Chapter 7

Geometric Relationships

Chapter 7 Get Ready

Chapter 7 Get Ready Question 1 Page 362

a) Two sides are equal. The triangle is isosceles.

b) No two sides are equal. The triangle is scalene.

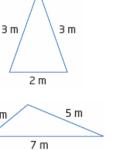


a) All three angles are equal and acute. The triangle is equilateral and acute.

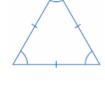
b) Two angles are equal, and one angle is obtuse. The triangle is isosceles and obtuse.

Chapter 7 Get Ready Question 3 Page 363

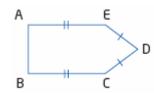
- a) Polygon ABCDE is an irregular pentagon.
- **b**) Polygon PQRSTU is a regular hexagon.

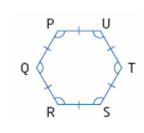


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Chapter 7 Get Ready Question 4 Page 363

a) Opposite pairs of sides are parallel. WXYZ is a parallelogram.

b) All four sides are equal, but no angles are right angles. ABCD is a rhombus.

Chapter 7 Get Ready Question 5 Page 363 a) $\angle Z + 72^\circ + 58^\circ = 180^\circ$

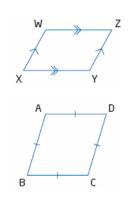
 $\angle Z = 180^{\circ} - 72^{\circ} - 58^{\circ}$ $\angle Z = 50^{\circ}$

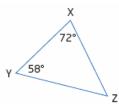
b)
$$\angle R + 90^\circ + 35^\circ = 180^\circ$$

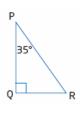
 $\angle R = 180^\circ - 90^\circ - 35^\circ$
 $\angle R = 55^\circ$

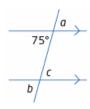
Chapter 7 Get Ready Question 6 Page 363

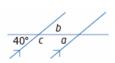
- a) $a = 75^{\circ}$, opposite angles.
- $b = 75^{\circ}$, corresponding angles.
- $c = 75^{\circ}$, alternate angles.
- **b**) $a = 40^{\circ}$, corresponding angles.
- $b = 40^{\circ}$, opposite angles.
- $c = 140^{\circ}$, supplementary angles (also co-interior with *a*).

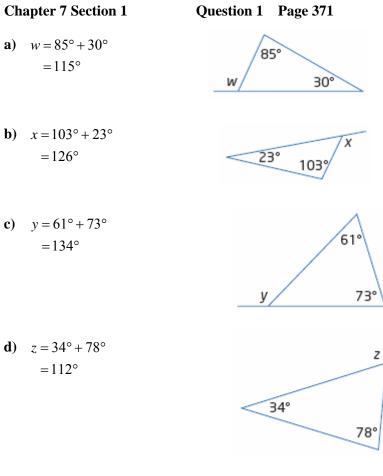










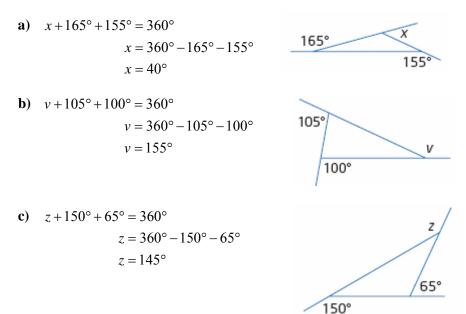


Chapter 7 Section 1: Angle Relationships in Triangles



Chapter 7 Section 1

Question 2 Page 371



Chapter 7 Section 1 Question 3 Page 371

$$x + 120^{\circ} + 70^{\circ} = 360^{\circ}$$
 Answer C.
 $x = 360^{\circ} - 120^{\circ} - 70^{\circ}$
 $x = 170^{\circ}$

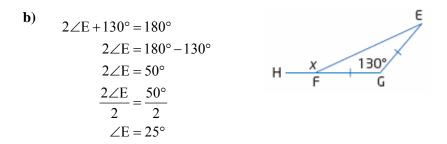
Chapter 7 Section 1

a)

Question 4 Page 372

$$\angle A + 75^{\circ} + 75^{\circ} = 180^{\circ}$$

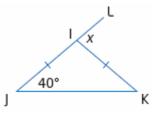
 $\angle A = 180^{\circ} - 75^{\circ} - 75^{\circ}$
 $\angle A = 30^{\circ}$
 $x = 75^{\circ} + 30^{\circ}$
 $= 105^{\circ}$
B
 75°
C
D



$$x = 25^{\circ} + 130^{\circ}$$
$$= 155^{\circ}$$

c)
$$x = 40^{\circ} + 40^{\circ}$$

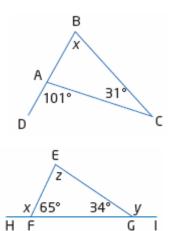
= 80°



Question 5 Page 372

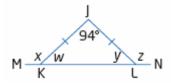
a)
$$x + 31^{\circ} = 101^{\circ}$$

 $x = 101^{\circ} - 31^{\circ}$
 $x = 70^{\circ}$



b)

$$z + 65^{\circ} + 34^{\circ} = 180^{\circ}$$
$$z = 180^{\circ} - 65^{\circ} - 34^{\circ}$$
$$z = 81^{\circ}$$
$$x = 81^{\circ} + 34^{\circ}$$
$$= 115^{\circ}$$
$$y = 81^{\circ} + 65^{\circ}$$
$$= 146^{\circ}$$





$$w = y$$

$$2w + 94^{\circ} = 180^{\circ}$$

$$2w = 180^{\circ} - 94^{\circ}$$

$$2w = 86^{\circ}$$

$$\frac{2w}{2} = \frac{86^{\circ}}{2}$$

$$w = 43^{\circ}$$

$$y = 43^{\circ}$$

$$x = 94^{\circ} + 43^{\circ}$$

$$= 137^{\circ}$$

$$z = 94^{\circ} + 43^{\circ}$$

$$= 137^{\circ}$$

$$= 136^{\circ}$$

$$a = c$$

$$2a + 136^{\circ} = 180^{\circ}$$

$$2a = 180^{\circ} - 136^{\circ}$$

$$2a = 44^{\circ}$$

$$\frac{2a}{2} = \frac{44^{\circ}}{2}$$

$$a = 22^{\circ}$$

$$c = 22^{\circ}$$

$$V \xrightarrow{c} \frac{d}{x} \frac{d}{d} \frac{d}{$$

$$e = 44^{\circ}$$

 $b + 44^{\circ} + 44^{\circ} = 180^{\circ}$

 $b + 88^{\circ} = 180^{\circ}$

$$z = 92^{\circ} + 44^{\circ}$$

=136°

 $b = 180^{\circ} - 88^{\circ}$

 $d = 92^\circ + 44^\circ$

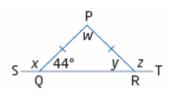
b = 92°

$$w + 88^{\circ} = 180^{\circ}$$
$$w = 180^{\circ} - 88^{\circ}$$
$$w = 92^{\circ}$$
$$x = 92^{\circ} + 44^{\circ}$$
$$= 136^{\circ}$$

 $w + 44^{\circ} + 44^{\circ} = 180^{\circ}$

. . .

y = 44°



d)

e)

Case #1:

$$a = 180^{\circ} - 140^{\circ}$$

= 40°
$$b + 40^{\circ} + 40^{\circ} = 180^{\circ}$$

$$b + 80^{\circ} = 180^{\circ}$$

$$b = 180^{\circ} - 80^{\circ}$$

$$b = 100^{\circ}$$

$$c = 40^{\circ} + 40^{\circ}$$

= 80°
$$d = 100^{\circ} + 40^{\circ}$$

= 140°

The other exterior angles measure 80° and 140° .

Case #2:

$$b = 180^{\circ} - 140^{\circ}$$

= 40°
$$2a + 40^{\circ} = 180^{\circ}$$

$$2a + 180^{\circ} - 40^{\circ}$$

$$2a = 140^{\circ}$$

$$\frac{2a}{2} = \frac{140^{\circ}}{2}$$

$$a = 70^{\circ}$$

$$c = 40^{\circ} + 70^{\circ}$$

= 110°
$$d = 70^{\circ} + 40^{\circ}$$

= 110°

The other exterior angles measure 110° and 110° .

Chapter 7 Section 1 Question 7 Page 372

 $mean = \frac{360^{\circ}}{3}$ $= 120^{\circ}$

Chapter 7 Section 1 Question 8 Page 372

Isosceles triangles have 2 exterior angles equal. Equilateral triangles have 3 exterior angles equal.

