Chapter 1

Mathematical Processes

Chapter 1 Get Ready

Chapter 1 Get Ready

Question 1 Page 4

a) $\frac{3}{10} + \frac{9}{10} = \frac{12}{10}$ $= \frac{6}{5}$ $= 1\frac{1}{5}$ b) $\frac{3}{8} + \frac{1}{4} = \frac{3}{8} + \frac{1 \times 2}{4 \times 2}$ $= \frac{3}{8} + \frac{2}{8}$ $= \frac{5}{8}$ d) $1\frac{7}{9} - \frac{2}{5} = \frac{16}{9} - \frac{2}{5}$ $= \frac{16 \times 5}{9 \times 5} - \frac{2 \times 9}{5 \times 9}$ $= \frac{13}{30}$ $= \frac{13}{45}$

Chapter 1 Get Ready

Question 2 Page 4

a)
$$\frac{5}{12} \times \frac{3}{10} = \frac{\frac{1}{5}}{\frac{12}{4}} \times \frac{3}{\frac{10}{2}}$$

 $= \frac{1}{8}$

b) $\left(\frac{3}{4} \div \frac{2}{3}\right) = \frac{3}{4} \times \frac{3}{2}$
 $= \frac{9}{8}$
 $= 1\frac{1}{8}$

c) $2\frac{7}{8} \times 6\frac{1}{2} = \frac{23}{8} \times \frac{13}{2}$
 $= \frac{299}{16}$
 $= 18\frac{11}{16}$

d) $\frac{2}{9} \div 2\frac{2}{7} = \frac{2}{9} \div \frac{16}{7}$
 $= \frac{2}{9} \times \frac{7}{16}$
 $= \frac{1}{2}\frac{1}{9} \times \frac{7}{16}$
 $= \frac{7}{72}$

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Chapter 1 Get Ready Question 3 Page 4

The fraction of pizza left for Brad can be determined by subtracting the fractions that the others have eaten from the 2 original pizzas.

$$2 - \left(\frac{1}{3} + \frac{3}{8} + \frac{1}{4} + \frac{1}{2}\right) = 2 - \left(\frac{1 \times 8}{3 \times 8} + \frac{3 \times 3}{8 \times 3} + \frac{1 \times 6}{4 \times 6} + \frac{1 \times 12}{2 \times 12}\right)$$
$$= 2 - \left(\frac{8}{24} + \frac{9}{24} + \frac{6}{24} + \frac{12}{24}\right)$$
$$= 2 - \frac{35}{24}$$
$$= \frac{48}{24} - \frac{35}{24}$$
$$= \frac{13}{24}$$

Brad has $\frac{13}{24}$ of one pizza left.

Chapter 1 Get Ready Question 4 Page 5

- **a**) 13 + (-5) = 8 **b**) -7 + 2 = -5
- c) -8 + (-15) = -23d) 7 - 11 = 7 + (-11)= -4
- e) 2-16 = 2 + (-16)= -14 f) 8 - (-7) = 8 + 7= 15
- **g**) -5 (-9) = -5 + 9= 4 **h**) $100 \times (-4) = -400$
- i) $-7 \times 7 = -49$ j) $-3 \times (-14) = 42$
- **k**) $42 \div (-6) = -7$ **l**) $-28 \div 7 = -4$

Chapter 1 Get Ready Question 5 Page 5

a)
$$-16 \times 6 \div (-2) = -96 \div (-2)$$

= 48
b) $-3 + 5 \times (-1) = -3 + (-5)$
= -8

c)
$$-15 + (-12) - 4 - (-8) = -15 + (-12) + (-4) + 8$$

= -23

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Chapter 1 Get Ready Question 6 Page 5

Net Earnings = -\$6200 - \$2150 + \$4780=-\$3570

Chapter 1 Get Ready

The business lost \$3570.00 during the 3-year period.

Chapter 1 Get Ready
a)
$$-3(9+11) = -3(20)$$

 $= -60$
b) $2+3(10-4)^2 = 2+3(6)^2$
 $= 2+3(36)$
 $= 2+108$
 $= 110$
c) $(7-15) \div (4+4) = -8 \div 8$
 $= -1$
d) $-5(-3) + (-8)(10) = 15 + (-80)$
 $= 15-80$

e)
$$\left[2 - (6+3)^2\right]^2 = \left[2 - (9)^2\right]^2$$

= $\left[2 - 81\right]^2$
= $(-79)^2$
= 6241

$$= -65$$
f) $-15 + 8 \times 7 - 32 \div 16 = -15 + 56 - 2$

Chapter 1 Get Ready

Question 9 Page 5

Net = $6 \times \$56 - 8 \times \43 = \$336 - \$344 =-\$8

Jason's net loss was \$8.

Chapter 1 Get Ready Question 10 Page 5

Answers will vary.

Chapter 1 Section 1: Focus on Problem Solving

Chapter 1 Section 1	Question 1	Page 8
a) 1, 3, 5, 7, 9, 11, 13, 15	Add 2 to the	previous term.
b) 4, 17, 30, 43, 56, 69, 82	Add 13 to the	e previous term.
c) 2, 4, 8, 16, 32, 64, 128	Multiply the	previous term by 2.
d) 1, 1, 2, 3, 5, 8, 13, 21, 34	Add the two	previous terms.
Chapter 1 Section 1	Question 2	Page 8
One nickel:	5¢	
One dime:	10¢	
One nickel, one dime:	15¢	
One quarter:	25¢	
One nickel, one quarter:	30¢	
One dime, one quarter:	35¢	
One nickel, one dime, one quar	ter: 40¢	
Two quarters:	50¢	
One nickel, two quarters:	55¢	
One dime, two quarters:	60¢	

You can make 11 different sums of money with one nickel, one dime and two quarters.

65¢

a) $1 \times 1 = 1$ $11 \times 11 = 121$ $111 \times 111 = 12321$ $1111 \times 1111 = 1234321$

One nickel, one dime, two quarters:

b) The digits increase from 1 up to the number of 1s in one of the factors, and then decrease to 1 so that the answer is symmetric.

c) There are nine 1s in the first factor. Following the pattern, the product is 12 345 678 987 654 321.

Chapter 1 Section 1 Question 4 Page 8

- a) $11 \times 37 = 407$ $22 \times 37 = 814$ $33 \times 37 = 1221$
- b) Add 407 to the previous term. The next three terms are 1628, 2035, and 2442.
- c) 99×37 is the eighth term after 11×37 . Add 407 eight times to obtain 3663.

