## Chapter 7

## Geometric Relationships

Chapter 7 Get Ready
Chapter 7 Get Ready
Question 1 Page 362
a) Two sides are equal. The triangle is isosceles.

b) No two sides are equal. The triangle is scalene.


## Chapter 7 Get Ready

Question 2 Page 362
a) All three angles are equal and acute. The triangle is equilateral and acute.

b) Two angles are equal, and one angle is obtuse. The triangle is isosceles and obtuse.


## Chapter 7 Get Ready Question 3 Page 363

a) Polygon ABCDE is an irregular pentagon.

b) Polygon PQRSTU is a regular hexagon.


## Chapter 7 Get Ready $\quad$ Question 4 Page 363

a) Opposite pairs of sides are parallel. WXYZ is a parallelogram.
b) All four sides are equal, but no angles are right angles. ABCD is a rhombus.


## Chapter 7 Get Ready

Question 5 Page 363
a) $\angle \mathrm{Z}+72^{\circ}+58^{\circ}=180^{\circ}$

$$
\begin{aligned}
& \angle \mathrm{Z}=180^{\circ}-72^{\circ}-58^{\circ} \\
& \angle \mathrm{Z}=50^{\circ}
\end{aligned}
$$


b) $\angle \mathrm{R}+90^{\circ}+35^{\circ}=180^{\circ}$

$$
\begin{aligned}
& \angle \mathrm{R}=180^{\circ}-90^{\circ}-35^{\circ} \\
& \angle \mathrm{R}=55^{\circ}
\end{aligned}
$$



## Chapter 7 Get Ready

## Question 6 Page 363

a) $a=75^{\circ}$, opposite angles.
$b=75^{\circ}$, corresponding angles.
$c=75^{\circ}$, alternate angles.

b) $a=40^{\circ}$, corresponding angles.
$b=40^{\circ}$, opposite angles.
$c=140^{\circ}$, supplementary angles (also co-interior with $a$ ).


## Chapter 7 Section 1: Angle Relationships in Triangles

## Chapter 7 Section 1

a) $w=85^{\circ}+30^{\circ}$

$$
=115^{\circ}
$$

## Question 1 Page 371


b) $x=103^{\circ}+23^{\circ}$

$$
=126^{\circ}
$$


c) $y=61^{\circ}+73^{\circ}$

$$
=134^{\circ}
$$


d) $z=34^{\circ}+78^{\circ}$

$$
=112^{\circ}
$$

## Chapter 7 Section 1

a) $x+165^{\circ}+155^{\circ}=360^{\circ}$
$x=360^{\circ}-165^{\circ}-155^{\circ}$
$x=40^{\circ}$

## Question 2 Page 371


b) $v+105^{\circ}+100^{\circ}=360^{\circ}$
$v=360^{\circ}-105^{\circ}-100^{\circ}$
$v=155^{\circ}$

c) $z+150^{\circ}+65^{\circ}=360^{\circ}$
$z=360^{\circ}-150^{\circ}-65^{\circ}$
$z=145^{\circ}$


## Chapter 7 Section $1 \quad$ Question 3 Page 371

$$
\begin{aligned}
x+120^{\circ}+70^{\circ} & =360^{\circ} \\
x & =360^{\circ}-120^{\circ}-70^{\circ} \\
x & =170^{\circ}
\end{aligned}
$$

Chapter 7 Section 1
Question 4 Page 372
a) $\angle \mathrm{A}+75^{\circ}+75^{\circ}=180^{\circ}$

$$
\angle \mathrm{A}=180^{\circ}-75^{\circ}-75^{\circ}
$$

$$
\angle \mathrm{A}=30^{\circ}
$$

$$
x=75^{\circ}+30^{\circ}
$$

$$
=105^{\circ}
$$


b)

$$
\begin{aligned}
2 \angle \mathrm{E}+130^{\circ} & =180^{\circ} \\
2 \angle \mathrm{E} & =180^{\circ}-130^{\circ} \\
2 \angle \mathrm{E} & =50^{\circ} \\
\frac{2 \angle \mathrm{E}}{2} & =\frac{50^{\circ}}{2} \\
\angle \mathrm{E} & =25^{\circ} \\
x & =25^{\circ}+130^{\circ} \\
& =155^{\circ}
\end{aligned}
$$

c) $x=40^{\circ}+40^{\circ}$

$$
=80^{\circ}
$$



## Chapter 7 Section 1

a) $x+31^{\circ}=101^{\circ}$

$$
\begin{aligned}
& x=101^{\circ}-31^{\circ} \\
& x=70^{\circ}
\end{aligned}
$$

Question 5 Page 372
b)

$$
\begin{aligned}
z+65^{\circ}+34^{\circ} & =180^{\circ} \\
Z & =180^{\circ}-65^{\circ}-34^{\circ} \\
Z & =81^{\circ} \\
x & =81^{\circ}+34^{\circ} \\
& =115^{\circ} \\
y & =81^{\circ}+65^{\circ} \\
& =146^{\circ}
\end{aligned}
$$

c)

$$
\begin{aligned}
w & =y \\
2 w+94^{\circ} & =180^{\circ} \\
2 w & =180^{\circ}-94^{\circ} \\
2 w & =86^{\circ} \\
\frac{2 w}{2} & =\frac{86^{\circ}}{2} \\
w & =43^{\circ} \\
y & =43^{\circ} \\
x & =94^{\circ}+43^{\circ} \\
& =137^{\circ} \\
z & =94^{\circ}+43^{\circ} \\
& =137^{\circ}
\end{aligned}
$$


d)

$$
y=44^{\circ}
$$

$$
\begin{aligned}
w+44^{\circ}+44^{\circ} & =180^{\circ} \\
w+88^{\circ} & =180^{\circ} \\
w & =180^{\circ}-88^{\circ} \\
w & =92^{\circ} \\
x & =92^{\circ}+44^{\circ} \\
& =136^{\circ} \\
z & =92^{\circ}+44^{\circ} \\
& =136^{\circ}
\end{aligned}
$$

e)

$$
\begin{aligned}
e & =44^{\circ} \\
b+44^{\circ}+44^{\circ} & =180^{\circ} \\
b+88^{\circ} & =180^{\circ} \\
b & =180^{\circ}-88^{\circ} \\
b & =92^{\circ} \\
d & =92^{\circ}+44^{\circ} \\
& =136^{\circ} \\
a & =c \\
2 a+136^{\circ} & =180^{\circ} \\
2 a & =180^{\circ}-136^{\circ} \\
2 a & =44^{\circ} \\
\frac{2 a}{2} & =\frac{44^{\circ}}{2} \\
a & =22^{\circ} \\
c & =22^{\circ}
\end{aligned}
$$

## Chapter 7 Section 1

Case \#1:

$$
\begin{aligned}
a & =180^{\circ}-140^{\circ} \\
& =40^{\circ} \\
b+40^{\circ}+40^{\circ} & =180^{\circ} \\
b+80^{\circ} & =180^{\circ} \\
b & =180^{\circ}-80^{\circ} \\
b & =100^{\circ} \\
c & =40^{\circ}+40^{\circ} \\
& =80^{\circ} \\
d & =100^{\circ}+40^{\circ} \\
& =140^{\circ}
\end{aligned}
$$

## Question 6 Page 372



The other exterior angles measure $80^{\circ}$ and $140^{\circ}$.
Case \#2:

$$
\begin{aligned}
b & =180^{\circ}-140^{\circ} \\
& =40^{\circ}
\end{aligned}
$$

$$
\begin{aligned}
2 a+40^{\circ} & =180^{\circ} \\
2 a+ & =180^{\circ}-40^{\circ} \\
2 a & =140^{\circ} \\
\frac{2 a}{2} & =\frac{140^{\circ}}{2} \\
a & =70^{\circ} \\
c & =40^{\circ}+70^{\circ} \\
& =110^{\circ}
\end{aligned}
$$

$$
\begin{aligned}
d & =70^{\circ}+40^{\circ} \\
& =110^{\circ}
\end{aligned}
$$

The other exterior angles measure $110^{\circ}$ and $110^{\circ}$.

