs + \pi r^2$

$= \pi \times 10 \times 30 + \pi \times 10^2$

$\doteq 1257$

The surface area is approximately 1257 cm^2.

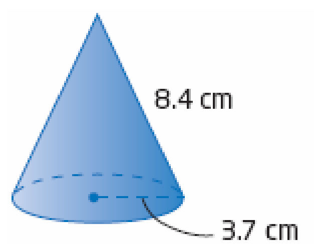


c) $SA = \pi rs + \pi r^2$

$= \pi \times 3.7 \times 8.4 + \pi \times 3.7^2$

$\doteq 141$

The surface area is approximately 141 cm^2.



Chapter 8 Section 4 Question 2 Page 447

a) $s^2 = 12^2 + 5^2$

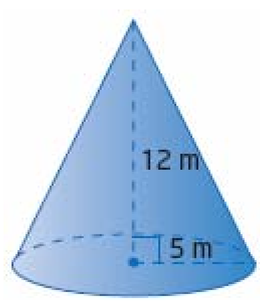
$s^2 = 144 + 25$

$s^2 = 169$

$s = \sqrt{169}$

$s = 13$

The slant height is 13 m.



b) $SA = \pi rs + \pi r^2$

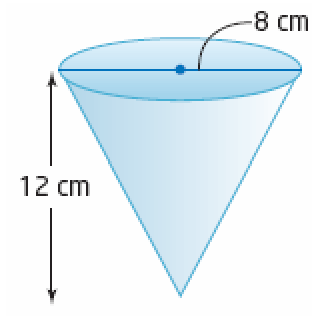
$= \pi \times 5 \times 13 + \pi \times 5^2$

$\doteq 283$

The surface area is approximately 283 m^2.

Chapter 8 Section 4 Question 3 Page 447

a)



$$s^2 = 12^2 + 4^2$$
$$s^2 = 144 + 16$$
$$s^2 = 160$$
$$s = \sqrt{160}$$
$$s \doteq 12.6$$

$$\begin{aligned} SA_{\text{lateral}} &= \pi r s \\ &= \pi \times 4 \times 12.6 \\ &\doteq 158 \end{aligned}$$

The area of paper required is about 158 cm^2.

b) Answers will vary. Assume that there is no paper being overlapped.

Chapter 8 Section 4 Question 4 Page 448

a) The cones have the same slant height. Both form triangles with the same side measurements.

b) The cones do not have the same surface area. The second cone has the greater surface area. The slant height is the same for both, but in the expression $SA = \pi rs + \pi r^2$, the second cone has the greater radius.

c)

$$s^2 = 6^2 + 4^2$$
$$s^2 = 36 + 16$$
$$s^2 = 52$$
$$s = \sqrt{52}$$
$$s \doteq 7.2$$

First cone:

$$\begin{aligned} SA &= \pi r s + \pi r^2 \\ &= \pi \times 4 \times 7.2 + \pi \times 4^2 \\ &\doteq 141 \end{aligned}$$

Second cone:

$$\begin{aligned} SA &= \pi r s + \pi r^2 \\ &= \pi \times 6 \times 7.2 + \pi \times 6^2 \\ &\doteq 249 \end{aligned}$$

The second cone has the greater surface area. The prediction was correct.

Chapter 8 Section 4 Question 5 Page 448

a)

$$
\begin{aligned}
SA_{\text{lateral}} &= \pi rs \\
60 &= \pi \times 4 \times s \\
60 &= 4\pi s \\
\frac{60}{4\pi} &= \frac{4\pi s}{4\pi} \\
5 &\doteq s
\end{aligned}
$$

The slant height is approximately 5 cm.

b)

$$
\begin{aligned}
5^2 &= 4^2 + h^2 \\
25 &= 16 + h^2 \\
9 &= h^2 \\
\sqrt{9} &= h \\
3 &= h
\end{aligned}
$$

The height of the cone is 3 cm.

Chapter 8 Section 4 Question 6 Page 448

Doubling the height of a cone does not double the surface area. Answers will vary. A sample answer is shown.

The formula for the surface area of the cone is $SA = \pi rs + \pi r^2$. When the height is doubled only the term πrs is changed. The term πr^2 remains unaltered. Hence, doubling the height of a cone does not double the surface area.

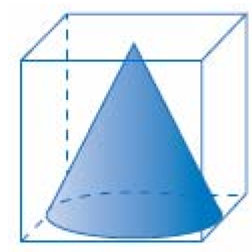
Chapter 8 Section 4 Question 7 Page 448

Doubling the radius of a cone does not double the surface area. Answers will vary. A sample answer is shown.

The formula for the surface area of a cone is $SA = \pi rs + \pi r^2$. When the radius is doubled, the term πr^2 will quadruple and the term πrs will more than double. Hence, the surface area of the new cone will be more than double the original cone.

Chapter 8 Section 4 Question 8 Page 448

a) The radius of the largest cone that will fit into the box is 5 cm, while the height is 10 cm.



b)

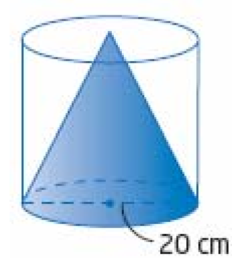
$$s^2 = 10^2 + 5^2$$
$$s^2 = 100 + 25$$
$$s^2 = 125$$
$$s = \sqrt{125}$$
$$s \doteq 11.2$$

$$SA = \pi rs + \pi r^2$$
$$= \pi \times 5 \times 11.2 + \pi \times 5^2$$
$$\doteq 254$$

The surface area is about 254 cm^2.

Chapter 8 Section 4 Question 9 Page 448

First, find the height of the cylinder. Then, find the slant height of the cone and finally its surface area.



$$V = \pi r^2 h$$
$$9425 = \pi \times 10^2 \times h$$
$$9425 = 100\pi h$$
$$\frac{9425}{100\pi} = \frac{100\pi h}{100\pi}$$
$$30.0 \doteq h$$

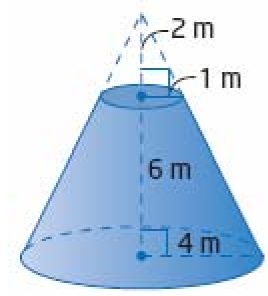
$$s^2 = 10^2 + 30.0^2$$
$$s^2 = 100 + 900$$
$$s^2 = 1000$$
$$s = \sqrt{1000}$$
$$s \doteq 31.6$$

$$SA = \pi rs + \pi r^2$$
$$= \pi \times 10 \times 31.6 + \pi \times 10^2$$
$$\doteq 1307$$

The surface area is about 1307 cm^2.

Chapter 8 Section 4 Question 10 Page 448

To find the surface area of the frustum, first find the surface area of the original cone, and then subtract the surface area of the top portion that has been removed.



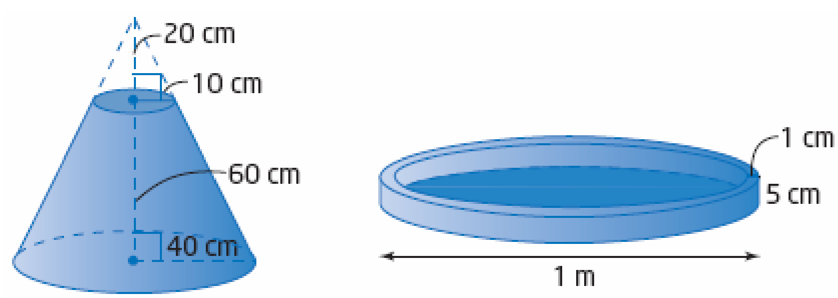
$$s_{cone}^2 = 4^2 + 8^2$$
$$s_{cone}^2 = 16 + 64$$
$$s_{cone}^2 = 80$$
$$s_{cone} = \sqrt{80}$$
$$s_{cone} \doteq 8.9$$

$$s_{top}^2 = 1^2 + 2^2$$
$$s_{top}^2 = 1 + 4$$
$$s_{top}^2 = 5$$
$$s_{top} = \sqrt{5}$$
$$s_{top} \doteq 2.2$$

$$\begin{aligned} SA_{frustum} &= \text{lateral } SA_{cone} - \text{lateral } SA_{top} + A_{\text{base of cone}} + A_{\text{base of top}} \\ &= \pi \times 4 \times 8.9 - \pi \times 1 \times 2.2 + \pi \times 4^2 + \pi \times 1^2 \\ &\doteq 158 \end{aligned}$$

The surface area of the frustum is about 158 m^2.