Chapter 1 Section 1 Question 5 Page 8

a) $\frac{1}{9} = 0.\overline{1}$ $\frac{2}{9} = 0.\overline{2}$ $\frac{3}{9} = 0.\overline{3}$

If the numerator is less than 9, then the decimal will be that digit repeated infinitely after the decimal place.

b) $\frac{1}{99} = 0.\overline{01}$ $\frac{12}{99} = 0.\overline{12}$ $\frac{23}{99} = 0.\overline{23}$

If the numerator is less than 99, then the decimal will be that number written as a two-digit number, repeated infinitely after the decimal place.

c) $\frac{1}{99999} = 0.\overline{00001}$ $\frac{123}{99999} = 0.\overline{00123}$ $\frac{12345}{99999} = 0.\overline{12345}$

If the numerator is less than 99 999 then the decimal will be that number written as a five-digit number, repeating infinitely after the decimal point.

Chapter 1 Section 1 Question 6 Page 9

Use the numbers in the given rows and columns to determine which ones are missing. As an example, consider the fourth column. The numbers 1 and 2 are missing from this column. Now, consider the fourth row. The numbers 2 and 3 are missing from this row. So, the blank at the intersection of the fourth row and the fourth column must contain the number 2. Continue this process to determine the other missing numbers.

The solution is shown.

2	4	З
9	5	8
1	6	7

4	8	9	5	З	2	6	1	7
2	6	1	8	7	9	5	4	3
7	5	З	4	1	6	9	8	2
6	7	8		4		1	5	9
1	3	2	9		8	4	7	6
5	9	4		6		Z	З	8
8	2	5	7	9	4	З	6	1
9	4	6	З	8	1	7	2	5
З	1	7	6	2	5	8	9	4

Chapter 1 Section 1 Question 7 Page 9

a) Gina is 22 years, 3 months, and 17 days old on January 1, 2020. Sam is 25 years, 11 months, and 9 days old on January 1, 2020.

b) To find the number of years, subtract the birth year from 2019, since 2020 has just started. To find the number of months, subtract the month number from 12, for December of 2019. To find the number of days, subtract the birthday number from 31, the number of days in December, and add 1 for January 1st.

Chapter 1 Section 1 Question 8 Page 9

- a) Since there are only 2 players, there is only 1 game required.
- **b**) Since there are 3 players, there are 3 pairs: 1-2, 1-3, and 2-3. Three games are required.

c) With 4 players, there are 6 pairs: 1-2, 1-3, 1-4, 2-3, 2-4, and 3-4. Six games are required.

d) You could list all possible pairs, as in the first three parts of this question. Alternatively, you can use reasoning and multiplication. With 10 players, each of the 10 can be matched with 9 others, resulting in $10 \times 9 = 90$ pairs. However, each pair has been counted twice. For example,

the pair 1-2 is the same as the pair 2-1. Hence there are $\frac{90}{2} = 45$ pairs of players, requiring 45

games.

Chapter 1 Section 1 Question 9 Page 9

The prime factors of 8820 are $2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 7$.

You can make these perfect squares using these factors:

 $2 \times 2 = 4$ $3 \times 3 = 9$ $2 \times 2 \times 3 \times 3 = 36$ $7 \times 7 = 49$ $2 \times 2 \times 7 \times 7 = 196$ $3 \times 3 \times 7 \times 7 = 441$ $2 \times 2 \times 3 \times 3 \times 7 \times 7 = 1764$

All of these divide evenly into 8820. There are 7 perfect squares that divide evenly into 8820.

Chapter 1 Section 1 Question 10 Page 9

You are given that F is half of C. However, C can also only be one more than F, carrying the 1 from the previous column. Therefore, C must be 2 and F must be 1. Looking at the last column, T must be a 0. E cannot be 3, 4 or 5, since that makes A a 0, 1 or 2, which have already been used. This leaves 6, 8 or 9 for E. Try combinations, to find those that work.

There are two possible solutions.

630	860
1766	<u>1788</u>
2396	2648

Chapter 1 Section 1 Question 11 Page 9

Use the clues to place limits on what each letter may represent. For example, H and D are perfect squares between 1 and 16, limiting their values to 1, 4, 9 or 16. B, C, N, and R are greater than 12, limiting their values to 13, 14, 15 or 16. Draw a large grid, and place the possible values in each square. Then, eliminate those that lead to inconsistent sums. The solution is shown.

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

Chapter 1 Section 2 Focus on Communicating

Chapter 1 Section 2 Question 1 Page 12

a) Subtract 5 from the previous term. The next two terms are 0 and -5.

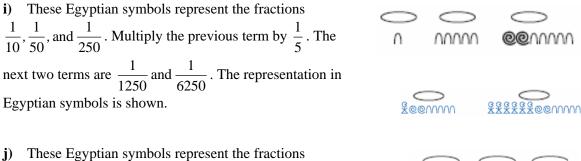
- **b**) Subtract 4 from the previous term. The next two terms are -18 and -22.
- c) Add $\frac{1}{4}$ to the previous term. The next two terms are 1 and $1\frac{1}{4}$.

d) Subtract
$$\frac{2}{5}$$
 from the previous term. The next two terms are $\frac{6}{5}$ and $\frac{4}{5}$.

- e) Multiply the previous term by -2. The next two terms are 48 and -96.
- f) Divide the previous term by 2. The next two terms are -12 and -6.

g) Subtract descending multiples of 5 from the previous term, starting with 20. The next two terms are 50 and 50 (subtract 5, then 0).

h) Multiply the previous term by integers starting from 1, and increasing by 1 each time. The next two terms are 360 (multiply by 5) and 2160 (multiply by 6).



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 $\frac{1}{400}$, $\frac{1}{200}$, and $\frac{1}{100}$. Multiply the previous term by 2. The next two eeee terms are $\frac{1}{50}$ and $\frac{1}{25}$. The representation in Egyptian symbols is shown.

Chapter 1 Section 2 Question 2 Page 12

Answers will vary.

Question 3 Page 12

The area of the semicircle on the hypotenuse is $\frac{1}{2}\pi(5)^2 = \frac{25}{2}\pi$. The areas of the semicircles on the other two sides are $\frac{1}{2}\pi(3)^2 = \frac{9}{2}\pi$ and

$$\frac{1}{2}\pi(4)^2 = \frac{16}{2}\pi$$
. Note that $\frac{9}{2}\pi + \frac{16}{2}\pi = \frac{25}{2}\pi$. The area of the semicircle

on the hypotenuse equals the sum of the areas of the semicircles on the other two sides.

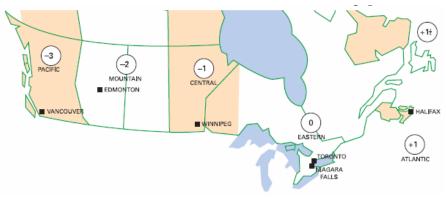
Chapter 1 Section 2 Question 4 Page 12

Diagram B is correct. As the wheel moves forward, the height of the light will increase and decrease smoothly.