Chapter 7 Section 1 Question 9 Page 372

a) A triangle cannot have two obtuse interior angles. The sum of two obtuse angles is greater than 180°.

b) Any acute triangle will have three obtuse exterior angles.

Chapter 7 Section 1 Question 10 Page 373



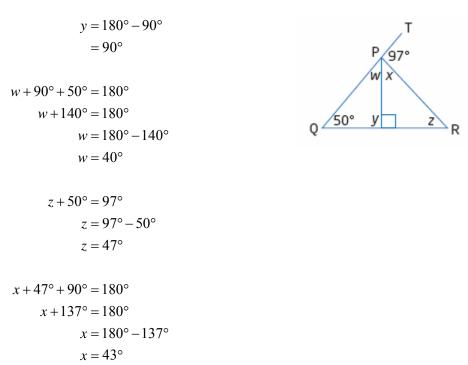
a)
$$\angle DAC = 180^{\circ} - 5^{\circ}$$

= 175°

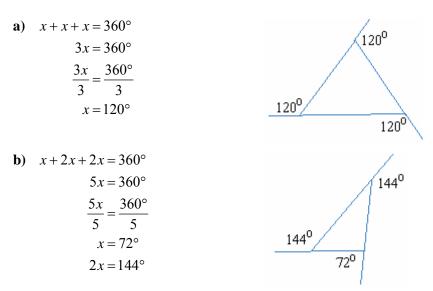
b)
$$x + 90^{\circ} + 5^{\circ} = 180^{\circ}$$

 $x + 95^{\circ} = 180^{\circ}$
 $x = 180^{\circ} - 95^{\circ}$
 $x = 85^{\circ}$
 $y = 90^{\circ} + 5^{\circ}$
 $= 95^{\circ}$

The interior angle at the top of the ramp measures 85°, while the exterior angle measures 95°.



Question 12 Page 373



c)
$$x + 2x + 3x = 360^{\circ}$$
$$6x = 360^{\circ}$$
$$\frac{6x}{6} = \frac{360^{\circ}}{6}$$
$$x = 60^{\circ}$$
$$2x = 120^{\circ}$$
$$3x = 180^{\circ}$$

This triangle is not possible. An exterior angle cannot equal 180°.

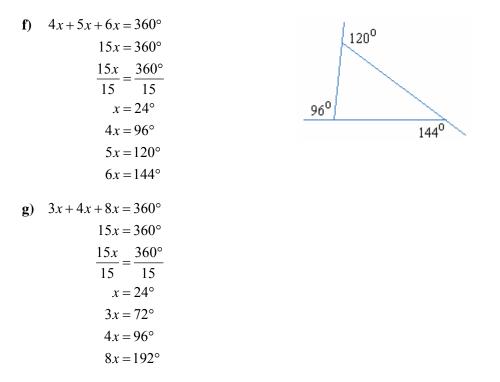
d)
$$x + x + 2x = 360^{\circ}$$
$$4x = 360^{\circ}$$
$$\frac{4x}{4} = \frac{360^{\circ}}{4}$$
$$x = 90^{\circ}$$
$$2x = 180^{\circ}$$

This triangle is not possible. An exterior angle must be less than 180°.

e)
$$3x + 4x + 5x = 360^{\circ}$$

 $12x = 360^{\circ}$
 $\frac{12x}{12} = \frac{360^{\circ}}{12}$
 $x = 30^{\circ}$
 $3x = 90^{\circ}$
 $4x = 120^{\circ}$
 $5x = 150^{\circ}$

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This triangle is not possible. An exterior angle cannot exceed 180°.

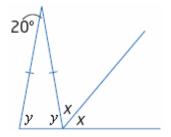
Chapter 7 Section 1 Question 13 Page 373

Hexaflexagons are paper hexagons folded from strips of paper which reveal different faces as they are flexed. You can download templates and instructions for making a hexaflexagon.

Question 14 Page 373

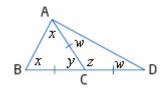
Chapter 7 Section 1

$2y + 20^{\circ} = 180^{\circ}$ $2y = 180^{\circ} - 20^{\circ}$ $2y = 160^{\circ}$ $\frac{2y}{2} = \frac{160^{\circ}}{2}$ $y = 80^{\circ}$ $2x = 20^{\circ} + 80^{\circ}$ $2x = 100^{\circ}$ $\frac{2x}{2} = \frac{100^{\circ}}{2}$ $x = 50^{\circ}$

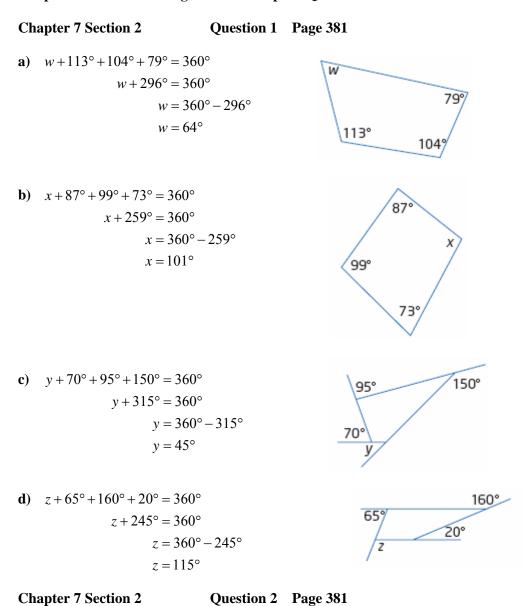


Answer B.

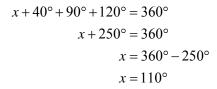
 $\angle ABC + \angle ADC = x + w$ $y + z = 180^{\circ}$ $2x + y + z + 2w = 360^{\circ}$ $2x + 180^{\circ} + 2w = 360^{\circ}$ $2x + 2w = 360^{\circ} - 180^{\circ}$ $2x + 2w = 180^{\circ}$ $\frac{2x + 2w}{2} = 180^{\circ}$ $\frac{2x + 2w}{2} = \frac{180^{\circ}}{2}$ $\frac{2x}{2} + \frac{2w}{2} = 90^{\circ}$ $x + w = 90^{\circ}$



Answer C.



Chapter 7 Section 2 Angle Relationships in Quadrilaterals



Answer A.

Chapter 7 Section 2 Question 3 Page 381

$$x + 80^{\circ} + 100^{\circ} + 120^{\circ} = 360^{\circ}$$

 $x + 300^{\circ} = 360^{\circ}$
 $x = 360^{\circ} - 300^{\circ}$
 $x = 60^{\circ}$

Answer B.

Chapter 7 Section 2 Question 4 Page 381

a)
$$\angle D + 100^{\circ} + 75^{\circ} + 50^{\circ} = 360^{\circ}$$

 $\angle D + 225^{\circ} = 360^{\circ}$
 $\angle D = 360^{\circ} - 225^{\circ}$
 $\angle D = 135^{\circ}$

b)
$$\angle C + 20^{\circ} + 35^{\circ} + 150^{\circ} = 360^{\circ}$$

 $\angle C + 205^{\circ} = 360^{\circ}$
 $\angle C = 360^{\circ} - 205^{\circ}$
 $\angle C = 155^{\circ}$

c)
$$\angle B + 70^{\circ} + 70^{\circ} + 70^{\circ} = 360^{\circ}$$

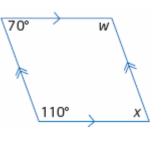
 $\angle B + 210^{\circ} = 360^{\circ}$
 $\angle B = 360^{\circ} - 210^{\circ}$
 $\angle B = 150^{\circ}$

d)
$$\angle A + 90^{\circ} + 90^{\circ} + 90^{\circ} = 360^{\circ}$$

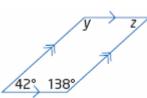
 $\angle A + 270^{\circ} = 360^{\circ}$
 $\angle A = 360^{\circ} - 270^{\circ}$
 $\angle A = 90^{\circ}$

Question 5 Page 381

- a) Opposite angles in a parallelogram are equal.
- $x = 70^{\circ}$
- $w = 110^{\circ}$



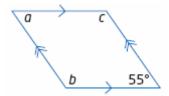
b) Opposite angles in a parallelogram are equal. $y = 138^{\circ}$ $z = 42^{\circ}$



c) Since opposite angles in a parallelogram are equal, $a = 55^{\circ}$ and b = c.

Adjacent angles in a parallelogram are supplementary.