## Chapter 7 Section $1 \quad$ Question 7 Page 372

$$
\begin{aligned}
\text { mean } & =\frac{360^{\circ}}{3} \\
& =120^{\circ}
\end{aligned}
$$

## Chapter 7 Section $1 \quad$ Question 8 Page 372

Isosceles triangles have 2 exterior angles equal. Equilateral triangles have 3 exterior angles equal.

## Chapter 7 Section $1 \quad$ Question 9 Page 372

a) A triangle cannot have two obtuse interior angles. The sum of two obtuse angles is greater than $180^{\circ}$.
b) Any acute triangle will have three obtuse exterior angles.

Chapter 7 Section $1 \quad$ Question 10 Page 373

a) $\angle \mathrm{DAC}=180^{\circ}-5^{\circ}$

$$
=175^{\circ}
$$

b) $x+90^{\circ}+5^{\circ}=180^{\circ}$

$$
x+95^{\circ}=180^{\circ}
$$

$$
x=180^{\circ}-95^{\circ}
$$

$$
x=85^{\circ}
$$

$$
y=90^{\circ}+5^{\circ}
$$

$$
=95^{\circ}
$$

The interior angle at the top of the ramp measures $85^{\circ}$, while the exterior angle measures $95^{\circ}$.

## Chapter 7 Section 1

$$
\begin{aligned}
y & =180^{\circ}-90^{\circ} \\
& =90^{\circ} \\
w+90^{\circ}+50^{\circ} & =180^{\circ} \\
w+140^{\circ} & =180^{\circ} \\
w & =180^{\circ}-140^{\circ} \\
w & =40^{\circ} \\
z+50^{\circ} & =97^{\circ} \\
z & =97^{\circ}-50^{\circ} \\
z & =47^{\circ} \\
x+47^{\circ}+90^{\circ} & =180^{\circ} \\
x+137^{\circ} & =180^{\circ} \\
x & =180^{\circ}-137^{\circ} \\
x & =43^{\circ}
\end{aligned}
$$

## Chapter 7 Section 1

a) $x+x+x=360^{\circ}$
$3 x=360^{\circ}$
$\frac{3 x}{3}=\frac{360^{\circ}}{3}$

$$
x=120^{\circ}
$$

## Question 12 Page 373


b) $x+2 x+2 x=360^{\circ}$

$$
5 x=360^{\circ}
$$

$$
\frac{5 x}{5}=\frac{360^{\circ}}{5}
$$

$$
x=72^{\circ}
$$

$$
2 x=144^{\circ}
$$


c) $x+2 x+3 x=360^{\circ}$
$6 x=360^{\circ}$
$\frac{6 x}{6}=\frac{360^{\circ}}{6}$

$$
x=60^{\circ}
$$

$$
2 x=120^{\circ}
$$

$$
3 x=180^{\circ}
$$

This triangle is not possible. An exterior angle cannot equal $180^{\circ}$.
d) $x+x+2 x=360^{\circ}$

$$
4 x=360^{\circ}
$$

$$
\frac{4 x}{4}=\frac{360^{\circ}}{4}
$$

$$
x=90^{\circ}
$$

$$
2 x=180^{\circ}
$$

This triangle is not possible. An exterior angle must be less than $180^{\circ}$.
e) $3 x+4 x+5 x=360^{\circ}$

$$
12 x=360^{\circ}
$$

$$
\frac{12 x}{12}=\frac{360^{\circ}}{12}
$$

$$
x=30^{\circ}
$$

$$
3 x=90^{\circ}
$$


f) $4 x+5 x+6 x=360^{\circ}$

$$
15 x=360^{\circ}
$$

$$
\frac{15 x}{15}=\frac{360^{\circ}}{15}
$$

$$
x=24^{\circ}
$$

$$
4 x=96^{\circ}
$$

$$
5 x=120^{\circ}
$$

$$
6 x=144^{\circ}
$$

g) $3 x+4 x+8 x=360^{\circ}$

$$
\begin{aligned}
15 x & =360^{\circ} \\
\frac{15 x}{15} & =\frac{360^{\circ}}{15} \\
x & =24^{\circ} \\
3 x & =72^{\circ} \\
4 x & =96^{\circ} \\
8 x & =192^{\circ}
\end{aligned}
$$

This triangle is not possible. An exterior angle cannot exceed $180^{\circ}$.

## Chapter 7 Section 1 <br> Question 13 Page 373

Hexaflexagons are paper hexagons folded from strips of paper which reveal different faces as they are flexed. You can download templates and instructions for making a hexaflexagon.

Chapter 7 Section 1

$$
\begin{aligned}
2 y+20^{\circ} & =180^{\circ} \\
2 y & =180^{\circ}-20^{\circ} \\
2 y & =160^{\circ} \\
\frac{2 y}{2} & =\frac{160^{\circ}}{2} \\
y & =80^{\circ}
\end{aligned}
$$

$$
2 x=20^{\circ}+80^{\circ}
$$

$$
2 x=100^{\circ}
$$

$$
\frac{2 x}{2}=\frac{100^{\circ}}{2}
$$

$$
x=50^{\circ}
$$

Answer B.

## Chapter 7 Section 1

$$
\begin{aligned}
\angle \mathrm{ABC}+\angle \mathrm{ADC} & =x+w \\
y+z & =180^{\circ} \\
2 x+y+z+2 w & =360^{\circ} \\
2 x+180^{\circ}+2 w & =360^{\circ} \\
2 x+2 w & =360^{\circ}-180^{\circ} \\
2 x+2 w & =180^{\circ} \\
\frac{2 x+2 w}{2} & =\frac{180^{\circ}}{2} \\
\frac{2 x}{2}+\frac{2 w}{2} & =90^{\circ} \\
x+w & =90^{\circ}
\end{aligned}
$$

Answer C.

## Chapter 7 Section 2 Angle Relationships in Quadrilaterals

## Chapter 7 Section 2 <br> Question 1 Page 381

a) $w+113^{\circ}+104^{\circ}+79^{\circ}=360^{\circ}$

$$
\begin{aligned}
w+296^{\circ} & =360^{\circ} \\
w & =360^{\circ}-296^{\circ} \\
w & =64^{\circ}
\end{aligned}
$$


b) $x+87^{\circ}+99^{\circ}+73^{\circ}=360^{\circ}$

$$
\begin{aligned}
x+259^{\circ} & =360^{\circ} \\
x & =360^{\circ}-259^{\circ} \\
x & =101^{\circ}
\end{aligned}
$$


c) $y+70^{\circ}+95^{\circ}+150^{\circ}=360^{\circ}$

$$
y+315^{\circ}=360^{\circ}
$$

$$
y=360^{\circ}-315^{\circ}
$$

$$
y=45^{\circ}
$$


d) $z+65^{\circ}+160^{\circ}+20^{\circ}=360^{\circ}$

$$
\begin{aligned}
z+245^{\circ} & =360^{\circ} \\
z & =360^{\circ}-245^{\circ} \\
z & =115^{\circ}
\end{aligned}
$$



## Chapter 7 Section $2 \quad$ Question 2 Page 381

$$
\begin{aligned}
x+40^{\circ}+90^{\circ}+120^{\circ} & =360^{\circ} \\
x+250^{\circ} & =360^{\circ} \\
x & =360^{\circ}-250^{\circ} \\
x & =110^{\circ}
\end{aligned}
$$

Answer A.