Chapter 1 Section 2

Question 5 Page 12

a) The map is divided into sections where time changes by onehour intervals. Starting at the original time zone, count how many time zones away the other one is using positive integers to the right and



10 cm

8 cm

6 cm

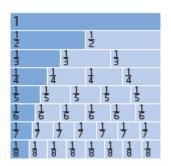
negative integers to the left. Add this integer value to the original time.

b) Halifax is +1 hour ahead of Toronto. If it is 3:00 P.M. in Toronto, it is 4:00 P.M. in Halifax.

c) Winnipeg is +2 hours ahead of Vancouver. If it is 2:30 A.M. in Vancouver, it is 4:30 A.M. in Winnipeg.

Chapter 1 Section 2 Question 6 Page 13

a) Use the last two rows. In the fraction strip made of $\frac{1}{7}$ pieces, shade three parts. In the fraction strip made of $\frac{1}{8}$ pieces, shade four parts. Compare the shaded parts. $\frac{4}{8} > \frac{3}{7}$ because four pieces of $\frac{1}{8}$ are wider than three pieces of $\frac{1}{7}$.



b) Place the $\frac{1}{2}$ piece and the $\frac{1}{3}$ piece side by side. They will have the same width as a $\frac{5}{6}$ piece.

c) Twelve rows are needed to illustrate $\frac{1}{3} + \frac{1}{4}$. The lowest common denominator is 12.

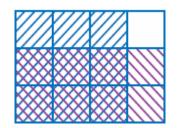
d) The dark blue bars are getting smaller. When 1 is divided by larger and larger numbers, the pieces become smaller and smaller. The number of pieces is the denominator of the fraction.

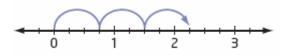
Chapter 1 Section 2 Question 7 Page 13

a) The rectangle is divided into thirds horizontally and two of these rows are shaded to show $\frac{2}{3}$. Then the rectangle is divided into quarters vertically and three of these columns are shaded to show $\frac{3}{4}$. The overlap of the shading shows the product. Since six

parts are double shaded, $\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$ or $\frac{1}{2}$.

b) The number line diagram models the product $3 \times \frac{3}{4} = 2\frac{1}{4}$.





Chapter 1 Section 2 Question 8 Page 13

- **a**) The sum of the first *n* odd numbers is n^2 .
- **b**) The sum of the first 5 odd numbers is 5^2 , or 25.

1 + 3 + 5 + 7 + 9 = 25

The sum of the first 6 odd numbers is 6^2 , or 36.

1 + 3 + 5 + 7 + 9 + 11 = 36

c) There are 50 odd numbers from 1 to 99 inclusive. The sum is $50^2 = 2500$.

d) There are 75 odd numbers from 1 to 150. There are 300 odd numbers from 1 to 600. The sum of the odd numbers from 150 to 600 is $300^2 - 75^2 = 84375$.

Chapter 1 Section 2 Question 9 Page 13

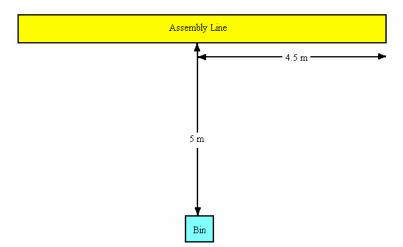
Answers will vary. A possible strategy is to pick an empty square, and use the row, column, and 3 by 3 square to narrow down what numbers may appear in that square. Then, eliminate those that are inconsistent.

6	1	9	З	7	8	4	5	2
4	7	5	2	6	9	З	8	1
8	2	З	4	5	1	6	7	9
1	6	7	5	2	З	8	9	4
5	З	8	1	9	4	2	6	7
9	4	2	6	8	7	1	З	5
7	9	1	8	3	2	5	4	6
3	5	4	7	1	6	9	2	8
2	8	6	9	4	5	7	1	3

Chapter 1 Section 2

Question 10 Page 13

The best location would be 4.5 m from the end of the assembly line toward the middle of the line and a perpendicular distance of 5 m from the line. This location minimizes the distances that workers must go to get their supplies.





Chapter 1 Section 3 Focus on Connecting

Chapter 1 Section 3 Question 1 Page 17

a)

 >
800 m
400 m
200 m
100 m
50 m
*
25 m

b) Total distance = 800 + 400 + 200 + 100 + 50 + 25= 1575 m

c) Distance from starting point = 800 - 400 + 200 - 100 + 50 - 25= 525 m

Chapter 1 Section 3

Question 2 Page 17

Quarters	Dimes	Nickels	Value (\$)
4	0	0	1.00
З	1	0	0.85
З	0	1	0.80
2	2	0	0.70
2	1	1	0.65
2	0	2	0.60
1	З	0	0.55
1	2	1	0.50
1	1	2	0.45
1	0	З	0.40
0	4	0	0.40
0	З	1	0.35
0	2	2	0.30
0	1	З	0.25
0	0	4	0.20

Chapter 1 Section 3 Question 3 Page 17

Answers will vary, depending on the size of the classroom. One approach is to treat the puck as a box of height 2.5 cm, and length and width both 7.4 cm. Find the volume of the puck, and divide this into the volume of the classroom.

Sample answer:

$$V_{\text{puck}} = 2.5 \times 7.4 \times 7.4$$

= 136.9 cm³

 $V_{\text{room}} = 1000 \times 500 \times 300$ = 150 000 000 cm³

Number of pucks = $\frac{150\ 000\ 000}{136.9}$ = 1 095 690

About one million pucks will fit into the classroom.

Chapter 1 Section 3 Question 4 Page 17

The round-trip distance on each ride is 16 km. Honi can make $\frac{2000}{16} = 125$ trips. Assuming 5

trips per week, the tires will last $\frac{125}{5} = 25$ weeks, or about half a year.

Chapter 1 Section 3 Question 5 Page 17

You can use the guess and test strategy. Start by guessing that Joe ate 1 slice. Then, Emily ate 2 slices, Samir ate 4 slices, Kendra ate 1 slice and Fong ate 5 slices. Add the slices: 1 + 2 + 3 + 1 + 5 = 12. These add correctly.

Emily ate
$$\frac{2}{12} = \frac{1}{6}$$
, Samir ate $\frac{3}{12} = \frac{1}{4}$, Joe ate $\frac{1}{12}$, Kendra ate $\frac{1}{12}$, and Fong ate $\frac{5}{12}$.

Chapter 1 Section 3

Question 6 Page 18

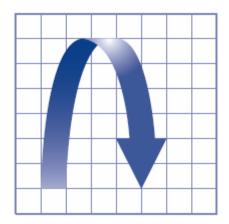
There are 16 triangles with side length 1. There are 7 triangles with side length 2. There are 3 triangles with side length 3. There is 1 triangle with side length 4.

The total number of triangles is 16 + 7 + 3 + 1 = 27.



Question 7 Page 18

The area of each square is $0.5 \times 0.5 = 0.25$ cm². Count squares to estimate the area of the arrow as about 12 squares, or 3 cm².