Chapter 8 Section 4 Question 11 Page 449



a) The area to be painted includes the base of the frustum, the lateral area of the frustum, the top of the frustum, the outer walls of the cylinder, the inner walls of the cylinder, the thin strip of the cylinder, the outer part of the base of the cylinder, and the inner part of the base of the cylinder.

b) To find the surface area of the frustum, first find the surface area of the original cone, and then subtract the surface area of the top portion that has been removed.

$$s_{\text{cone}}{}^2 = 40^2 + 80^2$$
$$s_{\text{cone}}{}^2 = 1600 + 6400$$
$$s_{\text{cone}}{}^2 = 8000$$
$$s_{\text{cone}} = \sqrt{8000}$$
$$s_{\text{cone}} \doteq 89.4$$

$$s_{\text{top}}{}^2 = 10^2 + 20^2$$
$$s_{\text{top}}{}^2 = 100 + 400$$
$$s_{\text{top}}{}^2 = 500$$
$$s_{\text{top}} = \sqrt{500}$$
$$s_{\text{top}} \doteq 22.4$$

$$\begin{aligned} SA_{\text{frustum}} &= \text{lateral } SA_{\text{cone}} - \text{lateral } SA_{\text{top}} + A_{\text{base of cone}} + A_{\text{base of top}} \\ &= \pi \times 40 \times 89.4 - \pi \times 10 \times 22.4 + \pi \times 40^2 + \pi \times 10^2 \\ &\doteq 15\,871 \end{aligned}$$

The area of the frustum is about 15 871 cm^2.

$$\begin{aligned} A_{\text{outer walls}} &= 2\pi \times 50 \times 5 \\ &\doteq 1571 \\ A_{\text{inner walls}} &= 2\pi \times 49 \times 4 \\ &\doteq 1232 \\ A_{\text{top strip}} &= \pi \times 50^2 - \pi \times 49^2 \\ &\doteq 311 \\ A_{\text{outside bottom}} &= \pi \times 50^2 \\ &\doteq 7854 \\ A_{\text{inside bottom}} &= \pi \times 49^2 \\ &= 7543 \\ SA_{\text{open cylinder}} &= 1571 + 1232 + 311 + 7854 + 7543 \\ &= 18\,511 \end{aligned}$$

The area of the cylinder is about 18 511 cm^2.

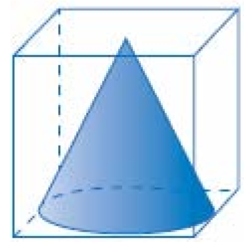
The total surface area is 15 871 + 18 511, or 34 382 cm^2 (about 3.4 m^2).

c) Emily will need 4 cans of paint to cover all surfaces.

Chapter 8 Section 4 Question 12 Page 449

Answers will vary.

Chapter 8 Section 4 Question 13 Page 449



a) The radius of the cone is $\frac{1}{2}x$, and the height is x.

b)

$$s^2 = x^2 + \left(\frac{1}{2}x\right)^2$$
$$s^2 = x^2 + \frac{1}{4}x^2$$
$$s^2 = \frac{5}{4}x^2$$
$$s = \sqrt{\frac{5}{4}x^2}$$
$$s = \frac{\sqrt{5}}{2}x$$

$$\begin{aligned} SA &= \pi r^2 + \pi rs \\ &= \pi\left(\frac{1}{2}x\right)^2 + \pi\left(\frac{1}{2}x\right)\left(\frac{\sqrt{5}}{2}x\right) \\ &= \frac{1}{4}\pi x^2 + \frac{\sqrt{5}}{4}\pi x^2 \end{aligned}$$

Chapter 8 Section 4 Question 14 Page 449

a) $\text{Lateral Area} = \pi rs$

$$\frac{\text{Lateral Area}}{\pi r} = \frac{\pi rs}{\pi r}$$
$$s = \frac{\text{Lateral Area}}{\pi r}$$

b) $$\begin{aligned} s &= \frac{\text{Lateral Area}}{\pi r} \\ &= \frac{100}{4\pi} \\ &\doteq 7.96 \end{aligned}$$

The slant height is 7.96 cm.

Chapter 8 Section 4 Question 15 Page 449

Answers will vary. A sample answer is shown.

The radius is about 4500 m.

$$s^2 = 4500^2 + 2351^2$$
$$s^2 = 25\,777\,201$$
$$s = \sqrt{25\,777\,201}$$
$$s \doteq 5077$$

$$SA_{\text{lateral}} = \pi r s$$
$$= \pi \times 4500 \times 5077$$
$$\doteq 71\,774\,397$$

The surface area is about 71 774 397 m^2.

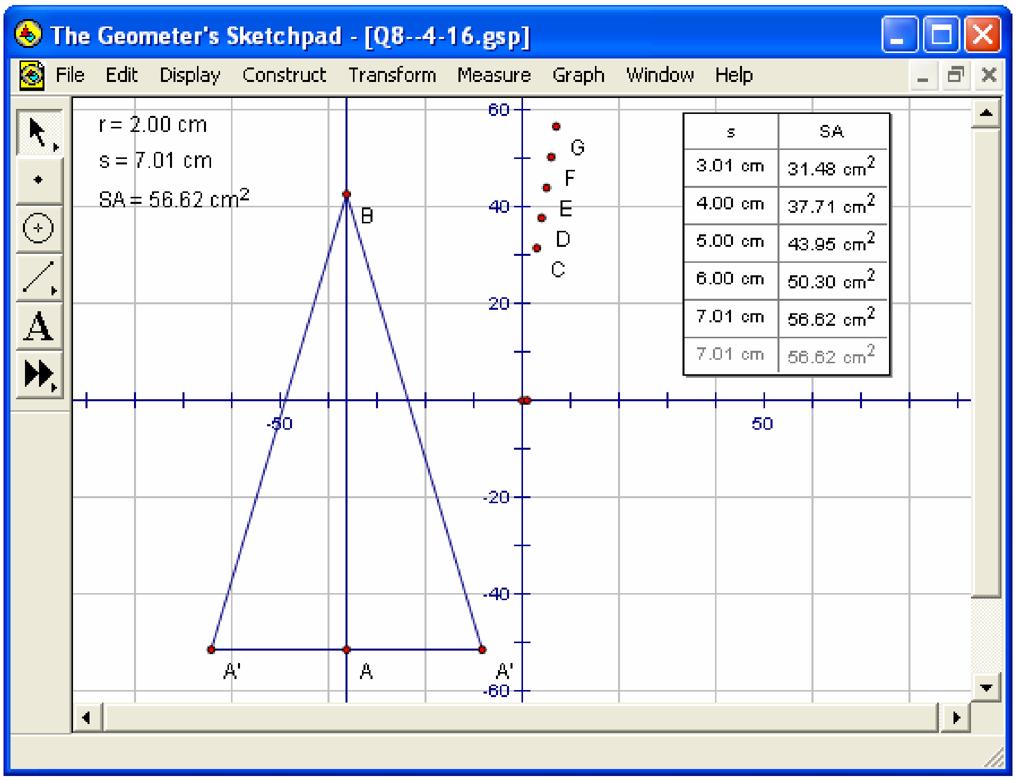
Chapter 8 Section 4 **Question 16** **Page 450**

a) $SA = \pi r^2 + \pi rs$

$= \pi(2)^2 + \pi(2)s$

$= 4\pi + 2\pi s$

b) Answers will vary. A sample sketch is shown. Click here to load the sketch.



c) Answers will vary. The relation appears to be linear.

Chapter 8 Section 5 Volume of a Cone

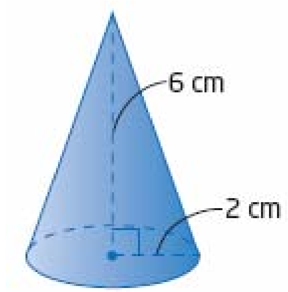
Chapter 8 Section 5 Question 1 Page 454

a) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 2^2 \times 6$

$\doteq 25$

The volume is approximately 25 cm^3.

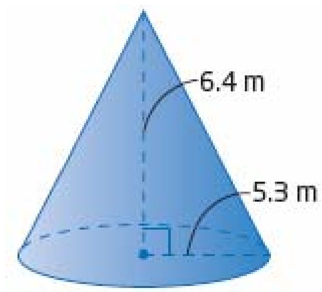


b) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 5.3^2 \times 6.4$

$\doteq 188$

The volume is approximately 188 m^3.

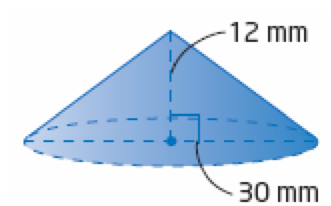


c) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 15^2 \times 12$

$\doteq 2827$

The volume is approximately 2827 mm^3.

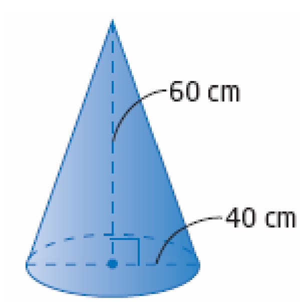


d) $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 20^2 \times 60$

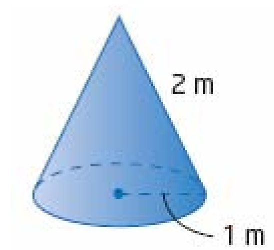
$\doteq 25\,133$

The volume is approximately 25 133 cm^3.