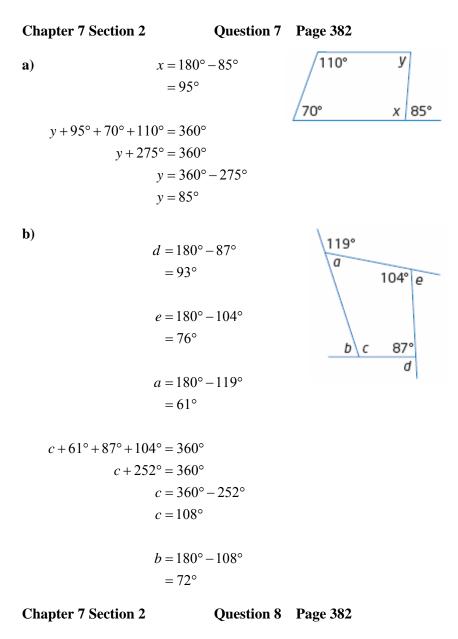
$$b + 55^{\circ} = 180^{\circ}$$
$$b = 180^{\circ} - 55^{\circ}$$
$$b = 125^{\circ}$$
$$c = b$$
$$= 125^{\circ}$$



Chapter 7 Section 2

Question 6 Page 381

For both triangles and quadrilateral, the sum of the exterior angles is 360°.



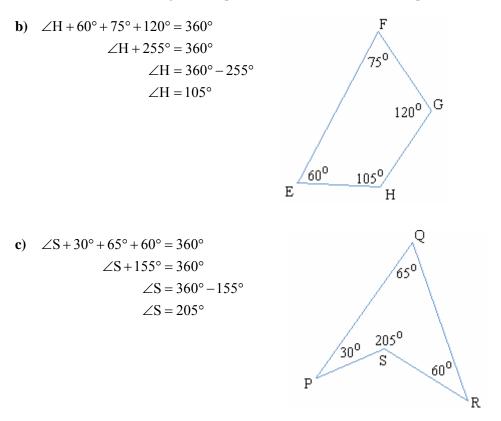
As shown in question 7 b), you need three angles, each at a different vertex; to calculate the measure of all of the interior and exterior angles of a quadrilateral. You can use angle relationships to calculate the others.

Chapter 7 Section 2 Question 9 Page 382

a)
$$\angle A + \angle B + \angle C = 170^{\circ} + 65^{\circ} + 160^{\circ}$$

= 395°

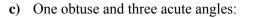
The sum of the interior angles of a quadrilateral must be 360°. This quadrilateral is not possible.

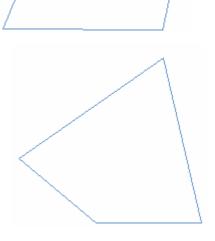


Chapter 7 Section 2 Question 10 Page 382

Answers will vary. Sample answers are shown.

- a) Four obtuse angles add to more than 360°. This quadrilateral is not possible.
- **b**) Exactly two obtuse angles:





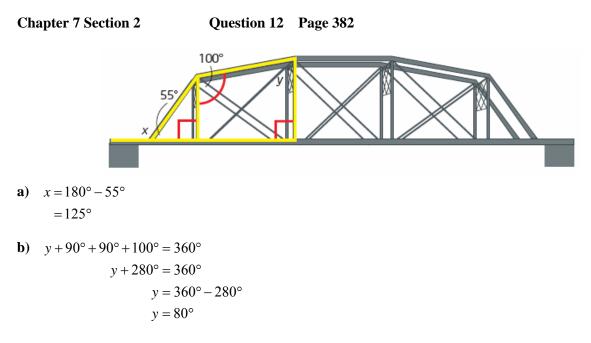
d) One obtuse angle, and two right angles:

e) If three of the angles are right angles, then the fourth must be a right angle as well. This quadrilateral is not possible.

Chapter 7 Section 2

Question 11 Page 382

 $mean = \frac{360^{\circ}}{4}$ $= 90^{\circ}$



c) Answers will vary. Sample answers are shown.

Triangles and quadrilaterals are easy to construct. Triangles are rigid.

$$x + 3x - 22^{\circ} = 180^{\circ}$$

$$4x - 22^{\circ} = 180^{\circ}$$

$$4x = 180^{\circ} + 22^{\circ}$$

$$4x = 202^{\circ}$$

$$\frac{4x}{4} = \frac{202^{\circ}}{4}$$

$$x = 50.5^{\circ}$$

$$3x - 22^{\circ} = 3(50.5^{\circ}) - 22^{\circ}$$

$$= 151.5^{\circ} - 22^{\circ}$$

$$= 129.5^{\circ}$$

$$2x - 10^{\circ} = 2(50.5^{\circ}) - 10^{\circ}$$

$$= 101^{\circ} - 10^{\circ}$$

$$= 91^{\circ}$$

$$y = 180^{\circ} - 91^{\circ}$$

$$= 89^{\circ}$$

$$x + 15^{\circ} = 50.5^{\circ} + 15^{\circ}$$

$$= 65.5^{\circ}$$

$$z = 180^{\circ} - 65.5^{\circ}$$

$$= 114.5^{\circ}$$

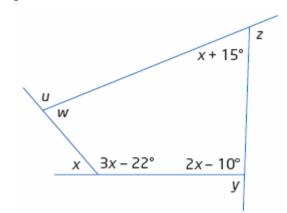
$$w + 129.5^{\circ} + 65.5^{\circ} + 91^{\circ} = 360^{\circ}$$

$$w = 360^{\circ} - 286^{\circ}$$

$$w = 74^{\circ}$$

$$u = 180^{\circ} - 74^{\circ}$$

$$= 106^{\circ}$$

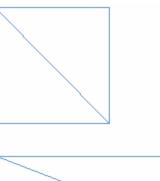


Chapter 7 Section 2 Question 14 Page 383

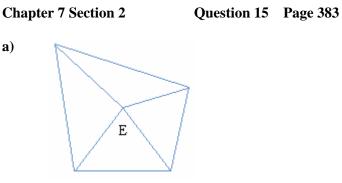
a) Each diagonal divides the quadrilateral into two congruent triangles.

b) The diagonal is a line of symmetry for the square, but not for the rectangle.

c) The diagonal bisects the corner angles in the square. The diagonal does not bisect the corner angles in the rectangle.







b) The sum of the four angles at point E is 360°.

c) The sum of all of the interior angles of the four triangles inside the quadrilateral is $4 \times 180^\circ = 720^\circ$.

d) The sum of the interior angles of quadrilateral is equal to the sum of the interior angles of the four triangles less the sum of the angles at E: $720^\circ - 360^\circ = 360^\circ$.

a)
$$x + x + x + x = 360^{\circ}$$
$$4x = 360^{\circ}$$
$$\frac{4x}{4} = \frac{360^{\circ}}{4}$$
$$x = 90^{\circ}$$

The angles are 90° , 90° , 90° , and 90° .

b)
$$x + x + 2x + 2x = 360^{\circ}$$

 $6x = 360^{\circ}$
 $\frac{6x}{6} = \frac{360^{\circ}}{6}$
 $x = 60^{\circ}$

The angles are 60° , 60° , 120° , and 120° .

c)
$$x + 2x + 3x + 4x = 360^{\circ}$$

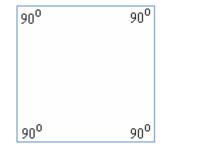
 $10x = 360^{\circ}$
 $\frac{10x}{10} = \frac{360^{\circ}}{10}$
 $x = 36^{\circ}$

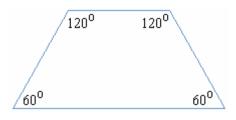
The angles are 36°, 72°, 108°, and 144°.

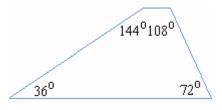
d)
$$3x + 4x + 5x + 6x = 360^{\circ}$$

 $18x = 360^{\circ}$
 $\frac{18x}{18} = \frac{360^{\circ}}{18}$
 $x = 20^{\circ}$

The angles are 60° , 80° , 100° , and 120° .









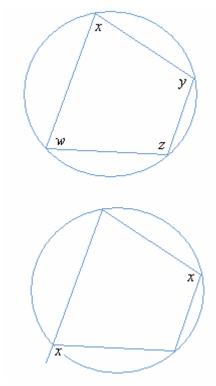
Question 17 Page 383

Answers will vary. Sample answers are shown.

In a cyclic quadrilateral, opposite angles are supplementary.

 $x + z = 180^{\circ}$ $w + y = 180^{\circ}$

Any external angle is equal to the interior and opposite internal angle.