Chapter 1 Section 3 Question 8 Page 18

The snail must climb a total of 27 m. Each day, it climbs a net 1 m. It will take 23 days to reach a point 4 m from the top of the pipe. On the 24th day, it will reach the top of the pipe.

Chapter 1 Section 3 Question 9 Page 18

Answers will vary. A sample answer is given.

A cat lives about 15 years, and has a heart rate of about 150 beats/min.

Number of beats in a lifetime = $150 \times 60 \times 24 \times 365 \times 15$ = $1180\ 000\ 000$

A cat's heart will beat about 1.18 billion times during its lifetime.

Chapter 1 Section 3

Question 10 Page 18

9	6	1	7	5	З	2	8	4
2	8	7	6	1	4	5	9	З
4	З	5	8	2	9	6	1	7
1	7	6	4	8	5	9	З	2
З	2	8	9	6	1	4	7	5
5	4	9	2	З	7	1	6	8
7	9	З	1	4	Ζ	8	5	6
8	1	Ζ	5	7	6	З	4	9
6	5	4	З	9	8	7	2	1

Question 11 Page 18

Answers will vary.

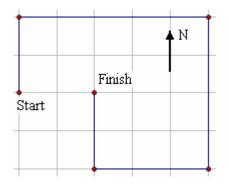
Chapter 1 Section 3 Question 12 Page 18

A polygon with 20 sides has 20 vertices. A diagonal is formed by connecting a vertex to another vertex, excluding itself and the vertices on either side. Therefore, each of the 20 vertices can be joined to 17 others. This implies that you have $20 \times 17 = 340$ diagonals. However, each diagonal has been counted twice. Divide by 2 to obtain the correct answer of 170 diagonals.

Chapter 1 Section 4 Focus on Representing

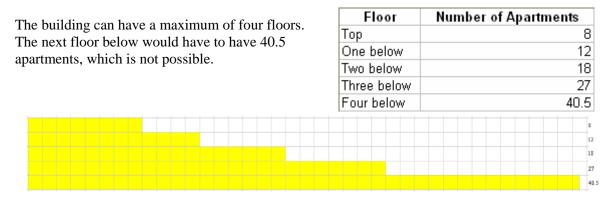
Chapter 1 Section 4 Question 1 Page 21

The treasure is 2 km east of the starting point.



Chapter 1 Section 4

Question 2 Page 21

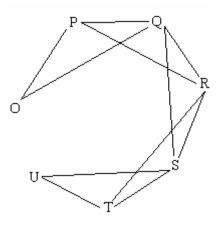


Chapter 1 Section 4

Question 3 Page 21

O must call P and Q. P must call Q and R. Q must call R and S. R must call S and T. S must call T and U. T must call U.

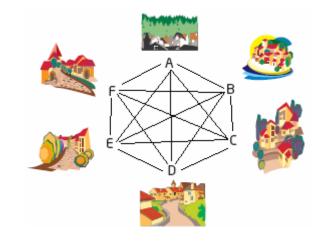
This adds to 11 conversations.



Question 4 Page 21

Question 5 Page 21

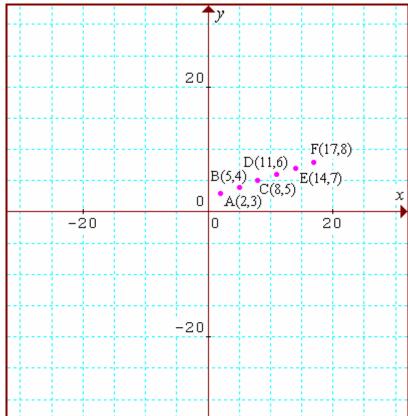
15 roads must be built.



Chapter 1 Section 4

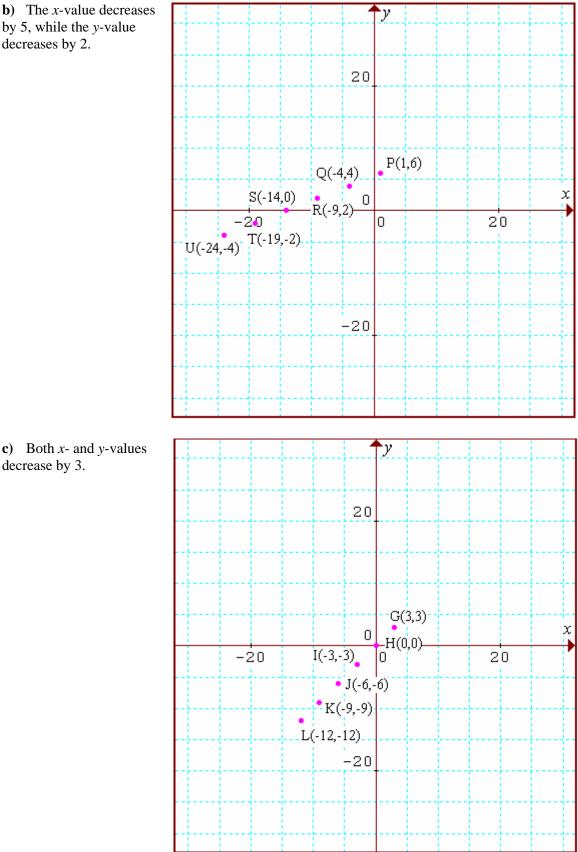
a) The *x*-value increases by 3 while the *y*-value

by 3, while the *y*-value increases by 1.



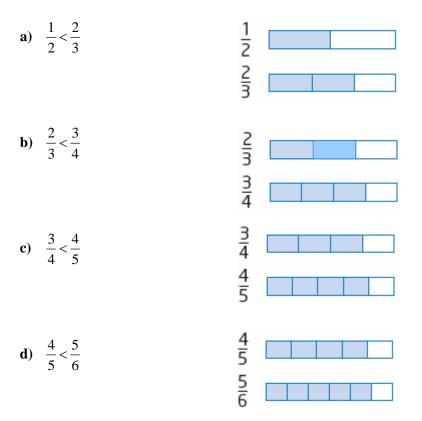
b) The *x*-value decreases by 5, while the *y*-value decreases by 2.

decrease by 3.



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For each part, the numerator and denominator of the fraction are two consecutive integers. The numerator of the second fraction equals the denominator of the first. In each case the value of the first fraction is less than that of the second.



Question 7 Page 22



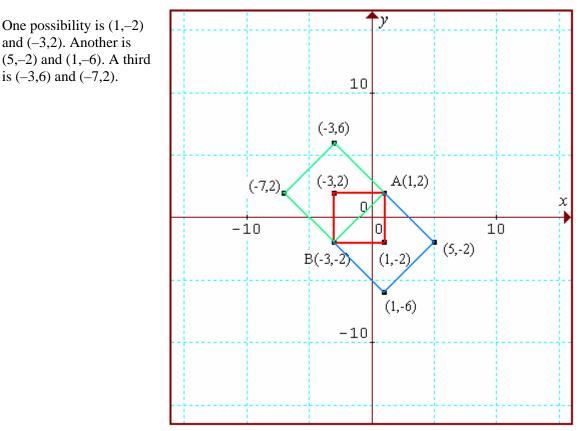
b) If the driver cog makes half a turn, it turns through 15 teeth. The driving cog must make $\frac{15}{20}$, or $\frac{3}{4}$ turns.