Chapter 8 Section 5 **Question 2** **Page 454**

a)

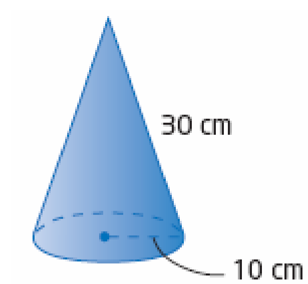


$$2^2 = 1^2 + h^2$$
$$4 = 1 + h^2$$
$$3 = h^2$$
$$\sqrt{3} = h$$
$$1.7 \doteq h$$

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi \times 1^2 \times 1.7 \\ &\doteq 2 \end{aligned}$$

The volume is about 2 m^3.

b)



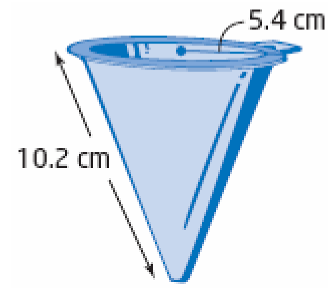
$$30^2 = 10^2 + h^2$$
$$900 = 100 + h^2$$
$$800 = h^2$$
$$\sqrt{800} = h$$
$$28.3 \doteq h$$

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi \times 10^2 \times 28.3 \\ &\doteq 2964 \end{aligned}$$

The volume is about 2964 cm^3.

Chapter 8 Section 5 **Question 3** **Page 454**

$$10.2^2 = 5.4^2 + h^2$$
$$104.04 = 29.16 + h^2$$
$$74.88 = h^2$$
$$\sqrt{74.88} = h$$
$$8.65 \doteq h$$



$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi \times 5.4^2 \times 8.65$$
$$\doteq 264.1$$

The funnel can hold about 264.1 cm^3 of oil.

Chapter 8 Section 5 **Question 4** **Page 455**

$$V = \frac{1}{3}\pi r^2 h$$
$$67 = \frac{1}{3}\pi \times 3^2 \times h$$
$$67 = 3\pi h$$
$$\frac{67}{3\pi} = \frac{3\pi h}{3\pi}$$
$$7.1 \doteq h$$

The height of the paper cup is approximately 7.1 cm^2.

Chapter 8 Section 5 **Question 5** **Page 455**

The volume of the cone is $\frac{1}{3} \times 300$, or 100 cm^3.

Chapter 8 Section 5 **Question 6** **Page 455**

Answers will vary.

Chapter 8 Section 5 **Question 7** **Page 455**

The volume of the cylinder is 3×150, or 450 cm^3.

Chapter 8 Section 5 Question 8 Page 455

a) Answers will vary. A possible estimate is 18 m.

b)

$$V = \frac{1}{3}\pi r^2 h$$

$$4000 = \frac{1}{3}\pi \times 15^2 \times h$$

$$4000 = 75\pi h$$

$$\frac{4000}{75\pi} = \frac{75\pi h}{75\pi}$$

$$16.98 \doteq h$$

The height of the storage unit is approximately 16.98 m.

c) Answers will vary. The estimate in part a) was about 1 m too high.

Chapter 8 Section 5 Question 9 Page 455

a) Answers will vary. A sample answer is shown.

The cone with base radius of 4 cm has the greater volume. The formula for the volume of a cone contains two factors of *r* and only one factor of *h*. Hence, the volume is more dependent on *r* than on *h*.

b)

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \times 3^2 \times 4$$

$$\doteq 38$$

$$V = \frac{1}{3}\pi r^2 h$$

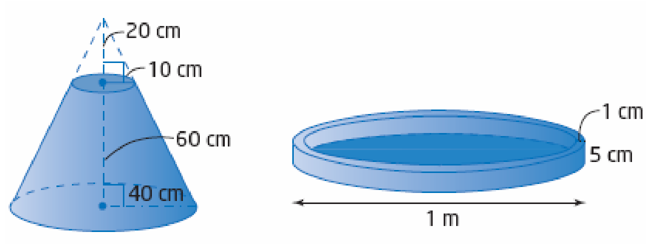
$$= \frac{1}{3}\pi \times 4^2 \times 3$$

$$\doteq 50$$

The prediction was correct. The cone with a radius of 3 cm has a volume of 38 m^3, while the cone with a radius of 4 cm has a volume of 50 cm^3.

Chapter 8 Section 5 Question 10 Page 455

To find the volume of the frustum, first find the volume of the original cone, and then subtract the volume of the top portion that has been removed.



$$\begin{aligned} V_{\text{frustum}} &= V_{\text{cone}} - V_{\text{top}} \\ &= \frac{1}{3}\pi \times 40^2 \times 80 - \frac{1}{3}\pi \times 10^2 \times 20 \\ &\doteq 131\,947 \end{aligned}$$

The volume of the frustum is approximately 131 947 cm^3.

$$\begin{aligned} V_{\text{cylinder}} &= V_{\text{wall}} + V_{\text{base}} \\ &= \left(\pi \times 50^2 \times 5 - \pi \times 49^2 \times 5\right) + \pi \times 49^2 \times 1 \\ &\doteq 9098 \end{aligned}$$

The volume of the cylinder is approximately 9098 cm^3.

The total volume of concrete required is 131 947 + 9098, or 141 045 cm^3.

Chapter 8 Section 5 Question 11 Page 455

a)
$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ 3 \times V &= 3 \times \frac{1}{3}\pi r^2 h \\ 3V &= \pi r^2 h \\ \frac{3V}{\pi r^2} &= \frac{\pi r^2 h}{\pi r^2} \\ h &= \frac{3V}{\pi r^2} \end{aligned}$$

b) 1 L = 1000 cm^3

$$\begin{aligned} h &= \frac{3V}{\pi r^2} \\ &= \frac{3 \times 1000}{\pi \times 4^2} \\ &\doteq 59.7 \end{aligned}$$

The height of the cone is approximately 59.7 cm.

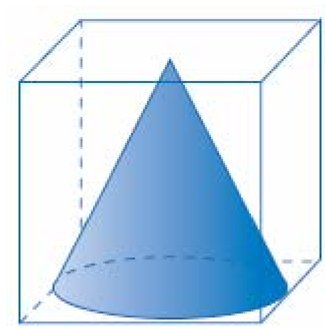
Chapter 8 Section 5 Question 12 Page 455

120 mL = 120 cm^3

$$V = \frac{1}{3}\pi r^2 h$$
$$120 = \frac{1}{3}\pi r^2 (15)$$
$$120 = 5\pi r^2$$
$$\frac{120}{5\pi} = \frac{5\pi r^2}{5\pi}$$
$$\frac{120}{5\pi} = r^2$$
$$\sqrt{\frac{120}{5\pi}} = r$$
$$2.8 \doteq r$$

The radius of the cone is approximately 2.8 cm.

Chapter 8 Section 5 Question 13 Page 456



a) The radius of the cone is 5 cm, and the height is 10 cm.

b) Estimates will vary. A possible estimate is 1:4.

c) $$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi \times 5^2 \times 10$$
$$\doteq 262$$

The volume of the cone is approximately 262 cm^3.

d) The ratio of the volume of the cone to the volume of the cube is 262:1000, or about 1:3.82.

e) Answers will vary. The estimate in part b) was close to the correct ratio.

Chapter 8 Section 5 Question 14 Page 456

$$V = \frac{1}{3}\pi r^2 h$$
$$200 = \frac{1}{3}\pi r^2 (2r)$$
$$200 = \frac{2\pi}{3} r^3$$
$$\frac{3}{2\pi} \times 200 = \frac{3}{2\pi} \times \frac{2\pi}{3} r^3$$
$$\frac{300}{\pi} = r^3$$
$$4.57 \doteq r$$

$$h = 2 \times 4.57$$
$$\doteq 9.1$$

The height of the cone is about 9.1 m.

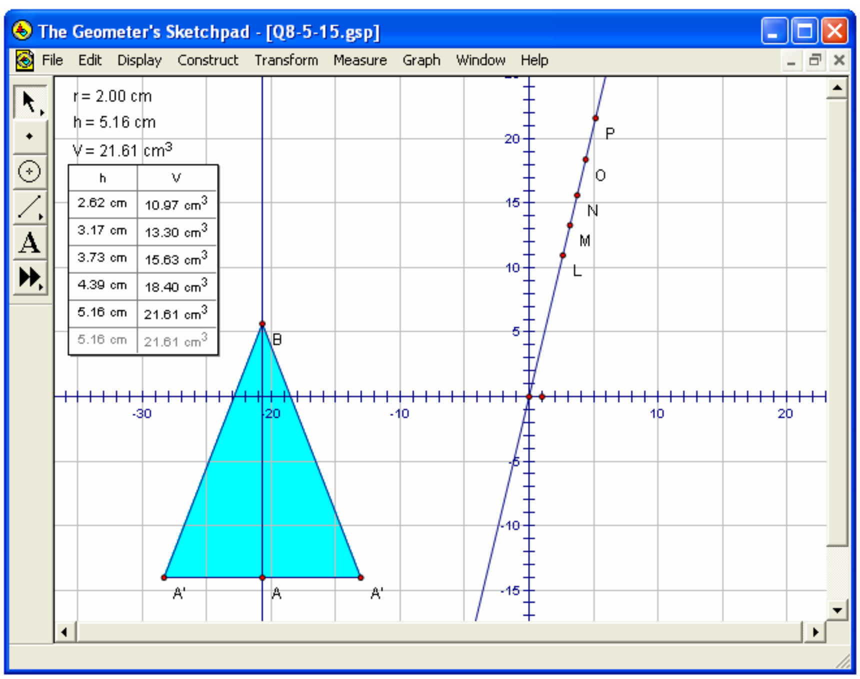
Chapter 8 Section 5 Question 15 Page 456

Answers will vary. A sample answer is shown. Click here to load the sketch.