Chapter 7 Section 2

Question 18 Page 383

$$y = 90^{\circ}$$
$$z = 60^{\circ}$$
$$y + z = 150^{\circ}$$
$$2x + 150^{\circ} = 180^{\circ}$$
$$2x = 180^{\circ} - 15$$
$$2x = 30^{\circ}$$
$$\frac{2x}{2} = \frac{30^{\circ}}{2}$$
$$x = 15^{\circ}$$
$$\angle CEB = 15^{\circ}$$

$$2x + 150^{\circ} = 180^{\circ}$$
$$2x = 180^{\circ} - 150^{\circ}$$
$$2x = 30^{\circ}$$
$$\frac{2x}{2} = \frac{30^{\circ}}{2}$$
$$x = 15^{\circ}$$
$$\angle \text{CEB} = 15^{\circ}$$

Answer B.

D A E y^2 С В

Question 19 Page 383

Let the sides measure *a*, *b*, *c*, and *d*. You can place them in the following orders:

a, *b*, *c*, and *d a*, *c*, *b*, and *d a*, *b*, *d*, and *c*

Any other arrangement will produce a quadrilateral congruent to one of these.

There are 3 non-congruent quadrilaterals you can make with four sides of unequal lengths.

Chapter 7 Section 3 Angle Relationships in Polygons

Note: Let *S* represent the sum of the interior angles.

Chapter 7 Section 3	Question 1 Pag	ge 391		
a) $S = 180(n-2)$	b) $S = 180(n-2)$	2)	c) $S = 180(n-2)$	
=180(10-2)	= 180(15 -	2)	=180(20-2)	
=180(8)	=180(13)		=180(18)	
=1440	= 2340		= 3240	
The sum of the interior angles is 1440°.	The sum of the ir angles is 2340°.	nterior	The sum of the interior angles is 3240°.	
Chapter 7 Section 3	Question 2 Pag	ge 391		
a) $S = 180(n-2)$	ł	b) $S = 180(n - 1)$	-2)	
=180(7-2)		=180(12-2)		
=180(5)		=180(10)		
= 900		=1800		
Each angle measures $\frac{900^{\circ}}{7}$, or	128.6°. I	Each angle mea	sures $\frac{1800^{\circ}}{12}$, or 150°.	
Chapter 7 Section 3	Question 3 Pa	ge 391		
a) $180(n-2) = 540$	ł	b) 180(a	(n-2) = 1800	
180n - 360 = 540		180 <i>n</i>	-360 = 1800	
180n - 360 + 360 = 540 + 360 = 560	60	180 <i>n</i> – 360	+360 = 1800 + 360	
180n = 900			180n = 2160	
$\frac{180n}{180} = \frac{900}{180}$			$\frac{180n}{180} = \frac{2160}{180}$	
n = 5			n = 12	
The polygon has 5 sides.]	The polygon ha	s 12 sides.	
c) $180(n-2) = 3060$				
180n - 360 = 3060				
180n - 360 + 360 = 3060 +	360			
180n = 3420				

The polygon has 19 sides.

 $\frac{180n}{180} = \frac{3420}{180}$

n = 19

Chapt	ter 7 Section 3	7 Section 3 Question 4 Page 391			
	Polygon	Number of Sides	Number of Diagonals From One Vertex	Number of Triangles in the Polygon	Sum of Interior Angles
	quadrilateral	4	1	2	360°
	pentagon	5	2	3	540°
	decagon	10	7	8	1440°
	icosagon	20	17	18	3240°

Question 5 Page 391

A regular polygon has equal interior angles, equal exterior angles, and equal sides.

Chapter 7 Section 3 Ques

Question 6 Page 391

$$S = 180(n-2)$$

= 180(4-2)
= 180(2)
= 360
Each angle in a square measures $\frac{360^{\circ}}{4}$, or 90°.

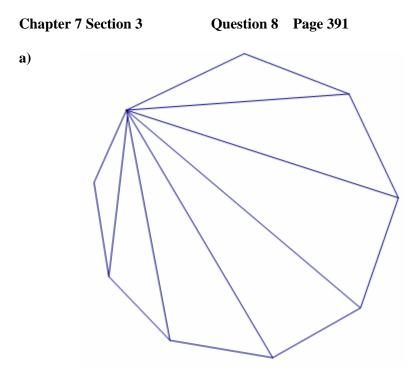
Chapter 7 Section 3 Question 7 Page 391

a) S = 180(n-2)= 180(6-2)= 180(4)= 720

Each angle in a regular hexagon measures $\frac{720^{\circ}}{6}$, or 120° . The adjacent sides of the table will meet at 120° .

- **b)** Answers will vary.
- c)

Changing the lengths of one pair of opposite sides by doubling them does not change the measures of the angles.



- **b**) There are 6 diagonals that can be drawn from any one vertex. Refer to the diagram in part a).
- c) S = 180(n-2)= 180(9-2)= 180(7)= 1260

The sum of the interior angles of the polygon is 1260°.

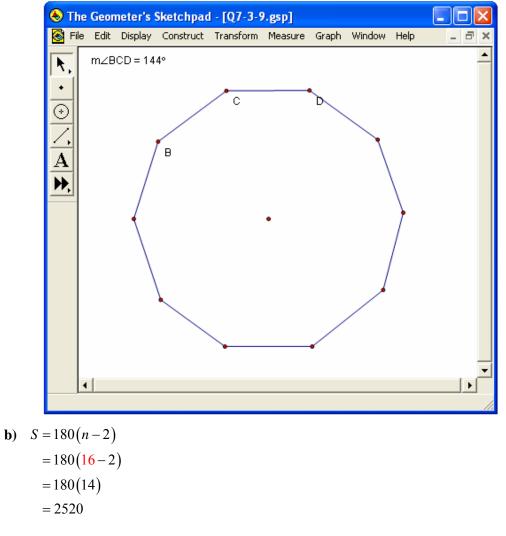
Question 9 Page 392

a) S = 180(n-2)= 180(10-2)= 180(8)= 1440

Each angle in a regular 10-sided polygon measures $\frac{1440^{\circ}}{10}$, or 144° .

Second methods may vary. A sample method is shown.

You can use *The Geometer's Sketchpad* \mathbb{R} to construct a model of a 10-sided regular polygon, and then measure one of the angles. Click <u>here</u> to load the sketch.



Each angle in a regular 16-sided polygon measures $\frac{2520^{\circ}}{16}$, or 157.5°.

c) S = 180(n-2)= 180(20-2)= 180(18)= 3240

Each angle in a regular 20-sided polygon measures $\frac{3240^{\circ}}{20}$, or 162° .

d) The measure of each interior angle of a regular polygon with *n* sides may be calculated from the expression $\frac{180(n-2)}{n}$.

Chapter 7 Section 3 Question 10 Page 392

a) A Canadian dollar coin has 11 sides.

b)
$$\frac{180(n-2)}{n} = \frac{180(11-2)}{11}$$

= $\frac{180(9)}{11}$
= 147.3

The angle between adjacent sides of the coin is about 147.3°.

c) Answers will vary. A sample answer is shown.

The Royal Canadian Mint may have chosen this shape to make it easier for blind people and vending machines to recognize, and harder to forge.

Chapter 7 Section 3 Question 11 Page 392

The sum of the exterior angles is 360° for all convex polygons. You cannot determine the number of sides from the sum of the exterior angles.

Chapter 7 Section 3 Question 12 Page 392

Three regular polygons whose interior angles divide evenly into 360° are triangles (60°), rectangles (90°), and hexagons (120°).

Question 13 Page 392

a) The gazebo has 12 sides.

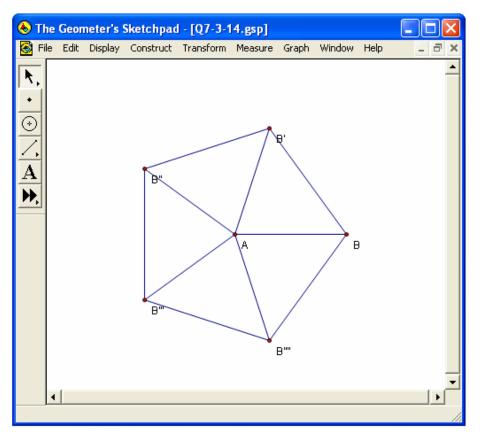
b)
$$\frac{180(n-2)}{n} = \frac{180(12-2)}{12}$$

= $\frac{180(10)}{12}$
= 150

The angle between adjacent sides is 150°.

- c) The angle between adjacent roof supports is $\frac{360^{\circ}}{12}$, or 30° .
- d) Answers will vary.
- e) The angle between adjacent roof supports in a gazebo with six sides is $\frac{360^{\circ}}{6}$, or 60° .

a) Click <u>here</u> to load the sketch.