## Chapter 7 Section 2 <br> Question 3 Page 381

$$
\begin{aligned}
x+80^{\circ}+100^{\circ}+120^{\circ} & =360^{\circ} \\
x+300^{\circ} & =360^{\circ} \\
x & =360^{\circ}-300^{\circ} \\
x & =60^{\circ}
\end{aligned}
$$

Answer B.

## Chapter 7 Section $2 \quad$ Question $4 \quad$ Page 381

a) $\angle \mathrm{D}+100^{\circ}+75^{\circ}+50^{\circ}=360^{\circ}$

$$
\begin{aligned}
\angle \mathrm{D}+225^{\circ} & =360^{\circ} \\
\angle \mathrm{D} & =360^{\circ}-225^{\circ} \\
\angle \mathrm{D} & =135^{\circ}
\end{aligned}
$$

b) $\angle \mathrm{C}+20^{\circ}+35^{\circ}+150^{\circ}=360^{\circ}$

$$
\angle \mathrm{C}+205^{\circ}=360^{\circ}
$$

$$
\angle \mathrm{C}=360^{\circ}-205^{\circ}
$$

$$
\angle \mathrm{C}=155^{\circ}
$$

c) $\angle \mathrm{B}+70^{\circ}+70^{\circ}+70^{\circ}=360^{\circ}$

$$
\begin{aligned}
\angle \mathrm{B}+210^{\circ} & =360^{\circ} \\
\angle \mathrm{B} & =360^{\circ}-210^{\circ} \\
\angle \mathrm{B} & =150^{\circ}
\end{aligned}
$$

d) $\angle \mathrm{A}+90^{\circ}+90^{\circ}+90^{\circ}=360^{\circ}$

$$
\begin{aligned}
\angle \mathrm{A}+270^{\circ} & =360^{\circ} \\
\angle \mathrm{A} & =360^{\circ}-270^{\circ}
\end{aligned}
$$

$$
\angle \mathrm{A}=90^{\circ}
$$

## Chapter 7 Section $2 \quad$ Question 5 Page 381

a) Opposite angles in a parallelogram are equal.

$$
\begin{gathered}
x=70^{\circ} \\
w=110^{\circ}
\end{gathered}
$$


b) Opposite angles in a parallelogram are equal.
$y=138^{\circ}$
$z=42^{\circ}$

c) Since opposite angles in a parallelogram are equal, $a=55^{\circ}$ and $b=c$.

Adjacent angles in a parallelogram are supplementary.

$$
\begin{aligned}
b+55^{\circ} & =180^{\circ} \\
b & =180^{\circ}-55^{\circ} \\
b & =125^{\circ} \\
c & =b \\
& =125^{\circ}
\end{aligned}
$$



## Chapter 7 Section 2 <br> Question 6 Page 381

For both triangles and quadrilateral, the sum of the exterior angles is $360^{\circ}$.

## Chapter 7 Section 2

Question 7 Page 382
a)

$$
\begin{aligned}
x & =180^{\circ}-85^{\circ} \\
& =95^{\circ}
\end{aligned}
$$



$$
\begin{aligned}
y+95^{\circ}+70^{\circ}+110^{\circ} & =360^{\circ} \\
y+275^{\circ} & =360^{\circ} \\
y & =360^{\circ}-275^{\circ} \\
y & =85^{\circ}
\end{aligned}
$$

b)

$$
\begin{aligned}
d & =180^{\circ}-87^{\circ} \\
& =93^{\circ} \\
e & =180^{\circ}-104^{\circ} \\
& =76^{\circ} \\
a & =180^{\circ}-119^{\circ} \\
& =61^{\circ}
\end{aligned}
$$

$$
\begin{aligned}
c+61^{\circ}+87^{\circ}+104^{\circ} & =360^{\circ} \\
c+252^{\circ} & =360^{\circ} \\
c & =360^{\circ}-252^{\circ} \\
c & =108^{\circ} \\
b & =180^{\circ}-108^{\circ} \\
& =72^{\circ}
\end{aligned}
$$



## Chapter 7 Section $2 \quad$ Question 8 Page 382

As shown in question 7 b), you need three angles, each at a different vertex; to calculate the measure of all of the interior and exterior angles of a quadrilateral. You can use angle relationships to calculate the others.

## Chapter 7 Section $2 \quad$ Question 9 Page 382

a) $\angle \mathrm{A}+\angle \mathrm{B}+\angle \mathrm{C}=170^{\circ}+65^{\circ}+160^{\circ}$

$$
=395^{\circ}
$$

The sum of the interior angles of a quadrilateral must be $360^{\circ}$. This quadrilateral is not possible.
b) $\angle \mathrm{H}+60^{\circ}+75^{\circ}+120^{\circ}=360^{\circ}$

$$
\begin{aligned}
\angle \mathrm{H}+255^{\circ} & =360^{\circ} \\
\angle \mathrm{H} & =360^{\circ}-255^{\circ} \\
\angle \mathrm{H} & =105^{\circ}
\end{aligned}
$$


c) $\angle \mathrm{S}+30^{\circ}+65^{\circ}+60^{\circ}=360^{\circ}$
$\angle \mathrm{S}+155^{\circ}=360^{\circ}$
$\angle \mathrm{S}=360^{\circ}-155^{\circ}$
$\angle \mathrm{S}=205^{\circ}$


## Chapter 7 Section $2 \quad$ Question $10 \quad$ Page 382

Answers will vary. Sample answers are shown.
a) Four obtuse angles add to more than $360^{\circ}$. This quadrilateral is not possible.
b) Exactly two obtuse angles:

c) One obtuse and three acute angles:

d) One obtuse angle, and two right angles:

e) If three of the angles are right angles, then the fourth must be a right angle as well. This quadrilateral is not possible.

## Chapter 7 Section $2 \quad$ Question 11 Page 382

$$
\begin{aligned}
\text { mean } & =\frac{360^{\circ}}{4} \\
& =90^{\circ}
\end{aligned}
$$

## Chapter 7 Section $2 \quad$ Question 12 Page 382


a) $x=180^{\circ}-55^{\circ}$

$$
=125^{\circ}
$$

b) $y+90^{\circ}+90^{\circ}+100^{\circ}=360^{\circ}$

$$
y+280^{\circ}=360^{\circ}
$$

$$
y=360^{\circ}-280^{\circ}
$$

$$
y=80^{\circ}
$$

c) Answers will vary. Sample answers are shown.

Triangles and quadrilaterals are easy to construct. Triangles are rigid.

## Chapter 7 Section 2

## Question 13 Page 383

$$
\begin{aligned}
x+3 x-22^{\circ} & =180^{\circ} \\
4 x-22^{\circ} & =180^{\circ} \\
4 x & =180^{\circ}+22^{\circ} \\
4 x & =202^{\circ} \\
\frac{4 x}{4} & =\frac{202^{\circ}}{4} \\
x & =50.5^{\circ}
\end{aligned}
$$

$$
3 x-22^{\circ}=3\left(50.5^{\circ}\right)-22^{\circ}
$$

$$
=151.5^{\circ}-22^{\circ}
$$

$$
=129.5^{\circ}
$$

$$
2 x-10^{\circ}=2\left(50.5^{\circ}\right)-10^{\circ}
$$

$$
=101^{\circ}-10^{\circ}
$$

$$
=91^{\circ}
$$

$$
y=180^{\circ}-91^{\circ}
$$

$$
=89^{\circ}
$$

$$
\begin{aligned}
x+15^{\circ} & =50.5^{\circ}+15^{\circ} \\
& =65.5^{\circ}
\end{aligned}
$$

$$
\begin{aligned}
z & =180^{\circ}-65.5^{\circ} \\
& =114.5^{\circ}
\end{aligned}
$$

$$
\begin{aligned}
w+129.5^{\circ}+65.5^{\circ}+91^{\circ} & =360^{\circ} \\
w+286^{\circ} & =360^{\circ} \\
w & =360^{\circ}-286^{\circ} \\
w & =74^{\circ} \\
u & =180^{\circ}-74^{\circ} \\
& =106^{\circ}
\end{aligned}
$$

## Chapter 7 Section $2 \quad$ Question 14 Page 383

a) Each diagonal divides the quadrilateral into two congruent triangles.
b) The diagonal is a line of symmetry for the square, but not for the rectangle.
c) The diagonal bisects the corner angles in the square. The diagonal does not bisect the corner angles in the rectangle.