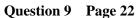
c) If the driving cog turns through 5 turns, it turns through 100 teeth. The driver cog must make $\frac{100}{30}$, or $3\frac{1}{3}$ turns.

d) If the driver cog makes half a turn, it turns through 12 teeth. The driving cog must turn through $\frac{12}{40}$, or $\frac{3}{10}$ turns.

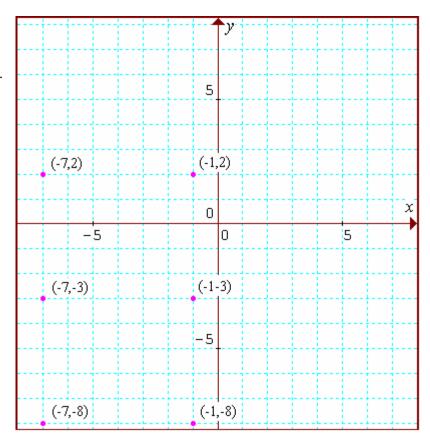
Chapter 1 Section 4

Question 8 Page 22

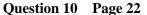




The base of the triangle measures 6 units. For an area of 15 square units, the height must be 5 units. Since the triangle is a right triangle, this point can be above or below either of the given points. The third vertex can be at (-7,2), (-7,-8), (-1,2) or (-1.-8.



Chapter 1 Section 4

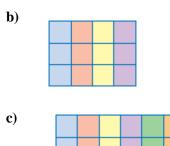


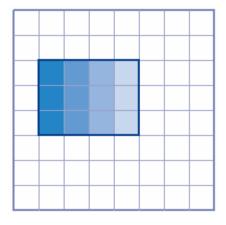
The same answers would result. The middle cog does not change the ratio of the driver cog to the driving cog.

Chapter 1 Section 5 Focus on Selecting Tools and Computational Strategies

Chapter 1 Section 5 Question 1 Page 25

a) 12 squares divided into 4 columns results in 3 squares in each column.





d) You cannot arrange the squares with a width of zero. The quotient $12 \div 0$ is undefined.

Chapter 1 Section 5 Question 2 Page 26

a) Start at 0, and subtract $\frac{2}{3}$ three times to arrive at -2.



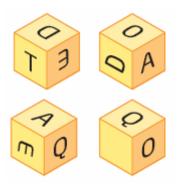


Chapter 1 Section 5 Question 3 Page 26

- a) The letter T belongs on the blank face. It is upside-down.
- **b**) You can make a physical model of the cube.
- c) Answers will vary.

Chapter 1 Section 5 Question 4 Page 26

Answers will vary. Some useful tools would be a survey, and a calculator.



Chapter 1 Section 5 Question 5 Page 26

Answers will vary. Sample answers are shown.

a) A calculator is appropriate for complicated math expressions involving square roots and fractions.

- **b**) Grid paper is useful when drawing a diagram to scale.
- c) A physical model is appropriate for solving real-life problems.
- d) A computer is useful for organizing data and generating complicated graphs.

Chapter 1 Section 5 Ques	stion 6 Page 26
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 $39^5 = 90\ 224\ 199$

Use a calculator for this problem.

Chapter 1 Section 5 Question 7 Page 26

- a) Subtract 6 to obtain the next term. The missing terms are -57 and -63.
- **b**) Multiply by 3 to obtain the next term. The missing terms are 1215 and 3645.
- c) Divide by -2 to obtain the next term. The missing terms are -4 and 2.
- d) Subtract 3 to obtain the next term. The missing terms are -158 and -161.
- e) Multiply by -2 to obtain the next term. The missing terms are -6144 and 12288.
- f) Add 24 to obtain the next term. The missing terms are 32 and 56.

Chapter 1 Section 5 Question 8 Page 26

a) Divide by increasing integers, starting with 2. The next three terms are $\frac{1}{3}$, $\frac{1}{21}$, and $\frac{1}{168}$.

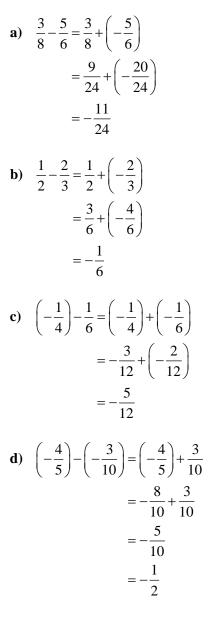
b) Subtract $\frac{1}{3}$ to obtain the next term. The next three terms are $-\frac{4}{3}$, $-\frac{5}{3}$, and -2.

c) Subtract $\frac{1}{4}$ to obtain the next term. The next three terms are $0, -\frac{1}{4}$, and $-\frac{1}{2}$.

d) Subtract $\frac{1}{12}$ to obtain the next term. The next three terms are $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$.

Question 9 Page 27

a) $-\frac{1}{2} + \left(-\frac{1}{2}\right) = -\frac{2}{2}$ = -1b) $-\frac{2}{3} + \left(-\frac{3}{4}\right) = -\frac{8}{12} + \left(-\frac{9}{12}\right)$ $= -\frac{17}{12}$ $= -1\frac{5}{12}$ c) $\frac{1}{7} + \left(-\frac{2}{5}\right) = \frac{5}{35} + \left(-\frac{14}{35}\right)$ $= -\frac{9}{35}$ d) $-\frac{2}{3} + \frac{3}{8} = -\frac{16}{24} + \frac{9}{24}$ $= -\frac{7}{24}$



Chapter 1 Section 5

Question 11 Page 27

Answers will vary. To multiply rational numbers in fraction form, multiply the numerators and multiply the denominators. To divide rational numbers in fraction form, multiply the dividend by the reciprocal of the divisor.