The shape formed is a pentagon.

b) To construct a regular octagon using this method, rotate the line segment 7 times through an angle of 45°.

c) Use an angle of $\frac{360^{\circ}}{20}$, or 18° for a regular 20-sided figure.

d) The angle of rotation is 360° divided by the number of sides.

Chapter 7 Section 3 Question 15 Page 393

Solutions for the Achievement Checks are shown in the Teacher's Resource.

Chapter 7 Section 3 Question 16 Page 393

All regular polygons are convex. The angle between adjacent sides must be less than 180°.

Chapter 7 Section 3 Question 17 Page 393

Answers will vary. A sample answer is shown.

The formula for the sum of the interior angles applies to concave polygons. An *n*-sided concave polygon can be divided into n - 2 triangles by diagonals from two or more vertices. Alternatively, you can use *The Geometer's Sketchpad*[®] to measure angle sums in various concave polygons.

Chapter 7 Section 3 Question 18 Page 393

Answers will vary. Sample answers are shown.

In the first diagram, angles on the same chord are equal. In the second diagram, the angle at the centre is double the angle at the circumference.

Chapter 7 Section 3	Question 19	Page 393
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Answers will vary.

Chapter 7 Section 3	Question 20	Page 393

 \angle ABC and \angle BCA both measure the same as angle *x*.

$$3x + x + x = 180^{\circ}$$
$$5x = 180^{\circ}$$
$$\frac{5x}{5} = \frac{180^{\circ}}{5}$$
$$x = 36^{\circ}$$

 $\angle BCA = 36^{\circ}$

Answer B.

Chapter 7 Section 3 Question 21 Page 393

Each diagonal requires one pair of vertices. There are 12×11 , or 132 pairs of vertices. However, each one has been counted twice. That leaves $\frac{132}{2}$, or 66. However, this also counts the edges of the polygon. The number of possible diagonals is 66-12, or 54. Answer A.

D

Chapter 7 Section 4 Midpoints and Medians in Triangles

Question 1 Page 398

Chapter 7 Section 4

a)
$$XY = \frac{1}{2}BC$$
$$= \frac{1}{2}(4)$$
$$= 2$$

b) XY = 2VW

A Y B 4 cm C

6 cm

W

7

=2(6)=12

The length of XY is 2 cm.

The length of XY is 12 cm.



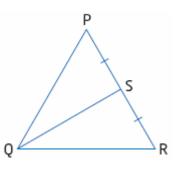
a) The area of $\triangle PQS$ is half the area of $\triangle PQR$. The area of $\triangle PQR$ is 16 cm². So, the area of $\triangle PQS$ is 8 cm².

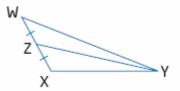
b) The area of \triangle QSR is half the area of \triangle PQR. The area of \triangle PQR is 16 cm². So, the area of \triangle QSR is 8 cm².

Chapter 7 Section 4 Question 3 Page 398

a) The area of Δ WZY is equal to the area of Δ XYZ. The area of Δ XYZ is 19 cm². So, the area of Δ WZY is 19 cm².

b) The area of Δ WXY is double the area of Δ XYZ. The area of Δ XYZ is 19 cm². So, the area of Δ WXY is 38 cm².





Chapter 7 Section 4 Qu

Question 4 Page 398

The length of the cross-brace AB is $\frac{1}{2} \times 5$, or 2.5 m.

3.0 m A 3.0 m 3.0 m 3.0 m

D

B

С

Chapter 7 Section 4

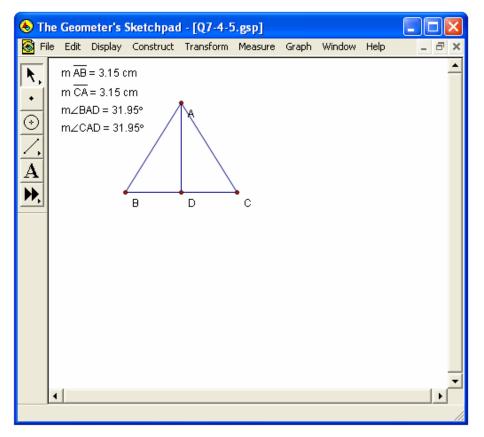
Question 5 Page 399

a) Answers will vary.

b) You can fold along the median and see if the equal sides line up.

c) You can construct the isosceles triangle and median, and then measure the angle on either side of the median.

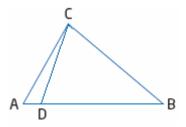
d) Click <u>here</u> to load the sketch.



The median bisects the angle.

Question 6 Page 399

If point D is moved close to vertex A, $\angle ADC$ is obtuse.



60⁰

Chapter 7 Section 4

Question 7 Page 399

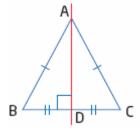
a) Refer to the diagram. In this case, the 60° angle is opposite the second-longest side.

b) Refer to the diagram. In this case, the 60° angle is opposite the second-longest side.

c) Since the angles sum to 180° , one of the angles must be larger than 60° and the third angle must be smaller. The largest angle is opposite the largest side, and the smallest angle is opposite the smallest side. Therefore, the 60° angle is opposite the second-longest side.

Chapter 7 Section 4 Question 8 Page 399

Since $\triangle ABD$ and $\triangle ACD$ are congruent (ASA or SAS), the perpendicular at D must pass through A.

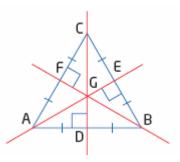


30⁰

Chapter 7 Section 4

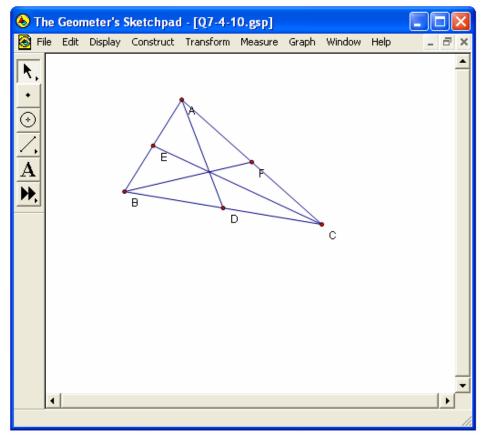
Question 9 Page 399

 \triangle AGC, \triangle CGB, and \triangle BGA are not equilateral triangles. The centre angle at G is obtuse for all three triangles.



Chapter 7 Section 4 Question 10 Page 399

Medians intersect at a point for all triangles. You can verify this using geometry software. A sample sketch is shown. Click <u>here</u> to load the sketch.



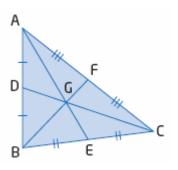
Chapter 7 Section 4

Question 11 Page 400

a) $\triangle BEG$ and $\triangle CEG$ have the same area because GE is a median of $\triangle BGC$.

b) The same logic applies as in part a), since DG and GF are also medians.

c) AE is a median, so $\triangle ABE$ has the same area as $\triangle ACE$. Since the areas of $\triangle BEG$ and $\triangle CEG$ are equal, the areas of $\triangle ABG$ and $\triangle ACG$ are also equal. The areas of the two triangles in $\triangle ABG$ are



equal, as are the areas of the two triangles in $\triangle ACG$. Therefore, $\triangle ADG$, $\triangle BDG$, $\triangle AFG$, and $\triangle CFG$ each have an area equal to half that of $\triangle ABG$. Comparing $\triangle BCF$ and $\triangle BAF$ shows that $\triangle BEG$ and $\triangle CEG$ also each have an area half that of $\triangle ABG$.

Chapter 7 Section 4 Question 12 Page 400

a) Answers will vary. Start with an equilateral triangle, shown in black. Connect the midpoints of the sides. Shade the smaller triangle formed, shown in red. Repeat for each of the three smaller black triangles. Continue the process.



b) After the first step, $\frac{1}{4}$ of the original triangle is shaded. After the second step, $\frac{1}{4} + \frac{1}{4}\left(\frac{3}{4}\right)$ is shaded. After the third step, $\frac{1}{4} + \frac{1}{4}\left(\frac{3}{4}\right) + \frac{1}{4}\left(\frac{3}{4}\right)^2$ is shaded.