## Chapter 7 Section 2 <br> Question 15 Page 383

a)

b) The sum of the four angles at point E is $360^{\circ}$.
c) The sum of all of the interior angles of the four triangles inside the quadrilateral is $4 \times 180^{\circ}=720^{\circ}$.
d) The sum of the interior angles of quadrilateral is equal to the sum of the interior angles of the four triangles less the sum of the angles at $\mathrm{E}: 720^{\circ}-360^{\circ}=360^{\circ}$.

## Chapter 7 Section 2

a) $x+x+x+x=360^{\circ}$

$$
\begin{aligned}
4 x & =360^{\circ} \\
\frac{4 x}{4} & =\frac{360^{\circ}}{4} \\
x & =90^{\circ}
\end{aligned}
$$

The angles are $90^{\circ}, 90^{\circ}, 90^{\circ}$, and $90^{\circ}$.
b) $x+x+2 x+2 x=360^{\circ}$

$$
\begin{aligned}
6 x & =360^{\circ} \\
\frac{6 x}{6} & =\frac{360^{\circ}}{6} \\
x & =60^{\circ}
\end{aligned}
$$

## Question 16 Page 383

| $90^{\circ}$ | $90^{\circ}$ |
| :---: | :---: |
|  |  |
| $90^{\circ}$ | $90^{\circ}$ |



The angles are $60^{\circ}, 60^{\circ}, 120^{\circ}$, and $120^{\circ}$.
c) $x+2 x+3 x+4 x=360^{\circ}$

$$
10 x=360^{\circ}
$$

$$
\frac{10 x}{10}=\frac{360^{\circ}}{10}
$$

$$
x=36^{\circ}
$$



The angles are $36^{\circ}, 72^{\circ}, 108^{\circ}$, and $144^{\circ}$.
d) $3 x+4 x+5 x+6 x=360^{\circ}$

$$
\begin{aligned}
18 x & =360^{\circ} \\
\frac{18 x}{18} & =\frac{360^{\circ}}{18} \\
x & =20^{\circ}
\end{aligned}
$$



The angles are $60^{\circ}, 80^{\circ}, 100^{\circ}$, and $120^{\circ}$.

## Chapter 7 Section $2 \quad$ Question 17 Page 383

Answers will vary. Sample answers are shown.
In a cyclic quadrilateral, opposite angles are supplementary.

$$
\begin{aligned}
x+z & =180^{\circ} \\
w+y & =180^{\circ}
\end{aligned}
$$



Any external angle is equal to the interior and opposite internal angle.


## Chapter 7 Section 2

$$
\begin{aligned}
y & =90^{\circ} \\
z & =60^{\circ} \\
y+z & =150^{\circ}
\end{aligned}
$$

$2 x+150^{\circ}=180^{\circ}$
$2 x=180^{\circ}-150^{\circ}$
$2 x=30^{\circ}$
$\frac{2 x}{2}=\frac{30^{\circ}}{2}$

$$
x=15^{\circ}
$$

$$
\angle \mathrm{CEB}=15^{\circ}
$$

Answer B.

## Chapter 7 Section $2 \quad$ Question 19 Page 383

Let the sides measure $a, b, c$, and $d$. You can place them in the following orders:
$a, b, c$, and $d$
$a, c, b$, and $d$
$a, b, d$, and $c$
Any other arrangement will produce a quadrilateral congruent to one of these.
There are 3 non-congruent quadrilaterals you can make with four sides of unequal lengths.

## Chapter 7 Section 3 Angle Relationships in Polygons

 Note: Let $S$ represent the sum of the interior angles.
## Chapter 7 Section 3

Question 1 Page 391
a) $S=180(n-2)$
$=180(10-2)$
$=180(8)$
$=1440$

The sum of the interior angles is $1440^{\circ}$.

$$
\text { b) } \quad \begin{aligned}
S & =180(n-2) \\
& =180(15-2) \\
& =180(13) \\
& =2340
\end{aligned}
$$

The sum of the interior angles is $2340^{\circ}$.

$$
\text { c) } \begin{aligned}
S & =180(n-2) \\
& =180(20-2) \\
& =180(18) \\
& =3240
\end{aligned}
$$

The sum of the interior angles is $3240^{\circ}$.

Chapter 7 Section 3

$$
\text { a) } \quad \begin{aligned}
S & =180(n-2) \\
& =180(7-2) \\
& =180(5) \\
& =900
\end{aligned}
$$

## Question 2 Page 391

b) $S=180(n-2)$

$$
=180(12-2)
$$

$$
=180(10)
$$

$$
=1800
$$

Each angle measures $\frac{900^{\circ}}{7}$, or $128.6^{\circ}$.
Each angle measures $\frac{1800^{\circ}}{12}$, or $150^{\circ}$.

## Chapter 7 Section 3

Question 3 Page 391

$$
\text { a) } \begin{aligned}
180(n-2) & =540 \\
180 n-360 & =540 \\
180 n-360+360 & =540+360 \\
180 n & =900 \\
\frac{180 n}{180} & =\frac{900}{180} \\
n & =5
\end{aligned}
$$

The polygon has 5 sides.
c)

$$
\begin{aligned}
180(n-2) & =3060 \\
180 n-360 & =3060 \\
180 n-360+360 & =3060+360 \\
180 n & =3420 \\
\frac{180 n}{180} & =\frac{3420}{180} \\
n & =19
\end{aligned}
$$

$$
\text { b) } \begin{aligned}
180(n-2) & =1800 \\
180 n-360 & =1800 \\
180 n-360+360 & =1800+360 \\
180 n & =2160 \\
\frac{180 n}{180} & =\frac{2160}{180} \\
n & =12
\end{aligned}
$$

The polygon has 12 sides.

The polygon has 19 sides.

## Chapter 7 Section $3 \quad$ Question 4 Page 391

| Number of <br> Solygon | Number of <br> Dlagonals from <br> One Vertex | Number of <br> Trlangles In <br> the Polygon | Sum of <br> Interlor <br> Angles |  |
| :--- | :---: | :---: | :---: | :---: |
| quadrilateral | 4 | 1 | 2 | $360^{\circ}$ |
| pentagon | 5 | 2 | 3 | $540^{\circ}$ |
| decagon | 10 | 7 | 8 | $1440^{\circ}$ |
| icosagon | 20 | 17 | 18 | $3240^{\circ}$ |

## Chapter 7 Section $3 \quad$ Question 5 Page 391

A regular polygon has equal interior angles, equal exterior angles, and equal sides.

## Chapter 7 Section 3 <br> Question 6 Page 391

$$
\begin{aligned}
S & =180(n-2) \\
& =180(4-2) \\
& =180(2) \\
& =360
\end{aligned}
$$

Each angle in a square measures $\frac{360^{\circ}}{4}$, or $90^{\circ}$.