Examples:

1	
2 3 6	2 3 2 4
-×-=	$-\div-=-\times-$
3 4 12	3 4 3 3
1	8
= -	= -
2	- 9

a) $-\frac{5}{6} \times \frac{3}{10} = -\frac{\cancel{3}}{\cancel{6}} \times \frac{\cancel{3}}{\cancel{10}} = -\frac{\cancel{3}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{3}}$ $=-\frac{1}{4}$ **b**) $\left(-\frac{1}{7}\right) \times \left(-\frac{3}{5}\right) = \frac{3}{35}$ c) $\left(-\frac{1}{8}\right) \times \frac{6}{11} = \left(-\frac{1}{8}\right) \times \frac{3}{11}$ $=-\frac{3}{44}$ $\mathbf{d}) \quad \frac{7}{8} \div \left(-\frac{5}{6}\right) = \frac{7}{8} \times \left(-\frac{6}{5}\right)$ $=\frac{7}{\cancel{8}}\times\left(-\frac{\cancel{6}}{5}\right)$ $=-\frac{21}{20}$ $=-1\frac{1}{20}$ e) $\left(-\frac{5}{2}\right) \div \left(-\frac{3}{2}\right) = \left(-\frac{5}{2}\right) \checkmark \left(-\frac{8}{2}\right)$

(a)
$$\left(-\frac{5}{12}\right) \div \left(-\frac{5}{8}\right) = \left(-\frac{5}{12}\right) \times \left(-\frac{5}{3}\right)$$
$$= \left(-\frac{5}{12}\right) \times \left(-\frac{8}{3}\right)$$
$$= \frac{10}{9}$$
$$= 1\frac{1}{9}$$

f)

$$\begin{pmatrix} -4\frac{2}{5} \end{pmatrix} \div 1\frac{4}{7} = \begin{pmatrix} -\frac{22}{5} \end{pmatrix} \div \frac{11}{7}$$

$$= \begin{pmatrix} -\frac{22}{5} \end{pmatrix} \times \frac{7}{11}$$

$$= \begin{pmatrix} -\frac{22}{5} \end{pmatrix} \times \frac{7}{11}$$

$$= -\frac{14}{5}$$

$$= -2\frac{4}{5}$$

Question 13 Page 27

- a) Answers will vary.
- **b**) The thickness of the two layers is 2×0.08 , or 0.16 mm.
- c) Use a calculator to multiply the thickness by 2 twenty times. The answer is 83 886.08 mm.

d) Answers will vary. As the number of folds increases, the thickness also increases, making it more difficult to fold the paper.

Chapter 1 Section 5 Question 14 Page 27

Answers will vary. You can use fraction strips or a grid diagram.

Chapter 1 Section 5

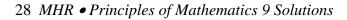
Question 15 Page 28

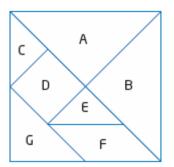
Length (cm)	Width (cm)	Perimeter (cm)	Area (cm²)
1	9	20	9
2	8	20	16
3	7	20	21
4	6	20	24
5	5	20	25
6	4	20	24
7	З	20	21
8	2	20	16
9	1	20	9

a) A and B are each $\frac{1}{4}$ of the whole square. C and E are each $\frac{1}{16}$ of the whole square.

D, F and G are each
$$\frac{1}{8}$$
 of the whole square.

b) **i**) $A + B = \frac{1}{4} + \frac{1}{4}$ $=\frac{1}{2}$ **ii)** C + G = $\frac{1}{16} + \frac{1}{8}$ $=\frac{1}{16}+\frac{2}{16}$ $=\frac{3}{16}$ **iii**) $D + E = \frac{1}{8} + \frac{1}{16}$ $=\frac{2}{16}+\frac{1}{16}$ $=\frac{3}{16}$ **iv**) $F - E = \frac{1}{8} - \frac{1}{16}$ $=\frac{2}{16}-\frac{1}{16}$ $=\frac{1}{16}$ **v**) $\frac{1}{4}A = \frac{1}{4}\left(\frac{1}{4}\right)$ $=\frac{1}{16}$ **vi**) $\frac{1}{2}$ D - F = $\frac{1}{2}\left(\frac{1}{8}\right) - \frac{1}{8}$ $=-\frac{1}{16}$





Chapter 1 Section 5 Question 17 Page 28

- a) F = C + E
- $\mathbf{b)} \quad \mathbf{B} = \mathbf{C} + \mathbf{E} + \mathbf{F}$

Chapter 1 Section 5 Question 18 Page 28

Subtract 4 to obtain the next term. To obtain the thousandth term, take 45, and subtract 4, 999 times to obtain 45 - 4(999) = -3951.

Chapter 1 Section 5 Question 19 Page 28

Subtract 7 to obtain the next term. From 100 to -600 is 700, or 100 subtractions of 7. Therefore, -600 is the 101st term.

Chapter 1 Section 5 Question 20 Page 28

Answers will vary. A sample answer is shown.

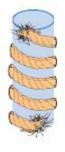
A cup is about 250 mL. A bathtub measures about 200 cm by 80 cm by 60 cm, for a volume of $200 \times 80 \times 60 = 960\ 000\ \text{mL}$. The number of cups required is $\frac{960\ 000}{250} = 3840$. About 4000 cups are required to fill a bathtub.

Chapter 1 Section 5 Question 21 Page 28

Use a physical model. If you fold it once, and then cut it, you will have 3 pieces of string. If you fold it twice, and cut it, you will have 5 pieces. If you fold it 3 times, and cut it, you will have 9 pieces. A general formula for the number of pieces after n folds is $2^n + 1$.

Chapter 1 Section 5 Question 22 Page 28

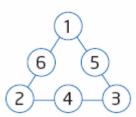
Make a physical model. The rope is about 47 m long.



Chapter 1 Section 6 Focus on Reasoning and Proving

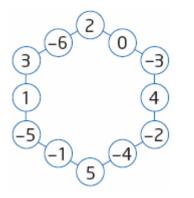
Chapter 1 Section 6

Question 1 Page 31



Chapter 1 Section 6

Question 2 Page 31



Chapter 1 Section 6

Question 3 Page 31

Let the first number be *n*. The next two numbers are n + 1 and n + 2. The sum of these numbers is n + n + 1 + n + 2 = 3n + 3. 3*n* is a multiple of 3, and must be divisible by 3. Adding 3 to 3*n* produces another multiple of 3, and is divisible by 3.

Chapter 1 Section 6 Question 4 Page 31

Answers will vary. Example: Since a newspaper is made by folding a sheet in half, there will always be an even number of pages because the number of pages equals two times the number of sheets.

Question 5 Page 32

Click <u>here</u> to load the spreadsheet.