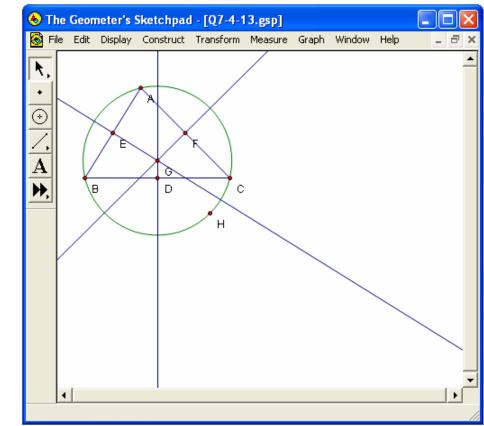
c) After the fourth step, $\frac{1}{4} + \frac{1}{4} \left(\frac{3}{4}\right) + \frac{1}{4} \left(\frac{3}{4}\right)^2 + \frac{1}{4} \left(\frac{3}{4}\right)^3$, or about 0.6836 (68.36%) is shaded.

Chapter 7 Section 4

Question 13 Page 400

a) The right bisectors of a triangle intersect at a single point. You can verify this using geometry software. A sample sketch is shown. Click <u>here</u> to load the sketch.

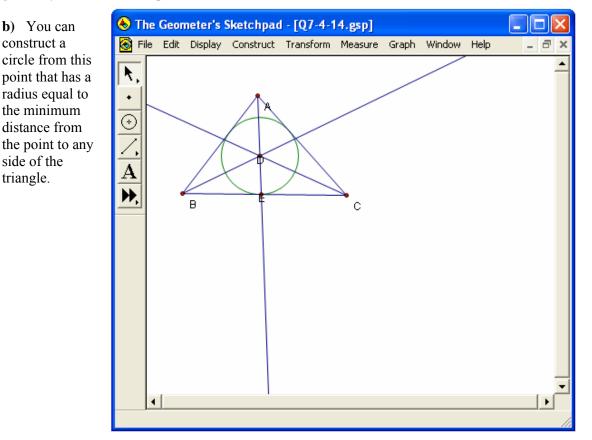
b) You can draw a circle from the point in part a) that passes through all three vertices of the triangle.



Chapter 7 Section 4

Question 14 Page 400

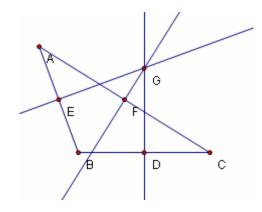
a) The angle bisectors of a triangle always intersect at a point. You can verify this using geometry software. A sample sketch is shown. Click <u>here</u> to load the sketch.



Chapter 7 Section 4

Question 15 Page 400

For an obtuse triangle, the intersection of the right bisectors of the sides is outside the triangle.



Chapter 7 Section 4

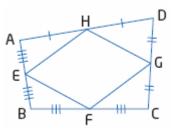
Question 16 Page 400

The longest side cannot be equal to or greater than the sum of the two shortest sides. Cases c), d) and g) are not possible.

Chapter 7 Section 5 Midpoints and Diagonals in Quadrilaterals

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Chapter 7 Section 5 Question 1 Page 405
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The midpoints of the sides of quadrilateral ABCD are joined to produce parallelogram EFGH. So, EF is parallel to HG, and EH is parallel to FG.



A B cm B C

Chapter 7 Section 5

Chapter 7 Section 5

Question 3 Page 405

Question 2 Page 405

The diagonals of parallelogram PQRS bisect each other.

The diagonals of parallelogram ABCD bisect each other.

Also, AC = 2AE, or 16 cm, and BD = 2DE, or 12 cm.

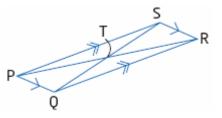
So, BE = DE, or 6 cm, and CE = AE, or 8 cm.

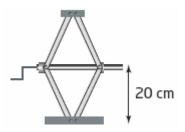
PT = $\frac{1}{2}$ PR = $\frac{1}{2}(14)$ = 7 The length of PT is 7 m. ST = $\frac{1}{2}$ QS = $\frac{1}{2}(10)$ = 5 The length of ST is 5 m.

Chapter 7 Section 5

Question 4 Page 405

The shaft and a line from the top of the jack to its base form diagonals of the parallelogram. Since the diagonals of a parallelogram bisect each other, the top of the jack will be 2(20), or 40 cm high when the shaft is 20 cm from the base.





Chapter 7 Section 5 Quest

Question 5 Page 405

a) The diagonals bisect each other in all four.

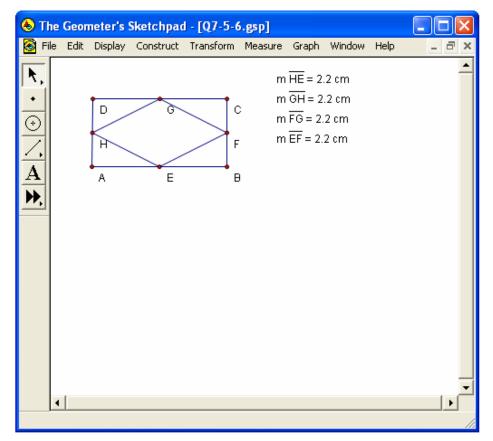
b) The diagonals have the same length in the rectangle and the square.

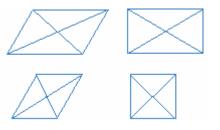
c) The diagonals intersect at 90° in the rhombus and the square.

d) The diagonals bisect each other at 90° in the rhombus and the square.

Chapter 7 Section 5 Question 6 Page 405

EFGH is a rhombus when ABCD is a rectangle. You can verify this using geometry software. A sample sketch is shown. Click <u>here</u> to load the sketch.

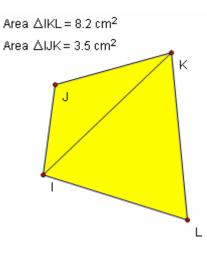




Chapter 7 Section 5 Question 7 Page 405

a) This is false. Any quadrilateral with four unequal sides is a counter-example. A sample is shown.

b) This is true. Any line segment joining opposite midpoints creates two parallelograms with equal heights and bases.

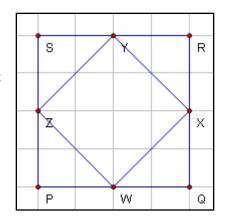


Chapter 7 Section 5

Question 8 Page 406

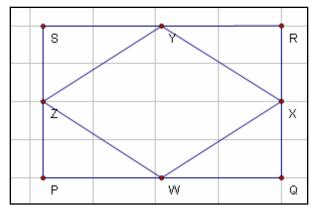
a) WXYZ is a square.

b) The area of WXYZ is half the area of PQRS. The diagonals of WXYZ form four triangles that are congruent to the triangles outside WXYZ.



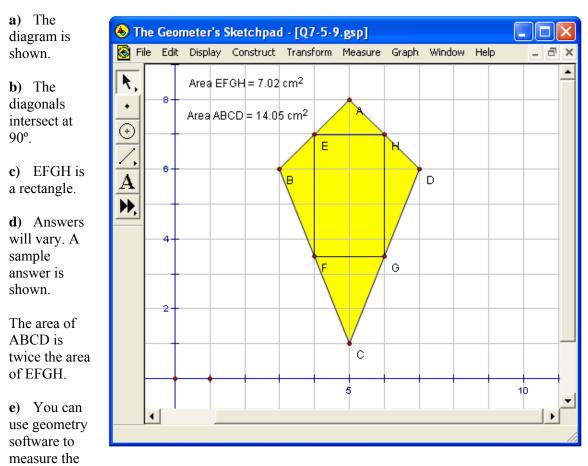
c) If PQRS is stretched into a rectangle, WXYZ becomes a rhombus.

d) The area relationship between WXYZ and PQRS will not change. All the triangles are still congruent.



Chapter 7 Section 5

Question 9 Page 406



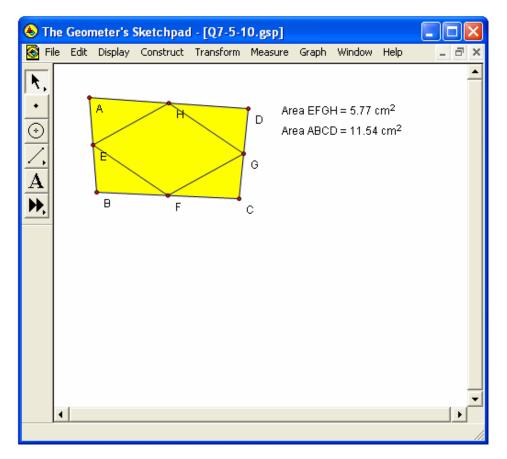
areas of ABCD and EFGH. A sample sketch is shown. Click here to load the sketch.