## Chapter 7 Section 3

Question 7 Page 391
a) $S=180(n-2)$

$$
\begin{aligned}
& =180(6-2) \\
& =180(4) \\
& =720
\end{aligned}
$$

Each angle in a regular hexagon measures $\frac{720^{\circ}}{6}$, or $120^{\circ}$. The adjacent sides of the table will meet at $120^{\circ}$.
b) Answers will vary.
c)


Changing the lengths of one pair of opposite sides by doubling them does not change the measures of the angles.

## Chapter 7 Section $3 \quad$ Question 8 Page 391

a)

b) There are 6 diagonals that can be drawn from any one vertex. Refer to the diagram in part a).
c) $S=180(n-2)$
$=180(9-2)$
$=180(7)$
$=1260$
The sum of the interior angles of the polygon is $1260^{\circ}$.

## Chapter 7 Section 3

Question 9 Page 392
a) $S=180(n-2)$

$$
\begin{aligned}
& =180(10-2) \\
& =180(8) \\
& =1440
\end{aligned}
$$

Each angle in a regular 10 -sided polygon measures $\frac{1440^{\circ}}{10}$, or $144^{\circ}$.

Second methods may vary. A sample method is shown.
You can use The Geometer's Sketchpad ${ }^{\circledR}$ to construct a model of a 10 -sided regular polygon, and then measure one of the angles. Click here to load the sketch.

b) $S=180(n-2)$

$$
\begin{aligned}
& =180(16-2) \\
& =180(14) \\
& =2520
\end{aligned}
$$

Each angle in a regular 16 -sided polygon measures $\frac{2520^{\circ}}{16}$, or $157.5^{\circ}$.
c) $S=180(n-2)$

$$
\begin{aligned}
& =180(20-2) \\
& =180(18) \\
& =3240
\end{aligned}
$$

Each angle in a regular 20 -sided polygon measures $\frac{3240^{\circ}}{20}$, or $162^{\circ}$.
d) The measure of each interior angle of a regular polygon with $n$ sides may be calculated from the expression $\frac{180(n-2)}{n}$.

## Chapter 7 Section $3 \quad$ Question $10 \quad$ Page 392

a) A Canadian dollar coin has 11 sides.
b) $\frac{180(n-2)}{n}=\frac{180(11-2)}{11}$

$$
=\frac{180(9)}{11}
$$

$$
\doteq 147.3
$$

The angle between adjacent sides of the coin is about $147.3^{\circ}$.
c) Answers will vary. A sample answer is shown.

The Royal Canadian Mint may have chosen this shape to make it easier for blind people and vending machines to recognize, and harder to forge.

## Chapter 7 Section $3 \quad$ Question $11 \quad$ Page 392

The sum of the exterior angles is $360^{\circ}$ for all convex polygons. You cannot determine the number of sides from the sum of the exterior angles.

## Chapter 7 Section $3 \quad$ Question 12 Page 392

Three regular polygons whose interior angles divide evenly into $360^{\circ}$ are triangles $\left(60^{\circ}\right)$, rectangles $\left(90^{\circ}\right)$, and hexagons $\left(120^{\circ}\right)$.

## Chapter 7 Section $3 \quad$ Question 13 Page 392

a) The gazebo has 12 sides.
b) $\frac{180(n-2)}{n}=\frac{180(12-2)}{12}$

$$
\begin{aligned}
& =\frac{180(10)}{12} \\
& =150
\end{aligned}
$$

The angle between adjacent sides is $150^{\circ}$.
c) The angle between adjacent roof supports is $\frac{360^{\circ}}{12}$, or $30^{\circ}$.
d) Answers will vary.
e) The angle between adjacent roof supports in a gazebo with six sides is $\frac{360^{\circ}}{6}$, or $60^{\circ}$.

## Chapter 7 Section 3 <br> Question 14 Page 392

a) Click here to load the sketch.


The shape formed is a pentagon.
b) To construct a regular octagon using this method, rotate the line segment 7 times through an angle of $45^{\circ}$.
c) Use an angle of $\frac{360^{\circ}}{20}$, or $18^{\circ}$ for a regular 20-sided figure.
d) The angle of rotation is $360^{\circ}$ divided by the number of sides.

## Chapter 7 Section $3 \quad$ Question 15 Page 393

Solutions for the Achievement Checks are shown in the Teacher's Resource.

All regular polygons are convex. The angle between adjacent sides must be less than $180^{\circ}$.
Chapter 7 Section $3 \quad$ Question 17 Page 393
Answers will vary. A sample answer is shown.
The formula for the sum of the interior angles applies to concave polygons. An $n$-sided concave polygon can be divided into $n-2$ triangles by diagonals from two or more vertices. Alternatively, you can use The Geometer's Sketchpad ${ }^{\Omega}$ to measure angle sums in various concave polygons.

## Chapter 7 Section $3 \quad$ Question 18 Page 393

Answers will vary. Sample answers are shown.
In the first diagram, angles on the same chord are equal. In the second diagram, the angle at the centre is double the angle at the circumference.


Chapter 7 Section $3 \quad$ Question 19 Page 393
Answers will vary.
Chapter 7 Section $3 \quad$ Question $20 \quad$ Page 393
$\angle \mathrm{ABC}$ and $\angle \mathrm{BCA}$ both measure the same as angle $x$.

$$
\begin{aligned}
3 x+x+x & =180^{\circ} \\
5 x & =180^{\circ} \\
\frac{5 x}{5} & =\frac{180^{\circ}}{5} \\
x & =36^{\circ}
\end{aligned}
$$


$\angle \mathrm{BCA}=36^{\circ}$

Answer B.

## Chapter 7 Section 3

Question 21 Page 393
Each diagonal requires one pair of vertices. There are $12 \times 11$, or 132 pairs of vertices. However, each one has been counted twice. That leaves $\frac{132}{2}$, or 66 . However, this also counts the edges of the polygon. The number of possible diagonals is $66-12$, or 54 . Answer A.

## Chapter 7 Section 4 Midpoints and Medians in Triangles

Chapter 7 Section 4
a) $X Y=\frac{1}{2} B C$

$$
\begin{aligned}
& =\frac{1}{2}(4) \\
& =2
\end{aligned}
$$

The length of XY is 2 cm .
b) $\mathrm{XY}=2 \mathrm{VW}$

$$
\begin{aligned}
& =2(6) \\
& =12
\end{aligned}
$$

The length of XY is 12 cm .

## Question 1 Page 398



## Chapter 7 Section 4

Question 2 Page 398
a) The area of $\triangle \mathrm{PQS}$ is half the area of $\triangle \mathrm{PQR}$. The area of $\triangle P Q R$ is $16 \mathrm{~cm}^{2}$. So, the area of $\triangle \mathrm{PQS}$ is $8 \mathrm{~cm}^{2}$.
b) The area of $\triangle \mathrm{QSR}$ is half the area of $\triangle \mathrm{PQR}$. The area of $\triangle \mathrm{PQR}$ is $16 \mathrm{~cm}^{2}$. So, the area of $\triangle \mathrm{QSR}$ is $8 \mathrm{~cm}^{2}$.


## Chapter 7 Section 4 <br> Question 3 Page 398

a) The area of $\triangle \mathrm{WZY}$ is equal to the area of $\triangle \mathrm{XYZ}$. The area of $\triangle X Y Z$ is $19 \mathrm{~cm}^{2}$. So, the area of $\triangle W Z Y$ is $19 \mathrm{~cm}^{2}$.
b) The area of $\triangle \mathrm{WXY}$ is double the area of $\triangle \mathrm{XYZ}$. The area of $\triangle \mathrm{XYZ}$ is $19 \mathrm{~cm}^{2}$. So, the area of $\triangle \mathrm{WXY}$ is $38 \mathrm{~cm}^{2}$.


## Chapter 7 Section 4

Question 4 Page 398
The length of the cross-brace $A B$ is $\frac{1}{2} \times 5$, or 2.5 m .