Use geometry software to construct a model of a cone with a fixed radius. Collect data on volume as the height is changed. Plot the data.

When the radius is constant, a change in height produces a proportional change in volume.

A sample screen shot is shown.



Chapter 8 Section 5 Question 16 Page 456

a) $$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi \times r^2 \times 20$$
$$= \frac{20}{3}\pi r^2$$

b)

c) Answers will vary. A sample answer is shown.

The relation is increasing for all values of *r* greater than 0 (since the radius cannot be negative). The growth rate is non-linear.

Chapter 8 Section 5 Question 17 Page 456

Cube: $$V = s^3$$
$$= 6^3$$
$$= 216$$

The volume of the cube is 216 cm^3.

Pyramid: $$V = \frac{1}{3}A_{base} \times h$$
$$= \frac{1}{3} \times 6^2 \times 12$$
$$= 144$$

The volume of the pyramid is 144 cm^3.

Cone: $$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi \times 3^2 \times 12$$
$$\doteq 113$$

The volume of the cone is approximately 113 cm^3.

Cylinder: $$V = \pi r^2 h$$
$$= \pi \times 3^2 \times 6$$
$$\doteq 170$$

The volume of the cone is approximately 170 cm^3.

From least to greatest, the volumes are cone, pyramid, cylinder and cube. Answer D.

Chapter 8 Section 6 Surface Area of a Sphere

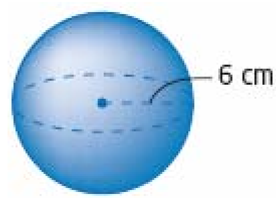
Chapter 8 Section 6 Question 1 Page 459

a) $SA = 4\pi r^2$

$= 4\pi \times 6^2$

$\doteq 452$

The volume is approximately 452 cm^2.



b) $SA = 4\pi r^2$

$= 4\pi \times 30.2^2$

$\doteq 11\,461$

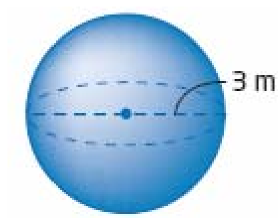
The volume is approximately 11 461 mm^2.

c) $SA = 4\pi r^2$

$= 4\pi \times 1.5^2$

$\doteq 28$

The volume is approximately 28 m^2.

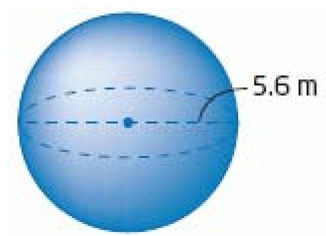


d) $SA = 4\pi r^2$

$= 4\pi \times 2.8^2$

$\doteq 99$

The volume is approximately 99 m^2.



Chapter 8 Section 6 Question 2 Page 459

a) Estimates will vary. A possible estimate is 4800 mm^2.

b) $SA = 4\pi r^2$

$= 4\pi \times 20^2$

$\doteq 5027$

The surface area is approximately 5027 mm^2.
Answers will vary. The estimate in part a) was close.

Chapter 8 Section 6   Question 3   Page 459

$$
\begin{aligned}
SA &= 4\pi r^2 \\
42.5 &= 4\pi r^2 \\
\frac{42.5}{4\pi} &= \frac{4\pi r^2}{4\pi} \\
\frac{42.5}{4\pi} &= r^2 \\
\sqrt{\frac{42.5}{4\pi}} &= r \\
1.8 &\doteq r
\end{aligned}
$$

The radius of the sphere is approximately 1.8 m.

Chapter 8 Section 6   Question 4   Page 459

a)
$$
\begin{aligned}
SA &= 4\pi r^2 \\
&= 4\pi \times 12.4^2 \\
&\doteq 1932.2
\end{aligned}
$$

The area of leather required is approximately 1932.2 cm^2, or 0.193 22 m^2.

b) It will cost 0.193 22 × \$28, or \$5.41 to cover the ball.

Chapter 8 Section 6   Question 5   Page 459

a)
$$
\begin{aligned}
SA &= 4\pi r^2 \\
&= 4\pi \times 6400^2 \\
&\doteq 514\,718\,540
\end{aligned}
$$

The surface area of the Earth is approximately 514 718 540 km^2.

b) Assume that the Earth is a sphere.

Chapter 8 Section 6 Question 6 Page 460

a) $SA = 4\pi r^2$

$= 4\pi \times 3400^2$

$\doteq 145\,267\,244$

The surface area of Mars is approximately 145 267 244 km^2.

b) The surface area of the Earth is $\frac{514\,718\,540}{145\,267\,244}$, or about 3.5 times greater than the surface area of Mars.

Chapter 8 Section 6 Question 7 Page 460

a) Estimates will vary. A possible estimate is 10 800 cm^2, or 1.08 m^2. This will require 2 jars of crystals.

b) $SA = 4\pi r^2$

$= 4\pi \times 30^2$

$\doteq 11\,310$

The surface area of the ball is approximately 11 310 cm^2, or 1.131 m^2.

c) Answers will vary. A sample answer is shown.

In this case, whether you use the approximate value or the exact value, two jars of reflective crystals are required to cover the gazing ball.

Chapter 8 Section 6 Question 8 Page 460

a) Predictions will vary. A possible prediction is 750 cm^2.

b) Change in $SA = 4\pi \times 17^2 - 4\pi \times 15^2$

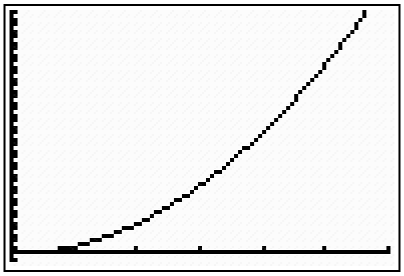
$\doteq 804$

The change in the surface area is about 804 cm^2.

c) Answers will vary. The prediction in part a) was close to the correct answer.

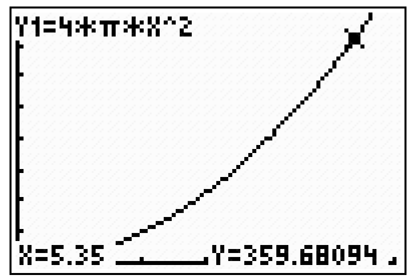
Chapter 8 Section 6 Question 9 Page 460

a)

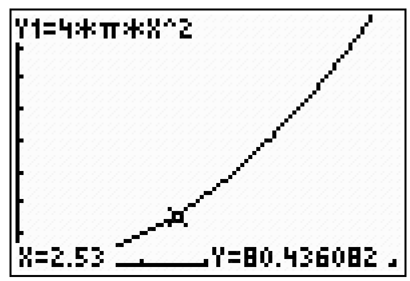


b) The radius must be greater than 0. As the radius increases, the surface area also increases in a non-linear pattern.

c) For a radius of 5.35 cm, the surface area is about 360 cm^2.



For a surface area of 80 cm^2, the radius is about 2.5 cm.



Chapter 8 Section 6 Question 10 Page 460

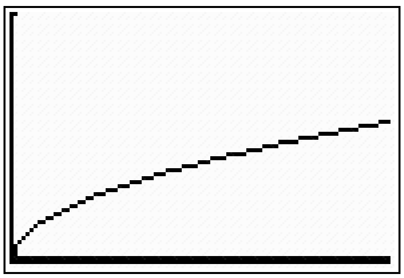
a) $SA = 4\pi r^2$

$$\frac{SA}{4\pi} = \frac{4\pi r^2}{4\pi}$$

$$\frac{SA}{4\pi} = r^2$$

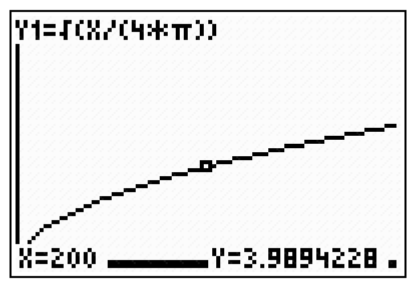
$$r = \sqrt{\frac{SA}{4\pi}}$$

b)



c) The radius and the surface area must be greater than 0. The trend between the two variables is non-linear with the radius increasing as the surface area increases but at a slower rate.

d) When the surface area is 200 cm^2, the radius is about 4 cm.



Chapter 8 Section 6 Question 11 Page 460

The surface area has increased by a factor of nine.

$$\begin{aligned} SA_{\text{old}} &= 4\pi r^2 \\ SA_{\text{new}} &= 4\pi(3r)^2 \\ &= 4\pi \times 9r^2 \\ &= 9(4\pi r^2) \end{aligned}$$

Chapter 8 Section 6 Question 12 Page 460

A cube with an edge length of $2r$ has a surface area of $6(2r)^2 = 24r^2$. A sphere of radius r has a surface area of $4\pi r^2$, or about $12.6r^2$. The cube has the larger surface area.