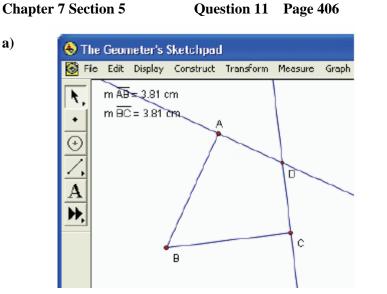
Chapter 7 Section 5 Question 10 Page 406

Answers will vary. Sample answers are shown.

a) The area of EFGH is half the area of ABCD.

b) Use geometry software to compare the areas. A sample sketch is shown. Click <u>here</u> to load the sketch.





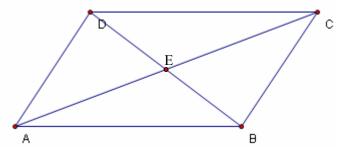
- **b**) By the Pythagorean theorem, $AD^2 + AB^2 = BD^2 = CD^2 + AB^2$. So, AD = CD.
- c) $\triangle ABD$ is congruent to $\triangle CBD$ (SSS), so $\angle ABD$ equals $\angle CBD$.

Chapter 7 Section 5 Question 12 Page 407

Solutions for the Achievements Checks are shown in the Teacher's Resource.

Chapter 7 Section 5 Question 13 Page 407

In any parallelogram ABCD, \triangle ABC and \triangle CDA are congruent (SSS), as are \triangle ABD and \triangle CDB. Thus, \angle CAB = \angle ACD, \angle CDB = \angle ABD, \angle ACB = \angle CAD, and \angle ADB = \angle CBD. \triangle ABE and \triangle CDE are congruent (ASA), so DE = BE and AE = CE.



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Chapter 7 Section 5

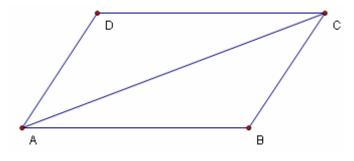
 \triangle ABC and \triangle CDA are congruent (SSS). So, \angle BCA = \angle DAC.

Therefore, AD is parallel to BC.

Similarly, $\angle BAC = \angle DCA$.

Therefore, AB is parallel to CD.

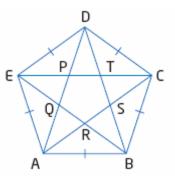
ABCD is a parallelogram.



Chapter 7 Section 5 Question 15 Page 407

a) The five triangles formed by two adjacent sides of PQRST, $\triangle ABC$, $\triangle BCD$, $\triangle CDE$, $\triangle DEA$ and $\triangle EAB$, are isosceles and congruent (SAS). So, all the acute angles in these triangles are equal.

 ΔABR , ΔBCS , ΔCDT , ΔDEP , and ΔEAQ are all congruent (ASA). The obtuse angles of these triangles are opposite to the interior angles of PQRST. Thus, these angles are all equal. ΔDTP , ΔEPQ , ΔAQR , ΔBRS , and ΔCST are all congruent (SAS), so the sides of PQRST are all equal. PQRST is a regular pentagon.

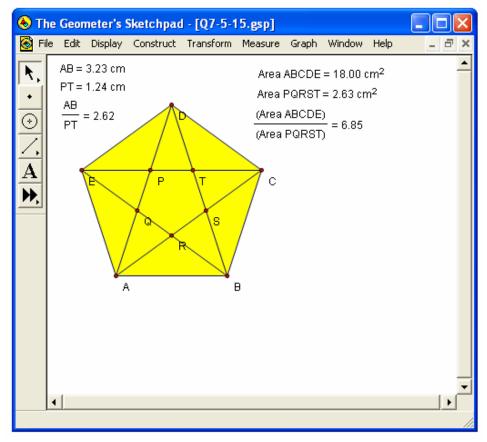


b) PQRST is similar to ABCDE. Both are regular pentagons.

c) Using direct measurement from the diagram, the ratio is about $\frac{1.6}{0.6}$, or about 2.7.

d) The ratio of areas is 2.7^2 , or about 7.

e) Geometry software produces results similar to the conjectures in parts c) and d). A sample sketch is shown. Click <u>here</u> to load the sketch.



Chapter 7 Section 5 Question 16 Page 407

a) There are 10 choices for the first point, and for each of these there are 9 choices for the second point. However, this counts each line segment twice. The number of line segments that can be constructed between 10 points is $\frac{10 \times 9}{2}$, or 45.

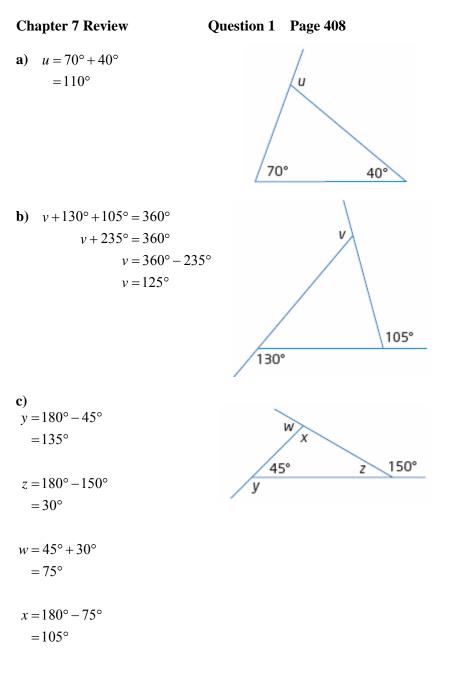
b) Using reasoning similar to part a), the number of handshakes is $\frac{12 \times 11}{2}$, or 66.

Chapter 7 Section 5 Question 17 Page 407

a) Using reasoning similar to question 10, the number of line segments is $\frac{n(n-1)}{2}$.

b) To obtain the number of diagonals, use the expression from part a), and subtract the line segments that form the edges of the polygon:

$$\frac{n(n-1)}{2} - n = \frac{n^2 - n}{2} - \frac{2n}{2}$$
$$= \frac{n^2 - 3n}{2}$$
$$= \frac{n(n-3)}{2}$$



Chapter 7 Review Question 2 Page 408 2x-15+3x-17 = 4x+12 5x-32 = 4x+12 5x-32+32-4x = 4x+12+32-4x x = 44 4x+12 = 4(44)+12= 188

Since the exterior angle must be less than 180°, this angle relationship is not possible.

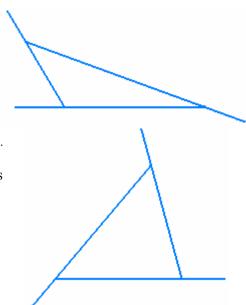
Chapter 7 Review Question 3 Page 408

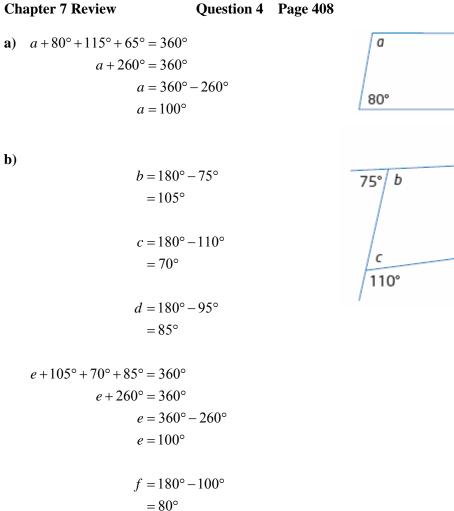
a) A triangle with an acute exterior angle occurs for any obtuse triangle.

b) It is not possible to have two acute exterior angles. In order to sum to 180°; the third exterior angle would have to be greater than 180°.

c) Any acute triangle has three obtuse exterior angles.

d) This is not possible. The sum of the exterior angles would be less than 360°.

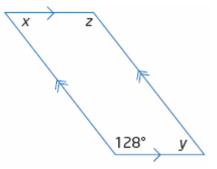




c) Since opposite angles in a parallelogram are equal, $z = 128^{\circ}$ and x = y.

Adjacent angles in a parallelogram are supplementary. $y + 128^{\circ} = 180^{\circ}$

$$y = 180^{\circ} - 128^{\circ}$$
$$y = 52^{\circ}$$
$$x = y$$
$$= 52^{\circ}$$



65°

115°

f

е

d

95°

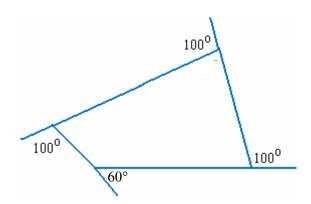
Chapter 7 Review Question 5 Page 408

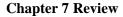
a) An example of a quadrilateral with three obtuse interior angles is one with three 110° angles and one 30° angle.