## Chapter 7 Section 4

Question 5 Page 399
a) Answers will vary.
b) You can fold along the median and see if the equal sides line up.
c) You can construct the isosceles triangle and median, and
 then measure the angle on either side of the median.
d) Click here to load the sketch.


The median bisects the angle.

If point D is moved close to vertex $\mathrm{A}, \angle \mathrm{ADC}$ is obtuse.


## Chapter 7 Section 4

Question 7 Page 399
a) Refer to the diagram. In this case, the $60^{\circ}$ angle is opposite the secondlongest side.
b) Refer to the diagram. In this case, the $60^{\circ}$ angle is opposite the secondlongest side.

c) Since the angles sum to $180^{\circ}$, one of the angles must be larger than $60^{\circ}$ and the third angle must be smaller. The largest angle is opposite the largest side, and the smallest angle is opposite the smallest side. Therefore, the $60^{\circ}$ angle is opposite the second-longest side.

Chapter 7 Section $4 \quad$ Question $8 \quad$ Page 399
Since $\triangle \mathrm{ABD}$ and $\triangle \mathrm{ACD}$ are congruent (ASA or SAS), the perpendicular at D must pass through A .


## Chapter 7 Section 4

Question 9 Page 399
$\Delta \mathrm{AGC}, \Delta \mathrm{CGB}$, and $\Delta \mathrm{BGA}$ are not equilateral triangles. The centre angle at G is obtuse for all three triangles.


## Chapter 7 Section 4

Question 10 Page 399
Medians intersect at a point for all triangles. You can verify this using geometry software. A sample sketch is shown. Click here to load the sketch.


Chapter 7 Section 4
Question 11 Page 400
a) $\triangle \mathrm{BEG}$ and $\triangle \mathrm{CEG}$ have the same area because GE is a median of $\triangle \mathrm{BGC}$.
b) The same logic applies as in part a), since DG and GF are also medians.
c) AE is a median, so $\triangle \mathrm{ABE}$ has the same area as $\triangle \mathrm{ACE}$. Since the areas of $\triangle \mathrm{BEG}$ and $\triangle \mathrm{CEG}$ are equal, the areas of $\triangle \mathrm{ABG}$ and
 $\triangle \mathrm{ACG}$ are also equal. The areas of the two triangles in $\triangle \mathrm{ABG}$ are equal, as are the areas of the two triangles in $\triangle \mathrm{ACG}$. Therefore, $\triangle \mathrm{ADG}, \triangle \mathrm{BDG}, \triangle \mathrm{AFG}$, and $\Delta \mathrm{CFG}$ each have an area equal to half that of $\triangle \mathrm{ABG}$. Comparing $\triangle \mathrm{BCF}$ and $\triangle \mathrm{BAF}$ shows that $\triangle \mathrm{BEG}$ and $\triangle \mathrm{CEG}$ also each have an area half that of $\triangle \mathrm{ABG}$.

## Chapter 7 Section $4 \quad$ Question 12 Page 400

a) Answers will vary. Start with an equilateral triangle, shown in black. Connect the midpoints of the sides. Shade the smaller triangle formed, shown in red. Repeat for each of the three smaller black triangles. Continue the process.

b) After the first step, $\frac{1}{4}$ of the original triangle is shaded. After the second step, $\frac{1}{4}+\frac{1}{4}\left(\frac{3}{4}\right)$ is shaded. After the third step, $\frac{1}{4}+\frac{1}{4}\left(\frac{3}{4}\right)+\frac{1}{4}\left(\frac{3}{4}\right)^{2}$ is shaded.
c) After the fourth step, $\frac{1}{4}+\frac{1}{4}\left(\frac{3}{4}\right)+\frac{1}{4}\left(\frac{3}{4}\right)^{2}+\frac{1}{4}\left(\frac{3}{4}\right)^{3}$, or about $0.6836(68.36 \%)$ is shaded.

## Chapter 7 Section 4 <br> Question 13 Page 400

a) The right bisectors of a triangle intersect at a single point. You can verify this using geometry software. A sample sketch is shown. Click here to load the sketch.
b) You can draw a circle from the point in part a) that passes through all three vertices of the triangle.


## Chapter 7 Section 4 <br> Question 14 Page 400

a) The angle bisectors of a triangle always intersect at a point. You can verify this using geometry software. A sample sketch is shown. Click here to load the sketch.
b) You can construct a circle from this point that has a radius equal to the minimum distance from the point to any side of the triangle.


## Chapter 7 Section 4 <br> Question 15 Page 400

For an obtuse triangle, the intersection of the right bisectors of the sides is outside the triangle.


The longest side cannot be equal to or greater than the sum of the two shortest sides. Cases c), d) and g ) are not possible.

## Chapter 7 Section 5 Midpoints and Diagonals in Quadrilaterals

Chapter 7 Section 5
Question 1 Page 405
The midpoints of the sides of quadrilateral ABCD are joined to produce parallelogram EFGH. So, EF is parallel to HG, and EH is parallel to FG.


## Chapter 7 Section $5 \quad$ Question 2 Page 405

The diagonals of parallelogram ABCD bisect each other. $\mathrm{So}, \mathrm{BE}=\mathrm{DE}$, or 6 cm , and $\mathrm{CE}=\mathrm{AE}$, or 8 cm .
Also, $\mathrm{AC}=2 \mathrm{AE}$, or 16 cm , and $\mathrm{BD}=2 \mathrm{DE}$, or 12 cm .


Chapter 7 Section 5
Question 3 Page 405
The diagonals of parallelogram PQRS bisect each other.

$$
\begin{aligned}
\mathrm{PT} & =\frac{1}{2} \mathrm{PR} \\
& =\frac{1}{2}(14) \\
& =7
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{ST} & =\frac{1}{2} \mathrm{QS} \\
& =\frac{1}{2}(10) \\
& =5
\end{aligned}
$$



The length of PT is 7 m . The length of ST is 5 m .

## Chapter 7 Section $5 \quad$ Question 4 Page 405

The shaft and a line from the top of the jack to its base form diagonals of the parallelogram. Since the diagonals of a parallelogram bisect each other, the top of the jack will be 2(20), or 40 cm high when the shaft is 20 cm from the base.


## Chapter 7 Section 5 <br> Question 5 Page 405

a) The diagonals bisect each other in all four.

b) The diagonals have the same length in the rectangle and the square.
c) The diagonals intersect at $90^{\circ}$ in the rhombus and the
 square.
d) The diagonals bisect each other at $90^{\circ}$ in the rhombus and the square.

Chapter 7 Section 5
Question 6 Page 405
EFGH is a rhombus when ABCD is a rectangle. You can verify this using geometry software. A sample sketch is shown. Click here to load the sketch.


## Chapter 7 Section $5 \quad$ Question $7 \quad$ Page 405

a) This is false. Any quadrilateral with four unequal sides is a counter-example. A sample is shown.
b) This is true. Any line segment joining opposite midpoints creates two parallelograms with equal heights and bases.


## Chapter 7 Section 5

Question 8 Page 406
a) WXYZ is a square.
b) The area of WXYZ is half the area of PQRS. The diagonals of WXYZ form four triangles that are congruent to the triangles outside WXYZ.

c) If PQRS is stretched into a rectangle, WXYZ becomes a rhombus.
d) The area relationship between WXYZ and PQRS will not change. All the triangles are still congruent.


## Chapter 7 Section 5 <br> Question 9 Page 406

a) The diagram is shown.
b) The diagonals intersect at $90^{\circ}$.
c) EFGH is a rectangle.
d) Answers will vary. A sample answer is shown.

The area of ABCD is twice the area of EFGH.
e) You can use geometry software to
 measure the
areas of ABCD and EFGH. A sample sketch is shown. Click here to load the sketch.