Chapter 8 Section 6 **Question 13** **Page 461**

a) Answers will vary. A possible estimate is $\frac{1}{2}$.

b)

$$
\begin{aligned}
SA_{\text{sphere}} &= 4\pi r^2 \\
&= 4\pi \times 5^2 \\
&= 100\pi \\
&= 314
\end{aligned}
$$

The surface area of the sphere is 100π, or about 314 cm^2.

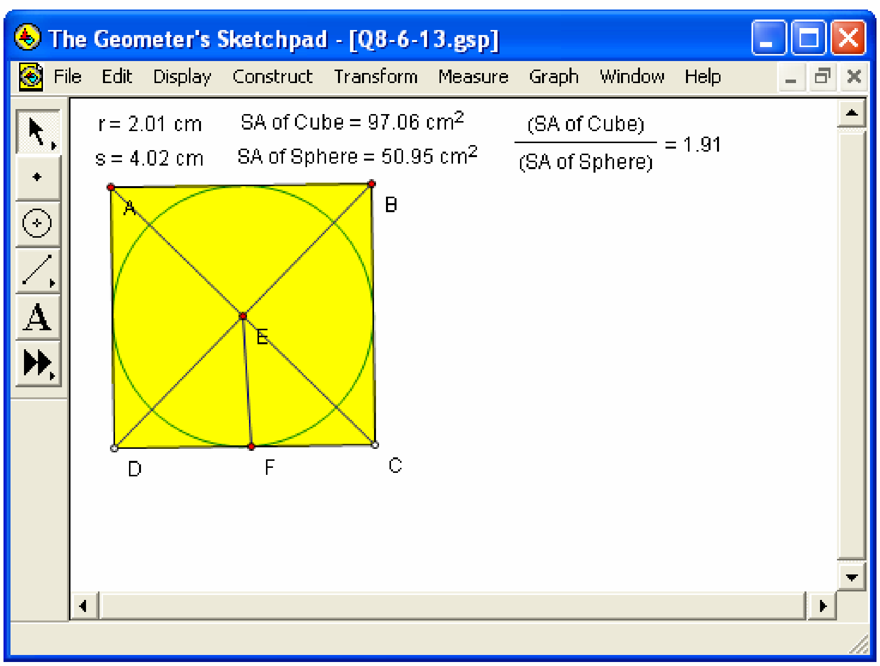
$$
\begin{aligned}
SA_{\text{cube}} &= 6s^2 \\
&= 6 \times 10^2 \\
&= 600
\end{aligned}
$$

The surface area of the cube is 600 cm^2.

The ratio of the surface areas is 100π:600, or π:6. Alternatively, the ratio is 314:600, or about 1:1.91.

c) Answers will vary. The estimate in part a) was close to the correct answer.

d) Answers will vary. A sample sketch is shown. The ratio of the surface areas of a cube and a sphere inscribed in the cube is constant at about 1.91. Click here to load the sketch.

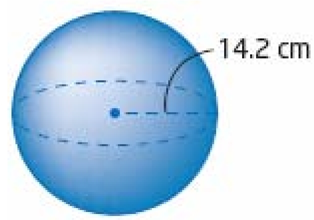


Chapter 8 Section 7 Volume of a Sphere

Chapter 8 Section 7 Question 1 Page 465

a)

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \times 14.2^3 \\ &\doteq 11\,994 \end{aligned}$$



The volume is approximately 11 994 cm^3.

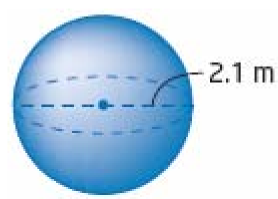
b)

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \times 32^3 \\ &\doteq 137\,258 \end{aligned}$$

The volume is approximately 137 258 mm^3.

c)

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \times 1.05^3 \\ &\doteq 5 \end{aligned}$$



The volume is approximately 5 m^3.

Chapter 8 Section 7 Question 2 Page 465

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \times 2.15^3 \\ &\doteq 42 \end{aligned}$$

The volume is approximately 42 cm^3.

Chapter 8 Section 7 Question 3 Page 465

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \times 4^3 \\ &\doteq 268 \end{aligned}$$

The volume of each hailstone is approximately 268 cm^3.

Chapter 8 Section 7 Question 4 Page 465

a) $$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \times 20^3 \\ &\doteq 33\,510 \end{aligned}$$

The volume of the ball is approximately 33 510 mm^3.

b) $$\begin{aligned} V &= s^3 \\ &= 40^3 \\ &= 64\,000 \end{aligned}$$

The volume of the cube is 64 000 mm^3.

c) The amount of empty space is 64 000 – 33 510, or 30 490 mm^3.

Chapter 8 Section 7 Question 5 Page 466

a) $V_{\text{small}} = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi \times 2^3$
$\doteq 33.5$

$V_{\text{large}} = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi \times 70.15^3$
$\doteq 1\,446\,011.1$

The volume of the small lollipop is approximately 33.5 cm^3, and the volume of the large lollipop is approximately 1 446 011.1 cm^3.

The volume of the large lollipop is $\frac{1\,446\,011.1}{33.5}$, or about 43 165 times the volume of the small lollipop. The mass of the large lollipop is $0.05 \times 43\,165$, or about 2158 kg.

b) Answers will vary. A sample answer is shown.

Assume that the largest lollipop had the same mass per cubic centimetre as the small lollipop.

Chapter 8 Section 7 **Question 6** **Page 466**

a) $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi \times 30^3$

$\doteq 113\,097$

The volume of the ball is approximately 113 097 cm^3.

b) $V = \pi r^2 h$

$= \pi \times 30^2 \times 60$

$\doteq 169\,646$

The volume of the cylindrical container is approximately 169 646 cm^3.

c) $\frac{V_{\text{sphere}}}{V_{\text{container}}} = \frac{113\,097}{169\,646}$

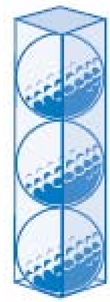
$\doteq 0.67 \text{ or } \frac{2}{3}$

The ratio of the volume of the sphere to the volume of the container is about 2:3.

d) This ratio is consistent for any sphere that just fits inside the cylinder, since $h = 2r$.

$$\begin{aligned}
\frac{V_{\text{sphere}}}{V_{\text{container}}} &= \frac{\frac{4}{3}\pi r^3}{\pi r^2 h} \\
&= \frac{\frac{4}{3}\pi r^3}{\pi r^2 (2r)} \\
&= \frac{\frac{4}{3}\pi r^3}{2\pi r^3} \\
&= \frac{\frac{4}{3}}{2} \\
&= \frac{2}{3}
\end{aligned}$$

Chapter 8 Section 7 Question 7 Page 466

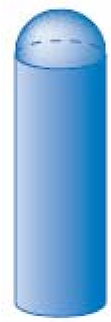


The box will measure 12.9 cm by 4.3 cm by 4.3 cm.

$$\begin{aligned} SA &= 4A_{\text{face}} + 2A_{\text{base}} \\ &= 4(12.9 \times 4.3) + 2(4.3^2) \\ &= 221.88 + 36.98 \\ &= 258.86 \end{aligned}$$

The amount of material needed to make the box is 258.86 cm^2.

Chapter 8 Section 7 Question 8 Page 466



a) Answers will vary. A possible estimate is 800 m^3.

b)
$$\begin{aligned} V_{\text{silo}} &= V_{\text{cylinder}} + V_{\text{hemisphere}} \\ &= \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) \\ &= \pi \times 3.25^2 \times 20 + \frac{1}{2} \times \frac{4}{3}\pi \times 3.25^3 \\ &\doteq 736 \end{aligned}$$

The volume of the silo is approximately 736 m^3.

c) The silo can hold 0.80×736, or about 589 m^3 of grain.

d)
$$\begin{aligned} V_{\text{bin}} &= 7 \times 3 \times 2.5 \\ &= 52.5 \end{aligned}$$

It will take $\frac{589}{52.5}$, or about 11.2 truckloads to fill the silo. So, 12 truckloads are needed.

Chapter 8 Section 7 Question 9 Page 466

The length of the cylinder is 10.2 – 4, or 6.2 m.

$$\begin{aligned} V_{\text{tank}} &= V_{\text{cylinder}} + V_{\text{sphere}} \\ &= \pi r^2 h + \frac{4}{3}\pi r^3 \\ &= \pi \times 2^2 \times 6.2 + \frac{4}{3}\pi \times 2^3 \\ &\doteq 111 \end{aligned}$$

The volume of the tank is approximately 111 m^3.

Chapter 8 Section 7 Question 10 Page 467

Answers will vary. A sample answer is shown.

Assume that the classroom measures 10 m by 5 m by 3 m. Assume that 3 basketballs line up on each metre. The number of balls that will fit into the classroom is about $30 \times 15 \times 9$, or 4050.

Chapter 8 Section 7 Question 11 Page 467

Solutions for the Achievement Checks are shown in the Teacher's Resource.