	A .				
Amount	Quarters	Dimes	Nickels		Number of Coins
1				1	1
2				2	2
3				3	3
4				4	4
5			1		1
6			1	1	2
7			1	2	3
8			1	3	4
9			1	4	5
10		1			1
11		1		1	2
12		1		2	3
13		1		3	4
14		1		4	5
15		1	1		2
16		1	1	1	3
17		1	1	2	4
17		1	1	3	5
10		1	1	4	6
20		2	1	4	2
20		2		1	
22		2		2	4
23		2		3	5
24		2		4	6
25	1				1
26	1			1	2
27	1			2	3
28	1			3	4
29	1			4	5
30	1		1		2
31	1		1	1	3
32	1		1	2	4
33	1		1	3	5
34	1		1	4	6
35	1	1			2
36	1	1		1	3
37	1	1		2	4
38	1	1		3	5
39	1	1		4	5
40	1	1	1		3
41	1	1	1	1	4
42	1	1	1	2	5
43	1	1	1	3	6
44	1	1	1	4	7
45	1	2	-		3
45	1	2		1	4
40	1	2		2	5
47	1	2		3	6
40	1	2		4	7
	2	2		4	2
50	2				2

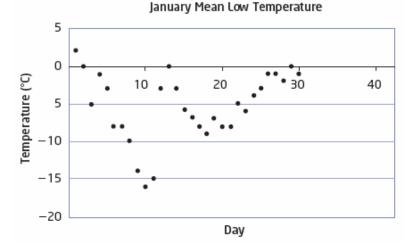
- a) $5 \times 2 + 8 3 = 15$
- **b**) $25 \div 5 + 11 = 25 9$

c)
$$\frac{1}{2} + \frac{1}{3} = \frac{11}{12} - \frac{1}{2}$$

d)
$$\frac{2}{3} \times \left(-\frac{1}{8}\right) = -\frac{1}{12}$$

Question 7 Page 32

a) The sum of the temperatures for the first 30 days is 30(-5) = -150. You are looking for a mean of -6°C over 31 days. The sum of the temperatures must be 31(-6) = -186. The difference is -186 - (-150) = -36. The temperature required on January 31 is -36 °C.



b) You are looking for a

mean of -4.5° C over 31 days. The sum of the temperatures must be 31(-4.5) = -139.5. The difference is -139.5 - (-150) = 10.5. The temperature required on January 31 is 10.5 °C.

Chapter 1 Section 6 Question 8 Page 32

a) 2 is a prime number, but is not odd.

Answers will vary.

- **b**) 1+2=3.3 is not negative.
- c) $\frac{3}{2}$ is a fraction, but is greater than 1.
- d) A trapezoid is a quadrilateral, but not a rectangle.

Ouestion 9 Page 32

Select each square in turn, and move the knight. You can always find a combination of moves that will land the knight on that square.

Chapter 1 Section 6

Question 10 Page 33

Three-quarters of a block of cheese plus three-quarters of a pound add to a complete block. The three-quarter pound weight makes up one-quarter of a block. The weight of a

complete block must be $4\left(\frac{3}{4}\right) = 3$ pounds.

Chapter 1 Section 6

Question 11 Page 33

Seven of the integers between -1 and -10 can be written as a difference of squares.



Integer	Difference of Squares
-1	0 ² - 1 ²
-2	
-3	1 ² - 2 ²
-4	0 ² - 2 ²
-5	2 ² - 3 ²
-6	
-7	3 ² - 4 ²
-8	1 ² - 3 ²
-9	0 ² - 3 ²
-10	

Chapter 1 Section 6

Use the "guess and check" method. A calculator or

Question 12 Page 33

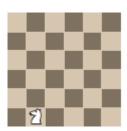
Use the "guess and check" method. A calculator or computer is a useful tool for this question.	Ride	Cost per Ride	Number of Times	Cost
	Roller Magic	\$3.25	1	\$3.25
	Death Drop	\$3.75	1	\$3.75
	The Amazing Loop	\$4.00	2	\$8.00
	Fire Pit	\$4.50	4	\$18.00
			Total Spent	\$33.00

Chapter 1 Section 6

Question 13 Page 33

Answers will vary. Divide the surface area of each hallway by the surface area of one tile. A sample answer is shown.

Fridgeway High School has two floors. Each has a hallway measuring 60 m long and 4 m wide. Each tile measures 0.4 m by 0.4 m. The total area of the hallways is $2 \times 60 \times 4 = 480 \text{ m}^2$. The area of a tile is $0.4 \times 0.4 = 0.16 \text{ m}^2$. The number of tiles required is $\frac{480}{0.16} = 3000$.



5	4	1	6	9	7	8	3	2
2	6	7	5	З	8	1	4	9
8	9	З	2	4	1	5	7	6
9	2	5	З	7	4	6	8	1
4	1	6	8	5	9	7	2	З
7	З	8	1	6	2	9	5	4
6	7	2	4	1	5	З	9	8
1	5	4	9	8	З	2	6	7
З	8	9	7	2	6	4	1	5

Chapter 1 Section 6 Question 15 Page 33

Answers will vary. A sample solution is shown.

The mass of a school bus is about 4800 kg. The average mass of a high school student is about 70 kg. The mass of a bus with 45 student passengers is 4800+45(70), or 7950 kg.

Chapter 1 Section 6 Question 16 Page 33

a) Construct a table to help you keep track of the sequence of numbers. Use the table to determine that the 50th term is -10. Click <u>here</u> to load the spreadsheet.

b) The 100th term is -14.

Number	Occurences	Total
1	1	1
-2 3	2	3
3	3	6
-4	4	10
5	5	15
-6 7	6	21 28
7	7	28
-8	8	36
9	9	45
-10	10	55
11	11	66
-12	12	78
13	13	91
-14	14	105
15	15	120
-16	16	136

c) The sum of the first 50 terms is

1 + 2(-2) + 3(3) + 4(-4) + 5(5) + 6(-6) + 7(7) + 8(-8) + 9(9) + 5(-10) = -5.

d) The sum of the first 50 terms is

$$1+2(-2)+3(3)+4(-4)+5(5)+6(-6)+7(7)+8(-8)+9(9)+10(-10)$$

+11(11)+12(-12)+13(13)+9(-14)=-35.

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Chapter 1 Section 7 Focus on Reflecting

Chapter 1 Section 7 Question 1 Page 35

You obtain two double Mobius strips that are looped together. The twist before taping results in one strip inside the other.

Chapter 1 Section 7 Question 2 Page 35

Work backwards:

Subtract -380 - 12 = -392. Then, divide $\frac{-392}{-7} = 56$. The number is 56.

Chapter 1 Section 7 Question 3 Page 35

Work backwards:

Add
$$\frac{7}{12} + \frac{3}{4} = \frac{7}{12} + \frac{9}{12}$$

= $\frac{16}{12}$
= $\frac{4}{3}$

Then, multiply
$$2 \times \frac{4}{3} = \frac{8}{3}$$
$$= 2\frac{2}{3}$$

The number is $2\frac{2}{3}$.

Chapter 1 Section 7 Question 4 Page 35

Answers will vary. Working backward is very effective in these kinds of questions.

Chapter 1 Section 7 Question 5 Page 36

Find the lowest common multiple of the integers.

 $9 \times 8 = 72$. This is also divisible by 1, 2, 3, 4, and 6. Now, include 5 and 7.

$$72 \times 7 \times 5 = 2520$$
.

The least number divisible by all of the integers 1 through 9 is 2520.

Question 6 Page 36

Strategies will vary. Start by choosing -3 for *x*. Use a table to add values of *y* and *z* that will work. Then, try -2 for *x*. Continue until you have found all combinations that work.