## Chapter 7 Section 5 Question 10 Page 406

Answers will vary. Sample answers are shown.
a) The area of EFGH is half the area of ABCD .
b) Use geometry software to compare the areas. A sample sketch is shown. Click here to load the sketch.


## Chapter 7 Section 5

Question 11 Page 406
a)

b) By the Pythagorean theorem, $\mathrm{AD}^{2}+\mathrm{AB}^{2}=\mathrm{BD}^{2}=\mathrm{CD}^{2}+\mathrm{AB}^{2}$. So, $\mathrm{AD}=\mathrm{CD}$.
c) $\triangle \mathrm{ABD}$ is congruent to $\triangle \mathrm{CBD}$ (SSS), so $\angle \mathrm{ABD}$ equals $\angle \mathrm{CBD}$.

## Chapter 7 Section $5 \quad$ Question 12 Page 407

Solutions for the Achievements Checks are shown in the Teacher's Resource.

## Chapter 7 Section 5 <br> Question 13 Page 407

In any parallelogram $\mathrm{ABCD}, \triangle \mathrm{ABC}$ and $\triangle \mathrm{CDA}$ are congruent (SSS), as are $\triangle \mathrm{ABD}$ and $\triangle \mathrm{CDB}$.
Thus, $\angle \mathrm{CAB}=\angle \mathrm{ACD}, \angle \mathrm{CDB}=$ $\angle \mathrm{ABD}, \angle \mathrm{ACB}=\angle \mathrm{CAD}$, and $\angle \mathrm{ADB}=\angle \mathrm{CBD} . \triangle \mathrm{ABE}$ and $\triangle \mathrm{CDE}$ are congruent (ASA), so $\mathrm{DE}=\mathrm{BE}$ and $\mathrm{AE}=\mathrm{CE}$.



## Chapter 7 Section 5 <br> Question 15 Page 407

a) The five triangles formed by two adjacent sides of PQRST, $\triangle \mathrm{ABC}, \triangle \mathrm{BCD}, \triangle \mathrm{CDE}, \triangle \mathrm{DEA}$ and $\triangle \mathrm{EAB}$, are isosceles and congruent (SAS). So, all the acute angles in these triangles are equal.
$\triangle \mathrm{ABR}, \triangle \mathrm{BCS}, \triangle \mathrm{CDT}, \triangle \mathrm{DEP}$, and $\triangle \mathrm{EAQ}$ are all congruent (ASA). The obtuse angles of these triangles are opposite to the interior angles of PQRST. Thus, these angles are all equal. $\triangle \mathrm{DTP}, \triangle \mathrm{EPQ}, \triangle \mathrm{AQR}, \triangle \mathrm{BRS}$, and $\Delta \mathrm{CST}$ are all congruent
 (SAS), so the sides of PQRST are all equal. PQRST is a regular pentagon.
b) PQRST is similar to ABCDE . Both are regular pentagons.
c) Using direct measurement from the diagram, the ratio is about $\frac{1.6}{0.6}$, or about 2.7.
d) The ratio of areas is $2.7^{2}$, or about 7 .
e) Geometry software produces results similar to the conjectures in parts c) and d). A sample sketch is shown. Click here to load the sketch.


## Chapter 7 Section $5 \quad$ Question 16 Page 407

a) There are 10 choices for the first point, and for each of these there are 9 choices for the second point. However, this counts each line segment twice. The number of line segments that can be constructed between 10 points is $\frac{10 \times 9}{2}$, or 45 .
b) Using reasoning similar to part a), the number of handshakes is $\frac{12 \times 11}{2}$, or 66 .

## Chapter 7 Section $5 \quad$ Question 17 Page 407

a) Using reasoning similar to question 10 , the number of line segments is $\frac{n(n-1)}{2}$.
b) To obtain the number of diagonals, use the expression from part a), and subtract the line segments that form the edges of the polygon:

$$
\begin{aligned}
\frac{n(n-1)}{2}-n & =\frac{n^{2}-n}{2}-\frac{2 n}{2} \\
& =\frac{n^{2}-3 n}{2} \\
& =\frac{n(n-3)}{2}
\end{aligned}
$$

## Chapter 7 Review

## Chapter 7 Review

a) $u=70^{\circ}+40^{\circ}$

$$
=110^{\circ}
$$

## Question 1 Page 408


b) $v+130^{\circ}+105^{\circ}=360^{\circ}$
$v+235^{\circ}=360^{\circ}$
$v=360^{\circ}-235^{\circ}$
$v=125^{\circ}$

c)

$$
\begin{aligned}
y & =180^{\circ}-45^{\circ} \\
& =135^{\circ} \\
z & =180^{\circ}-150^{\circ} \\
& =30^{\circ}
\end{aligned}
$$

$$
w=45^{\circ}+30^{\circ}
$$

$$
=75^{\circ}
$$

$$
x=180^{\circ}-75^{\circ}
$$

$$
=105^{\circ}
$$

Chapter 7 Review

$$
\begin{aligned}
2 x-15+3 x-17 & =4 x+12 \\
5 x-32 & =4 x+12 \\
5 x-32+32-4 x & =4 x+12+32-4 x \\
x & =44
\end{aligned}
$$

Question 2 Page 408

$$
\begin{aligned}
4 x+12 & =4(44)+12 \\
& =188
\end{aligned}
$$

Since the exterior angle must be less than $180^{\circ}$, this angle relationship is not possible.

## Chapter 7 Review <br> Question 3 Page 408

a) A triangle with an acute exterior angle occurs for any obtuse triangle.
b) It is not possible to have two acute exterior angles. In order to sum to $180^{\circ}$; the third exterior angle would have to be greater than $180^{\circ}$.

c) Any acute triangle has three obtuse exterior angles.
d) This is not possible. The sum of the exterior angles would be less than $360^{\circ}$.


## Chapter 7 Review

Question 4 Page 408
a) $a+80^{\circ}+115^{\circ}+65^{\circ}=360^{\circ}$

$$
\begin{aligned}
a+260^{\circ} & =360^{\circ} \\
a & =360^{\circ}-260^{\circ} \\
a & =100^{\circ}
\end{aligned}
$$


b)

$$
\begin{aligned}
b & =180^{\circ}-75^{\circ} \\
& =105^{\circ} \\
c & =180^{\circ}-110^{\circ} \\
& =70^{\circ}
\end{aligned}
$$

$$
d=180^{\circ}-95^{\circ}
$$

$$
=85^{\circ}
$$

$$
\begin{aligned}
e+105^{\circ}+70^{\circ}+85^{\circ} & =360^{\circ} \\
e+260^{\circ} & =360^{\circ} \\
e & =360^{\circ}-260^{\circ} \\
e & =100^{\circ} \\
f & =180^{\circ}-100^{\circ} \\
& =80^{\circ}
\end{aligned}
$$

c) Since opposite angles in a parallelogram are equal, $z=128^{\circ}$ and $x=y$.

Adjacent angles in a parallelogram are supplementary.

$$
\begin{aligned}
y+128^{\circ} & =180^{\circ} \\
y & =180^{\circ}-128^{\circ} \\
y & =52^{\circ} \\
x & =y \\
& =52^{\circ}
\end{aligned}
$$



## Chapter 7 Review

## Question 5 Page 408

a) An example of a quadrilateral with three obtuse interior angles is one with three $110^{\circ}$ angles and one $30^{\circ}$ angle.
b) It is not possible to have a quadrilateral with four obtuse interior angles. The sum of the interior angles would be greater than $360^{\circ}$.
c) An example of a quadrilateral with three obtuse exterior angles is one with three $100^{\circ}$ angles and one $60^{\circ}$ angle.
d) A quadrilateral with four obtuse exterior angles is not possible. The sum of the exterior angles would be greater than $360^{\circ}$.