Chapter 8 Section 7 Question 12 Page 467

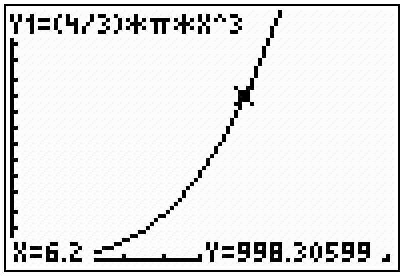
Estimates will vary. A possible estimate is 5 cm.

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ 600 &= \frac{4}{3}\pi r^3 \\ 3 \times 600 &= 3 \times \frac{4}{3}\pi r^3 \\ 1800 &= 4\pi r^3 \\ \frac{1800}{4\pi} &= \frac{4\pi r^3}{4\pi} \\ \frac{1800}{4\pi} &= r^3 \\ 5.23 &\doteq r \end{aligned}$$

The radius of the sphere is approximately 5.23 cm.

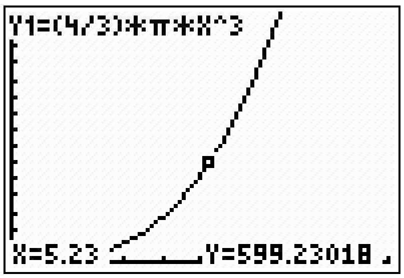
Chapter 8 Section 7 Question 13 Page 467

a)



The volume of a sphere with a radius of 6.2 cm is approximately 998.3 cm^3.

b)



The volume of a sphere with a radius of 5.23 cm is 599.2 cm^3. The answer checks.

Chapter 8 Section 7 Question 14 Page 467

$$\begin{aligned} SA &= 4\pi r^2 \\ 250 &= 4\pi r^2 \\ \frac{250}{4\pi} &= \frac{4\pi r^2}{4\pi} \\ \frac{250}{4\pi} &= r^2 \\ \sqrt{\frac{250}{4\pi}} &= r \\ 4.46 &\doteq r \end{aligned}$$

$$\begin{aligned} SA &= 4\pi r^2 \\ 500 &= 4\pi r^2 \\ \frac{500}{4\pi} &= \frac{4\pi r^2}{4\pi} \\ \frac{500}{4\pi} &= r^2 \\ \sqrt{\frac{500}{4\pi}} &= r \\ 6.31 &\doteq r \end{aligned}$$

$$\begin{aligned} V_{\text{old}} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \times 4.46^3 \\ &\doteq 372 \end{aligned}$$

$$\begin{aligned} V_{\text{new}} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \times 6.31^3 \\ &\doteq 1052 \end{aligned}$$

The volume increases by a factor of $\frac{1052}{372}$, or about 2.83.

Chapter 8 Section 7 Question 15 Page 467

a) Answers will vary. A possible estimate is 1:2.

b)

$$\begin{aligned} V_{\text{cube}} &= s^3 \\ &= 8^3 \\ &= 512 \end{aligned}$$

$$\begin{aligned} V_{\text{sphere}} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \times 4^3 \\ &= \frac{256\pi}{3} \\ &\doteq 268 \end{aligned}$$

The ratio of the volume of the sphere to the volume of the cube is 268:512, or about 1:0.52.
Note: the actual ratio is π:6.

$$\begin{aligned} \frac{\frac{256\pi}{3}}{512} &= \frac{256\pi}{1536} \\ &= \frac{\pi}{6} \end{aligned}$$

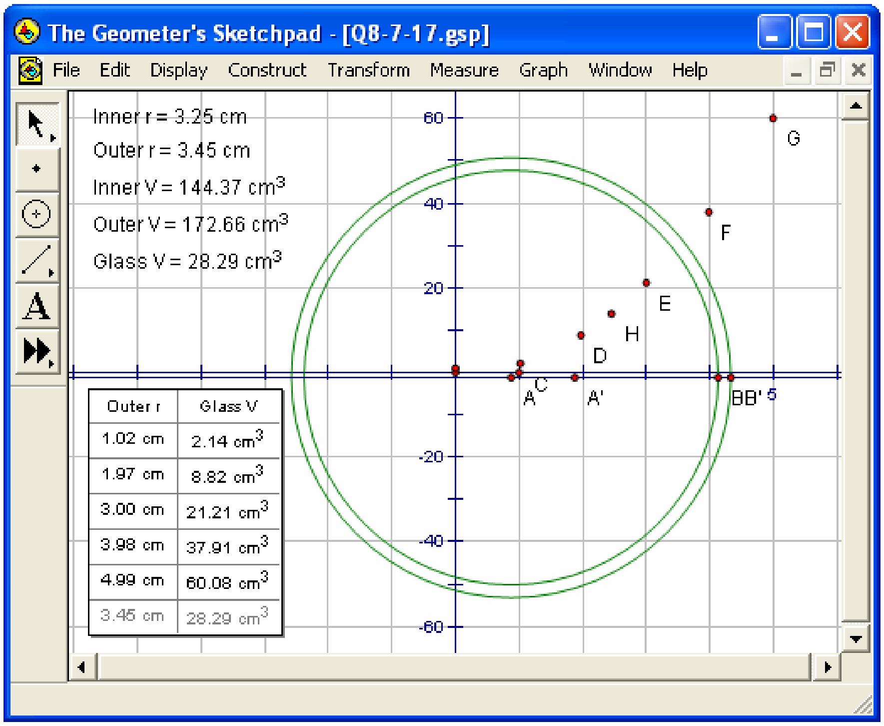
c) Answers will vary. The answer in part b) is close to the estimate in part a).

Chapter 8 Section 7 Question 16 Page 467

A cube with edges of length 2r has a larger volume than a sphere with a radius of r. The sphere will fit inside the cube.

Chapter 8 Section 7 Question 17 Page 468

Answers will vary. A sample sketch is shown. Click here to load the sketch.



The relationship is non-linear.

Chapter 8 Section 7 Question 18 Page 469

$$\begin{aligned} V_{\text{cylinder}} &= \pi r^2 h \\ &= \pi \times 6^2 \times 6 \\ &\doteq 679 \end{aligned}$$

The volume of the cylinder is approximately 679 cm^3.

$$\begin{aligned} V_{\text{cone}} &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi \times 6^2 \times 6 \\ &\doteq 226 \end{aligned}$$

The volume of the cone is approximately 226 cm^3.

$$\begin{aligned} V_{\text{sphere}} &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \times 6^3 \\ &\doteq 905 \end{aligned}$$

The volume of the sphere is approximately 905 cm^3.

From least volume to greatest volume the order is cone, cylinder, and sphere. Answer B.

Chapter 8 Section 7 Question 19 Page 469

$$\begin{aligned} V_{\text{box}} &= lwh \\ &= 4 \times 12 \times 16 \\ &= 768 \end{aligned}$$

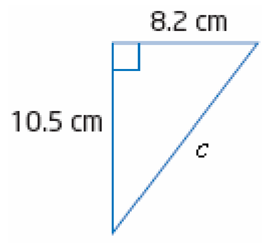
$$\begin{aligned} V_{\text{balls}} &= 12\left(\frac{4}{3}\pi r^3\right) \\ &= 12\left(\frac{4}{3}\pi \times 2^3\right) \\ &\doteq 402.12 \end{aligned}$$

The volume of empty space is 768 – 402.12, or 365.88 cm^3.

Chapter 8 Review

Chapter 8 Review **Question 1** **Page 470**

a)



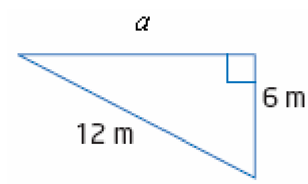
$$c^2 = 8.2^2 + 10.5^2$$
$$c^2 = 67.24 + 110.25$$
$$c^2 = 177.49$$
$$\sqrt{c^2} = \sqrt{177.49}$$
$$c \doteq 13.32$$

$$P = 13.32 + 8.2 + 10.5$$
$$\doteq 32.0$$

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 8.2 \times 10.5$$
$$\doteq 43.1$$

The perimeter is about 32.0 cm, and the area is about 43.1 cm^2.

b)



$$12^2 = 6^2 + a^2$$
$$144 = 36 + a^2$$
$$144 - 36 = 36 + a^2 - 36$$
$$108 = a^2$$
$$\sqrt{108} = \sqrt{a^2}$$
$$10.39 \doteq a$$

$$P = 10.39 + 12 + 6$$
$$\doteq 28.4$$
$$A = \frac{1}{2} \times 6 \times 10.39$$
$$\doteq 31.2$$

The perimeter is about 28.4 m, and the area is about 31.2 m^2.

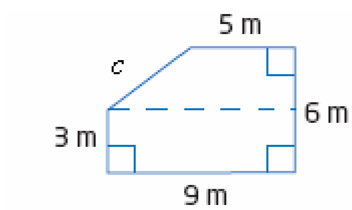
Chapter 8 Review **Question 2** **Page 470**

$$
\begin{aligned}
6^2 &= 2^2 + a^2 \\
36 &= 4 + a^2 \\
36 - 4 &= 4 + a^2 - 4 \\
32 &= a^2 \\
\sqrt{32} &= \sqrt{a^2} \\
5.7 &\doteq a
\end{aligned}
$$

The ladder reaches approximately 5.7 m up the wall.