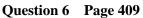
b) It is not possible to have a quadrilateral with four obtuse interior angles. The sum of the interior angles would be greater than 360°.

c) An example of a quadrilateral with three obtuse exterior angles is one with three 100° angles and one 60° angle.

d) A quadrilateral with four obtuse exterior angles is not possible. The sum of the exterior angles would be greater than 360°.







a) S = 180(n-2)= 180(6-2)= 180(4)= 720

The sum of the interior angles of a hexagon is 720° .

b)
$$S = 180(n-2)$$

= $180(8-2)$
= $180(6)$
= 1080

The sum of the interior angles of an octagon is 1080°.

c) S = 180(n-2)= 180(12-2)= 180(10)= 1800

The sum of the interior angles of a dodecagon is 1800°.

Chapter 7 Review

Question 7 Page 409

a)
$$\frac{180(n-2)}{n} = \frac{180(5-2)}{5}$$

= $\frac{180(3)}{5}$
= 108

Each interior angle of a pentagon measures 108°.

b)
$$\frac{180(n-2)}{n} = \frac{180(9-2)}{9}$$
$$= \frac{180(7)}{9}$$
$$= 140$$

Each interior angle of a nonagon measures 140°.

c)
$$\frac{180(n-2)}{n} = \frac{180(16-2)}{16}$$

= $\frac{180(14)}{16}$
= 157.5

Each interior angle of a hexadecagon measures 157.5°.

Chapter 7 Review

Question 8 Page 409

$$180(n-2) = 168n$$

$$180n - 360 = 168n$$

$$180n - 360 - 168n + 360 = 168n - 168n + 360$$

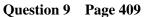
$$12n = 360$$

$$\frac{12n}{12} = \frac{360}{12}$$

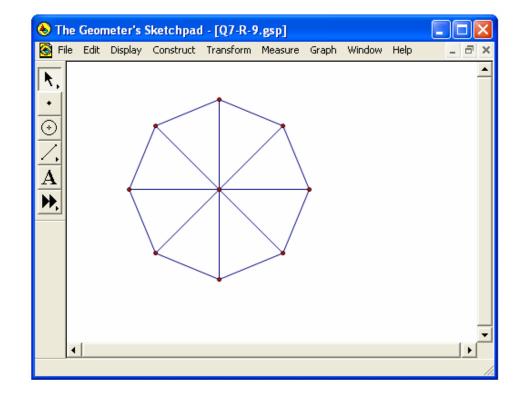
$$n = 30$$

The polygon has 30 sides.

Chapter 7 Review



a)



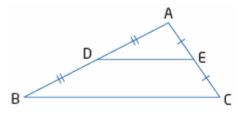
b) Answers will vary. A sample answer is shown.

Use geometry software. Construct a line segment, and rotate it around one end point 7 times at an angle of 45°. Join the ends of the segments. Click <u>here</u> to load the sketch.

Chapter 7 Review

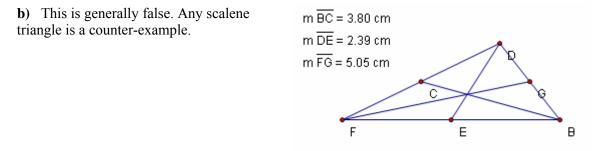
Question 10 Page 409

DE connects the midpoints of AB and AC. Therefore, the base and altitude of \triangle ADE are half those of \triangle ABC. The area of \triangle ADE is $\left(\frac{1}{2}\right)^2$, or $\frac{1}{4}$ the area of \triangle ABC.



Chapter 7 Review Question 11 Page 409

a) Each median divides the triangle into two triangles. All of these triangles are congruent (SAS). The medians are equal in length since they are sides of the congruent triangles.



Chapter 7 Review	Question 12	Page 409
Answers will vary.		
Chapter 7 Review	Question 13	Page 409

Answers will vary.

Chapter 7 Chapter Test

Chapter 7 Chapter Test Question 1 Page 410

 $x + 110^{\circ} + 110^{\circ} = 360^{\circ}$ $x + 220^{\circ} = 360^{\circ}$ $x = 360^{\circ} - 220^{\circ}$ $x = 140^{\circ}$

Answer C.

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The interior angle at B is $180^{\circ} - 119^{\circ}$, or 61° .

 $x + 51^{\circ} + 61^{\circ} = 180^{\circ}$ $x + 112^{\circ} = 180^{\circ}$ $x = 180^{\circ} - 112^{\circ}$ $x = 68^{\circ}$

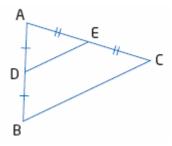
Answer B.

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The sum of the exterior angles of a convex polygon is always 360°. Answer B.

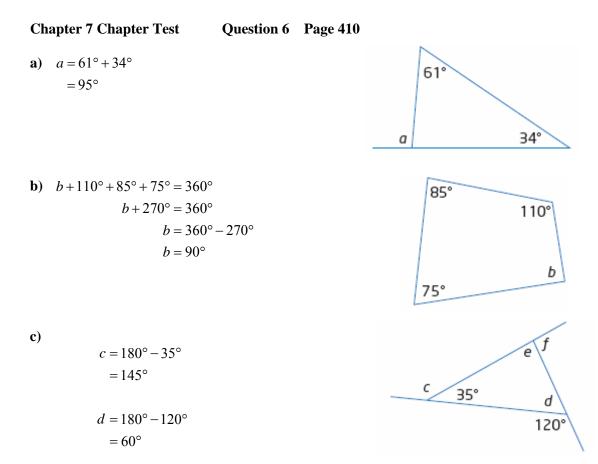
Chapter 7 Chapter Test Question 4 Page 410

The area of $\triangle ADE$ is one-quarter of the area of $\triangle ABC$, or one-third of the area of trapezoid DBCE. Answer D.



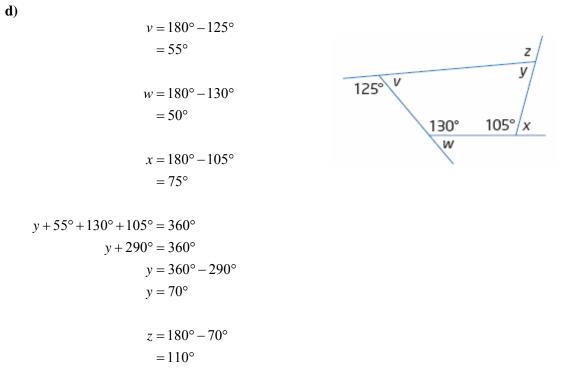
Chapter 7 Chapter Test Question 5 Page 410

The diagonals of a rectangle bisect each other. Answer B.



$$e + 60^{\circ} + 35^{\circ} = 180^{\circ}$$

 $e + 95^{\circ} = 180^{\circ}$
 $e = 180^{\circ} - 95^{\circ}$
 $e = 85^{\circ}$
 $f = 35^{\circ} + 60^{\circ}$
 $= 95^{\circ}$



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Answers will vary. Sample answers are shown.

a) For a parallelogram:

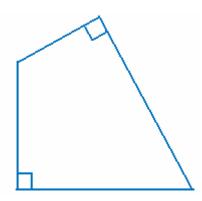
The sum of the interior angles is 360°. Opposite interior angles are equal. Adjacent interior angles are supplementary.

b) For a parallelogram:

The diagonals bisect each other and bisect the area of the parallelogram.

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A quadrilateral with a pair of equal opposite angles is shown. However, it is not a parallelogram.



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$$S = 180(n-2)$$

= 180(14-2)
= 180(12)
= 2160

The sum of the interior angles of a 14-sided polygon is 2160°.

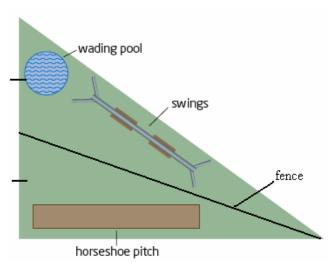
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180(n-2) = 2340 180n - 360 = 2340 180n - 360 + 360 = 2340 + 360 180n = 2700 $\frac{180n}{180} = \frac{2700}{180}$ n = 15

The polygon has 15 sides.

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Run the fence along the median from the right vertex of the lot.



Chapter 7 Chapter Test

a) The shape is a hexagon.

b) The hexagon is regular. The sides are equal, and measuring with a protractor shows that the interior angles are equal.

c)
$$S = 180(n-2)$$

= $180(6-2)$
= $180(4)$
= 720

Each interior angle measures $\frac{720^{\circ}}{6}$, or 120° .

d) For regular polygons, the measure of the interior angles increases as the number of sides increases. Manpreet should increase the measure of each interior angle.