Chapter 7 Review
Question 6 Page 409
a) $S=180(n-2)$

$$
\begin{aligned}
& =180(6-2) \\
& =180(4) \\
& =720
\end{aligned}
$$

The sum of the interior angles of a hexagon is $720^{\circ}$.
b) $S=180(n-2)$

$$
=180(8-2)
$$

$$
=180(6)
$$

$$
=1080
$$

The sum of the interior angles of an octagon is $1080^{\circ}$.
c) $S=180(n-2)$
$=180(12-2)$
$=180(10)$
$=1800$
The sum of the interior angles of a dodecagon is $1800^{\circ}$.
a) $\frac{180(n-2)}{n}=\frac{180(5-2)}{5}$

$$
\begin{aligned}
& =\frac{180(3)}{5} \\
& =108
\end{aligned}
$$

Each interior angle of a pentagon measures $108^{\circ}$.
b) $\frac{180(n-2)}{n}=\frac{180(9-2)}{9}$

$$
\begin{aligned}
& =\frac{180(7)}{9} \\
& =140
\end{aligned}
$$

Each interior angle of a nonagon measures $140^{\circ}$.
c) $\frac{180(n-2)}{n}=\frac{180(16-2)}{16}$

$$
\begin{aligned}
& =\frac{180(14)}{16} \\
& =157.5
\end{aligned}
$$

Each interior angle of a hexadecagon measures $157.5^{\circ}$.
Chapter 7 Review
Question 8 Page 409

$$
\begin{aligned}
180(n-2) & =168 n \\
180 n-360 & =168 n \\
180 n-360-168 n+360 & =168 n-168 n+360 \\
12 n & =360 \\
\frac{12 n}{12} & =\frac{360}{12} \\
n & =30
\end{aligned}
$$

The polygon has 30 sides.
a)

b) Answers will vary. A sample answer is shown.

Use geometry software. Construct a line segment, and rotate it around one end point 7 times at an angle of $45^{\circ}$. Join the ends of the segments. Click here to load the sketch.

## Chapter 7 Review

Question 10 Page 409
DE connects the midpoints of AB and AC .
Therefore, the base and altitude of $\triangle \mathrm{ADE}$ are half those of $\triangle \mathrm{ABC}$. The area of $\triangle \mathrm{ADE}$ is $\left(\frac{1}{2}\right)^{2}$, or $\frac{1}{4}$ the area of $\triangle \mathrm{ABC}$.


## Chapter 7 Review <br> Question 11 Page 409

a) Each median divides the triangle into two triangles. All of these triangles are congruent (SAS). The medians are equal in length since they are sides of the congruent triangles.
b) This is generally false. Any scalene triangle is a counter-example.


Chapter 7 Review
Answers will vary.
Chapter 7 Review
Question 13 Page 409

Answers will vary.

## Chapter 7 Chapter Test

Chapter 7 Chapter Test Question 1 Page 410

$$
\begin{aligned}
x+110^{\circ}+110^{\circ} & =360^{\circ} \\
x+220^{\circ} & =360^{\circ} \\
x & =360^{\circ}-220^{\circ} \\
x & =140^{\circ}
\end{aligned}
$$

Answer C.
Chapter 7 Chapter Test Question 2 Page 410
The interior angle at B is $180^{\circ}-119^{\circ}$, or $61^{\circ}$.

$$
\begin{aligned}
x+51^{\circ}+61^{\circ} & =180^{\circ} \\
x+112^{\circ} & =180^{\circ} \\
x & =180^{\circ}-112^{\circ} \\
x & =68^{\circ}
\end{aligned}
$$

Answer B.
Chapter 7 Chapter Test Question 3 Page 410
The sum of the exterior angles of a convex polygon is always $360^{\circ}$. Answer B.

## Chapter 7 Chapter Test $\quad$ Question 4 Page 410

The area of $\triangle \mathrm{ADE}$ is one-quarter of the area of $\triangle \mathrm{ABC}$, or onethird of the area of trapezoid DBCE. Answer D.


## Chapter 7 Chapter Test Question 5 Page 410

The diagonals of a rectangle bisect each other. Answer B.

## Chapter 7 Chapter Test Question 6 Page 410

a) $a=61^{\circ}+34^{\circ}$

$$
=95^{\circ}
$$


b) $b+110^{\circ}+85^{\circ}+75^{\circ}=360^{\circ}$

$$
\begin{aligned}
b+270^{\circ} & =360^{\circ} \\
b & =360^{\circ}-270^{\circ} \\
b & =90^{\circ}
\end{aligned}
$$


c)

$$
\begin{aligned}
C & =180^{\circ}-35^{\circ} \\
& =145^{\circ}
\end{aligned}
$$

$$
d=180^{\circ}-120^{\circ}
$$

$$
=60^{\circ}
$$

$$
\begin{aligned}
e+60^{\circ}+35^{\circ} & =180^{\circ} \\
e+95^{\circ} & =180^{\circ} \\
e & =180^{\circ}-95^{\circ} \\
e & =85^{\circ} \\
f & =35^{\circ}+60^{\circ} \\
& =95^{\circ}
\end{aligned}
$$

d)

$$
\begin{aligned}
v & =180^{\circ}-125^{\circ} \\
& =55^{\circ} \\
w & =180^{\circ}-130^{\circ} \\
& =50^{\circ} \\
x & =180^{\circ}-105^{\circ} \\
& =75^{\circ}
\end{aligned}
$$

$$
\begin{aligned}
y+55^{\circ}+130^{\circ}+105^{\circ} & =360^{\circ} \\
y+290^{\circ} & =360^{\circ} \\
y & =360^{\circ}-290^{\circ} \\
y & =70^{\circ} \\
z & =180^{\circ}-70^{\circ} \\
& =110^{\circ}
\end{aligned}
$$

## Chapter 7 Chapter Test $\quad$ Question $7 \quad$ Page 410

Answers will vary. Sample answers are shown.
a) For a parallelogram:

The sum of the interior angles is $360^{\circ}$. Opposite interior angles are equal. Adjacent interior angles are supplementary.
b) For a parallelogram:

The diagonals bisect each other and bisect the area of the parallelogram.

## Chapter 7 Chapter Test Question 8 Page 410

A quadrilateral with a pair of equal opposite angles is shown. However, it is not a parallelogram.


## Chapter 7 Chapter Test <br> Question 9 Page 410

$$
\begin{aligned}
S & =180(n-2) \\
& =180(14-2) \\
& =180(12) \\
& =2160
\end{aligned}
$$

The sum of the interior angles of a 14 -sided polygon is $2160^{\circ}$.

## Chapter 7 Chapter Test Question 10 Page 410

$$
\begin{aligned}
180(n-2) & =2340 \\
180 n-360 & =2340 \\
180 n-360+360 & =2340+360 \\
180 n & =2700 \\
\frac{180 n}{180} & =\frac{2700}{180} \\
n & =15
\end{aligned}
$$

The polygon has 15 sides.

## Chapter 7 Chapter Test <br> Question 11 Page 410

Run the fence along the median from the right vertex of the lot.


## Chapter 7 Chapter Test Question 12 Page 410

a) The shape is a hexagon.
b) The hexagon is regular. The sides are equal, and measuring with a protractor shows that the interior angles are equal.
c) $S=180(n-2)$

$$
\begin{aligned}
& =180(6-2) \\
& =180(4) \\
& =720
\end{aligned}
$$

Each interior angle measures $\frac{720^{\circ}}{6}$, or $120^{\circ}$. d) For regular polygons, the measure of the interior angles increases as the number of sides increases. Manpreet should increase the measure of each interior angle.