Chapter 8 Review **Question 3** **Page 470**

a)



$$
\begin{aligned}
c^2 &= 3^2 + 4^2 \\
c^2 &= 9 + 16 \\
c^2 &= 25 \\
c &= \sqrt{25} \\
c &= 5
\end{aligned}
$$

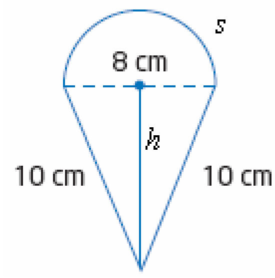
$$
\begin{aligned}
P &= 5 + 5 + 6 + 9 + 3 \\
&= 28
\end{aligned}
$$

$$
\begin{aligned}
A &= A_{\text{trapezoid}} + A_{\text{rectangle}} \\
&= \frac{1}{2} \times 3 \times (5 + 9) + 3 \times 9 \\
&= 21 + 27 \\
&= 48
\end{aligned}
$$

The perimeter is 28 m, and the area is 48 m^2.

b)

$$s = \frac{1}{2}\pi d$$
$$= \frac{1}{2}\pi \times 8$$
$$\doteq 12.57$$



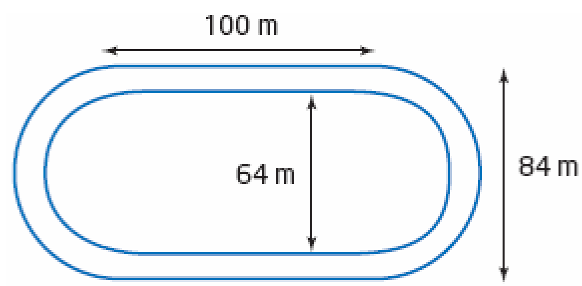
$$P = 12.57 + 10 + 10$$
$$\doteq 32.6$$

$$h^2 + 4^2 = 10^2$$
$$h^2 + 16 = 100$$
$$h^2 = 100 - 16$$
$$h^2 = 84$$
$$h = \sqrt{84}$$
$$h \doteq 9.17$$

$$A = A_{\text{triangle}} + A_{\text{semicircle}}$$
$$= \frac{1}{2} \times 8 \times 9.17 + \frac{1}{2}\pi \times 4^2$$
$$\doteq 61.8$$

The perimeter is about 32.6 cm, and the area is about 61.8 cm^2.

Chapter 8 Review **Question 4** **Page 470**



a) $d = 100 + 100 + \pi \times 64$
$\doteq 401.1$

Tyler runs about 401.1 m.

b) $d = 100 + 100 + \pi \times 84$
$\doteq 463.9$

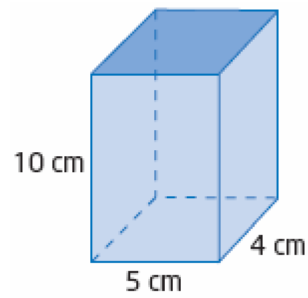
Dylan runs about 463.9 m.

c) Dylan runs 463.9 – 401.1, or 62.8 m farther than Tyler.

Chapter 8 Review **Question 5** **Page 470**

a)

$$\begin{aligned} SA &= 2A_{\text{bottom}} + 2A_{\text{sides}} + 2A_{\text{front}} \\ &= 2(5\times 4) + 2(10\times 4) + 2(10\times 5) \\ &= 40 + 80 + 100 \\ &= 220 \end{aligned}$$



The surface area is 220 cm^2.

b)

$$\begin{aligned} s^2 &= 115^2 + 147^2 \\ s^2 &= 13\,225 + 21\,609 \\ s^2 &= 34\,834 \\ s &= \sqrt{34\,834} \\ s &\doteq 186.6 \end{aligned}$$

$$\begin{aligned} SA &= A_{\text{base}} + 4A_{\text{triangle}} \\ &= 230\times 230 + 4\left(\frac{1}{2}\times 230\times 186.6\right) \\ &= 52\,900 + 85\,836 \\ &= 138\,736 \end{aligned}$$

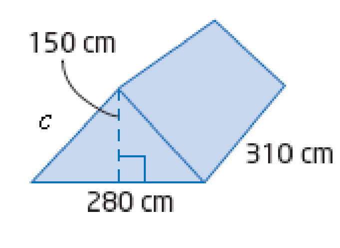
The surface area is about 138 736 m^2.

Chapter 8 Review **Question 6** **Page 471**

a) $V = A_{base} \times h$

$= \left(\frac{1}{2} \times 280 \times 150\right) \times 310$

$= 6\,510\,000$



The volume of the tent is 6 510 000 cm^3.

b)

$c^2 = 140^2 + 150^2$

$c^2 = 19\,600 + 22\,500$

$c^2 = 42\,100$

$c = \sqrt{42\,100}$

$c \doteq 205.2$

$SA = A_{bottom} + 2A_{sides} + 2A_{front}$

$= 280 \times 310 + 2 \times 205.2 \times 310 + 2\left(\frac{1}{2} \times 280 \times 150\right)$

$= 86\,800 + 127\,224 + 42\,000$

$= 256\,024$

The amount of nylon required to make the tent is 256 024 cm^2.

c) Answers will vary. A sample answer is shown.

Assume that the side walls of the tent are flat.

d) Answers will vary. A sample answer is shown.

The answer is fairly reasonable. When erecting a tent, you want the side walls to be as flat and stretched as possible.

Chapter 8 Review **Question 7** **Page 471**

500 mL = 500 cm^3

$V = \pi r^2 h$

$500 = \pi \times 4^2 \times h$

$500 = 16\pi h$

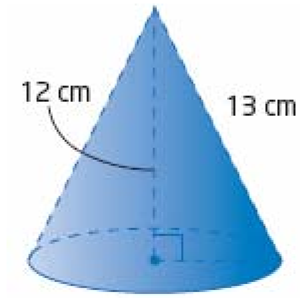
$\frac{500}{16\pi} = \frac{16\pi h}{16\pi}$

$\frac{500}{16\pi} = h$

$9.9 \doteq h$

The height of the can is 9.9 cm.

Chapter 8 Review **Question 8** **Page 471**

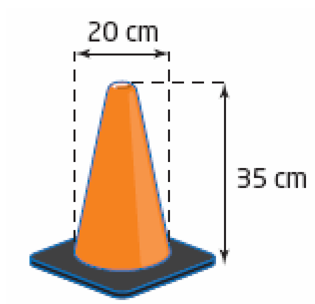


$$13^2 = 12^2 + r^2$$
$$169 = 144 + r^2$$
$$25 = r^2$$
$$\sqrt{25} = r$$
$$5 = r$$

$$SA = \pi rs + \pi r^2$$
$$= \pi \times 5 \times 13 + \pi \times 5^2$$
$$\doteq 283$$

The surface area is approximately 283 cm^2.

Chapter 8 Review **Question 9** **Page 471**



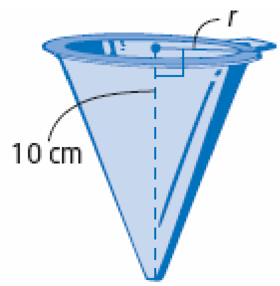
$$s^2 = 10^2 + 35^2$$
$$s^2 = 100 + 1225$$
$$s^2 = 1325$$
$$s = \sqrt{1325}$$
$$s \doteq 36.4$$

$$SA = \pi rs + \pi r^2$$
$$= \pi \times 10 \times 36.4 + \pi \times 10^2$$
$$\doteq 1458$$

The surface area is about 1458 cm^2.

Chapter 8 Review **Question 10** **Page 471**

100 mL = 100 cm^3



$$V = \frac{1}{3}\pi r^2 h$$

$$100 = \frac{1}{3}\pi r^2 (10)$$

$$100 = \frac{10\pi}{3}r^2$$

$$\frac{3}{10\pi} \times 100 = \frac{3}{10\pi} \times \frac{10}{3}\pi r^2$$

$$\frac{300}{10\pi} = r^2$$

$$\sqrt{\frac{300}{10\pi}} = r$$

$$3.1 \doteq r$$

The radius is approximately 3.1 cm.

Chapter 8 Review **Question 11** **Page 471**

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 h \\ &= \frac{1}{3}\pi \times 8^2 \times 10 \\ &\doteq 670 \end{aligned}$$

The volume of the cone is approximately 670 cm^3. The volume of the cone is $\frac{1}{3}$ of the volume of the cylinder.

Chapter 8 Review **Question 12** **Page 471**

$$\begin{aligned} SA &= 4\pi r^2 \\ &= 4\pi \times 10.9^2 \\ &\doteq 1493.0 \end{aligned}$$

The amount of leather required to cover the volleyball is approximately 1493.0 cm^2.

Chapter 8 Review **Question 13 Page 471**

a)

$$\begin{aligned} SA &= \frac{1}{2}\left(4\pi r^2\right) \\ &= \frac{1}{2} \times 4\pi \times 6400^2 \\ &\doteq 257\ 359\ 270 \end{aligned}$$

The area of the Northern Hemisphere is approximately 257 359 270 km^2.

b) Answers will vary. A sample answer is shown.

Assume that the Earth is a sphere.

c) The fraction of the Northern Hemisphere that Canada covers is $\frac{9\ 970\ 610}{257\ 359\ 270}$, or about 0.04. This is about $\frac{1}{25}$ of the Northern Hemisphere.

Chapter 8 Review **Question 14 Page 471**

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \times 11.15^3 \\ &\doteq 5806.5 \end{aligned}$$

The volume of the soccer ball is approximately 5806.5 cm^3.

Chapter 8 Review **Question 15 Page 471**

a) Answers will vary. A possible estimate is 5200 cm^3.

b)
$$
\begin{aligned}
V_{\text{emptyspace}} &= V_{\text{box}} - V_{\text{ball}} \\
&= 22.3^3 - 5806.5 \\
&= 5283.07
\end{aligned}
$$

c) Answers will vary. The estimate in part a) was close to the correct answer.

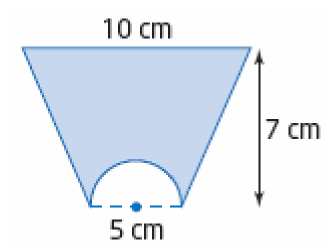
Chapter 8 Chapter Test

Chapter 8 Chapter Test Question 1 Page 472

$$V = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi \times 3^3$$
$$\doteq 113$$

The volume of the sphere is approximately 113 cm^3. Answer C.

Chapter 8 Chapter Test Question 2 Page 472



$$A = A_{trapezoid} - A_{semicircle}$$
$$= \frac{1}{2} \times 7 \times (10 + 5) - \frac{1}{2}\pi \times 2.5^2$$
$$\doteq 43$$

The area of the figure is approximately 43 cm^2. Answer A.

Chapter 8 Chapter Test Question 3 Page 472

$$V = \pi r^2 h$$
$$= \pi \times 3.75^2 \times 1.4$$
$$\doteq 61.850$$

The volume of the water is approximately 61.850 m^3, or 61 850 L. Answer A.

Chapter 8 Chapter Test Question 4 Page 472

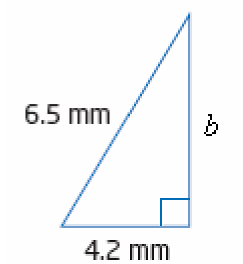
$$s^2 = 15^2 + 15^2$$
$$s^2 = 225 + 225$$
$$s^2 = 450$$
$$s = \sqrt{450}$$
$$s \doteq 21.2$$

$$Lateral\ Area = \pi rs$$
$$= \pi \times 15 \times 21.2$$
$$\doteq 999$$

The amount of plastic sheeting required is approximately 999 m^2. Answer D.

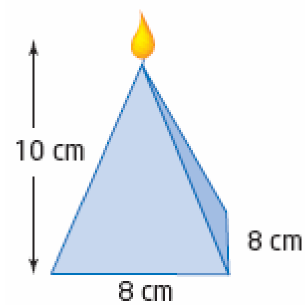
Chapter 8 Chapter Test Question 5 Page 472

$$6.5^2 = 4.2^2 + b^2$$
$$42.25 = 17.64 + b^2$$
$$24.61 = b^2$$
$$\sqrt{24.61} = b$$
$$5.0 \doteq b$$



The length of the unknown side is approximately 5.0 mm. Answer B.

Chapter 8 Chapter Test Question 6 Page 472



a) $V = \frac{1}{3}A_{base} \times h$

$$= \frac{1}{3} \times 8^2 \times 10$$
$$\doteq 213$$

The amount of wax required is approximately 213 cm^3.

b)

$$s^2 = 4^2 + 10^2$$
$$s^2 = 16 + 100$$
$$s^2 = 116$$
$$s = \sqrt{116}$$
$$s \doteq 10.77$$
$$SA = A_{base} + 4A_{triangle}$$
$$= 8 \times 8 + 4\left(\frac{1}{2} \times 8 \times 10.77\right)$$
$$= 64 + 172.32$$
$$\doteq 236.3$$

The area of plastic wrap needed is about 236.3 cm^2, assuming no overlap.

Chapter 8 Chapter Test Question 7 Page 472

Answers will vary. A sample answer is shown.

Assume that the paper towels are stacked in three columns with two rolls in each column. Then, the dimensions of the carton would be 10 cm by 30 cm by 56 cm.

$$\begin{aligned} SA &= 2A_{\text{bottom}} + 2A_{\text{sides}} + 2A_{\text{front}} \\ &= 2(10 \times 30) + 2(56 \times 30) + 2(10 \times 56) \\ &= 600 + 3360 + 1120 \\ &= 5080 \end{aligned}$$

The area of cardboard needed is 5080 cm^2.

Chapter 8 Chapter Test Question 8 Page 472

Doubling the radius of a sphere will increase the volume eight times. Doubling the radius of a cylinder will quadruple the volume.

Sphere:

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \times 1^3 \\ &= \frac{4}{3}\pi \end{aligned}$$

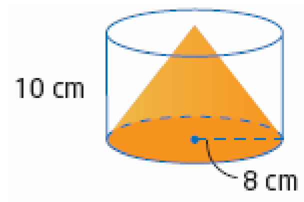
$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi \times 2^3 \\ &= 8 \times \frac{4}{3}\pi \end{aligned}$$

Cylinder:

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 1^2 \times 1 \\ &= \pi \end{aligned}$$

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 2^2 \times 1 \\ &= 4\pi \end{aligned}$$

Chapter 8 Chapter Test Question 9 Page 472

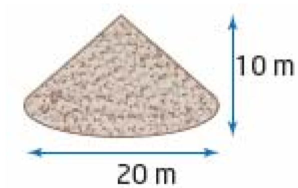


$s^2 = 8^2 + 10^2$
$s^2 = 64 + 100$
$s^2 = 164$
$s = \sqrt{164}$
$s \doteq 12.8$

$SA = \pi rs + \pi r^2$
$= \pi \times 8 \times 12.8 + \pi \times 8^2$
$\doteq 523$

The surface area of the cone is about 523 cm^2.

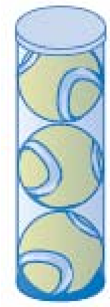
Chapter 8 Chapter Test Question 10 Page 472



$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi \times 10^2 \times 10$
$\doteq 1047$

The volume of the pile is approximately 1047 m^3.

Chapter 8 Chapter Test Question 11 Page 473



a) $V = \pi r^2 h$

$= \pi \times 4.2^2 \times 25.2$

$\doteq 1396.5$

The volume of the can is approximately 1396.5 cm^3.

b) $SA = 2\pi r^2 + 2\pi rh$

$= 2\pi \times 4.2^2 + 2\pi \times 4.2 \times 25.2$

$\doteq 776$

The amount of aluminum required to make the can is approximately 776 cm^2.

c) $A = \pi r^2$

$= \pi \times 4.2^2$

$\doteq 55$

The amount of plastic required for the lid is approximately 55 cm^2.

d) Answers will vary. A sample answer is shown.

Assume that the circular lid covers the top of the cylindrical can with no side parts.

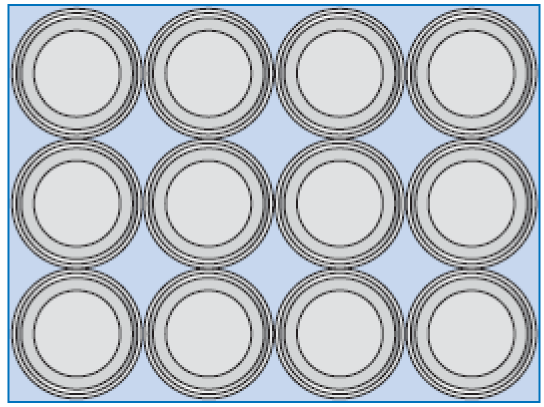
Chapter 8 Chapter Test Question 12 Page 473

a) $V_{\text{emptyspace}} = V_{\text{can}} - V_{\text{balls}}$

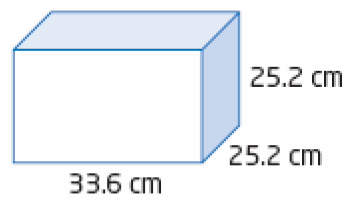
$$= 1396.5 - 3\left(\frac{4}{3}\pi \times 4.2^3\right)$$

$$\doteq 465.5$$

The empty space in each can is approximately 465.5 cm^3.



b)



c) $V_{\text{emptyspace}} = V_{\text{box}} - V_{\text{cans}} + V_{\text{empty space in cans}}$

$$= 25.2 \times 25.2 \times 33.6 - 12(1396.5) + 12(465.5)$$

$$\doteq 10\,165.3$$

The total empty space is about 10 165.3 cm^3.

d) $SA = 4(33.6 \times 25.2) + 2(25.2 \times 25.2)$

$$\doteq 4657$$

The area of cardboard needed to make the box is about 4657 cm^2.