X	Ŷ	Ζ
-3	-3	1
-3	-2	0
-3	-1	-1
-3	0	-2
-3	1	-3
-2	-3	0
-2	-2	-1
-2	-1	-2
-2	0	-3
-1	-3	-1
-1	-2	-2
-1	-1	-3
0	-3	-2
0	-2	-3
1	-3	-3

Chapter 1 Section 7

Question 7 Page 36

- $a) \quad 1 \times 2 \times 3 \times 4 = 24$
- **b**) $5^2 1 = 24$
- c) $2 \times 3 \times 4 \times 5 = 120$
- **d**) $11^2 1 = 120$
- $e) \quad 4 \times 5 \times 6 \times 7 = 840$
- **f**) $29^2 1 = 840$

g) When you multiply four consecutive natural numbers, the product equals one less than the square of one more than the product of the first and last number.

Example: in parts **a**) and **b**) the product of the first and last number plus 1 is $1 \times 4 + 1 = 5$. Square the 5 and subtract 1 to get the same number as in **a**).

h)
$$5 \times 6 \times 7 \times 8 = 1680$$

 $41^2 - 1 = 1680$
 $10 \times 11 \times 12 \times 13 = 17160$
 $131^2 - 1 = 17160$

i) The examples worked.

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-60+90 = 30 The character lands on zero after 5 moves. 30-75 = -45 -45+60 = 15 15-45 = -30-30+30 = 0

Chapter 1 Section 7

Question 9 Page 36

Do not include 1 or 100. 49 numbers are divisible by 2. 33 numbers are divisible by 3. However, 16 of these are numbers that are divisible by both 2 and 3. Subtract 16 from 49 + 33 to arrive at 66.

You can check your reasoning by using a hundred chart and selecting numbers that are divisible by 2 or 3.

1	21	41	61	81
2	22	42	62	82
3	23	43	63	83
4	24	44	64	84
5	25	45	65	85
6	26	46	66	86
7	27	47	67	87
8	28	48	68	88
9	29	49	69	89
10	30	50	70	90
11	31	51	71	91
12	32	52	72	92
13	33	53	73	93
14	34	54	74	94
15	35	55	75	95
16	36	56	76	96
17	37	57	77	97
18	38	58	78	98
19	39	59	79	99
20	40	60	80	100

Chapter 1 Section 7

Question 10 Page 36

a) Karen used too much milk and sugar. She assumed that 500 mL is 5 L, rather than 0.5 L for the milk. She also assumed that 125 g is 1.25 kg, rather than 0.125 kg.

b) Karen should have used 1.5 L of milk, and 0.375 kg of sugar.

Chapter 1 Section 7 Question 11 Page 36

Answers will vary. A sample answer is shown.

The population of Ontario is about 12 000 000. Assume that four people share a pizza, and have pizza once per month. This requires $\frac{12\ 000\ 000}{4} \times 12 = 36\ 000\ 000\ \text{pizzas}$. Assume that an average pizza measures about 0.6 m in diameter. The area is about $0.6 \times 0.6 = 0.36\ \text{m}^2$. The total area of the pizzas ordered in Ontario in a year is about $0.36 \times 36\ 000\ 000 = 12\ 960\ 000\ \text{m}^2$.

Question 12 Page 36

a) i) 2 7 6 9 5 1 4 3 8 ii) -3 2



b) Pair off the least and greatest numbers, moving toward the median. Then, put that single number in the centre square and arrange the pairs around it.

Chapter 1 Review Question 1 Page 37

a) Subtract 3 to obtain the next term. The next three terms are 0, -3, and -6.

b) Multiply by 2 to obtain the next term. The next three terms are 56, 112, and 224.

c) Add consecutive numbers starting from 1 to obtain the next term. The next three terms are 15, 20, and 26.

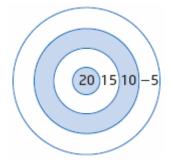
d) Subtract consecutive numbers starting from 4 to obtain the next term. The next three terms are -19, -27, and -36.

Chapter 1 Review Question 2 Page 37

The perimeter of the field is 2(100 + 70) = 340 m. The number of posts required is $\frac{340}{5} = 68$.

Chapter 1 Review Question 3 Page 37

a) To find the number of different scores with three arrows, make a table and list all possible patterns.



b) A possible table is shown. The different scores possible are 60, 55, 50, 45, 40, 35, 30, 25, 20, 15, 10, 5, 0, and -15.

20s	15s	10s	-5s	Score
3	0	0	0	60
2	1	0	0	55
2	0	1	0	50
2	0	0	1	35
1	2	0	0	50
1	1	1	0	45
1	1	0	1	30
1	0	2	0	40
1	0	1	1	25
1	0	0	2	10
0	3	0	0	45
0	2	1	0	40
0	2	0	1	25
0	1	2	0	35
0	1	1	1	20
0	1	0	2	5
0	0	З	0	30
0	0	2	1	15
0	0	1	2	0
0	0	0	З	-15

Question 4 Page 37

А

Ρ

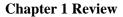
В

0

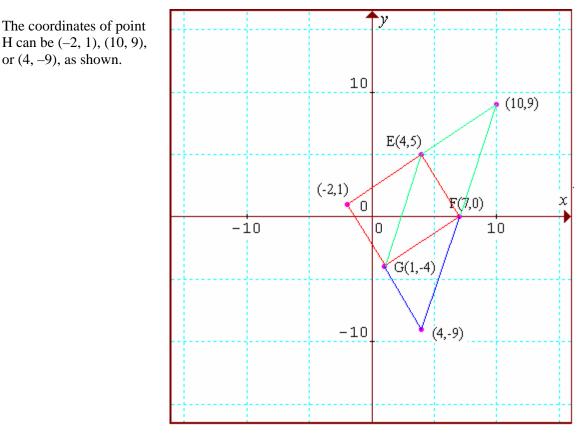
С

R

It cannot be done because one house will always be blocked from one of the services. You can connect two of the houses to all three services, and the third to two of the services. The last connection always requires crossing one of the previous connections.



Question 5 Page 37



Chapter 1 Review

Question 6 Page 37

There are 16 squares with a side length of 1. There are 9 squares with a side length of 2. There are 4 squares with a side length of 3. There is 1 square with a side length of 4.

The total number of squares is 30.

Question 7 Page 37

Calculate the time to travel 20 km, and then subtract this time from 7:30 P.M.

Time = $\frac{20}{28}$ $\doteq 0.714 \text{ h}$ $\doteq 43 \text{ min}$ Dave needs to catch the bus by 6:47 P.M.

Chapter 1 Review Question 8 Page 37

a) If each dimension is doubled, the area increases by a factor of 4, as shown.

b) Let the original area be A = lw. The new area is

New area = (2l)(2w)= 4lw

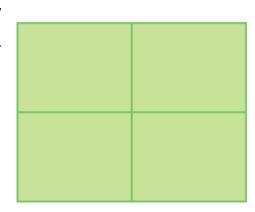
The new area is 4 times the old area.

Chapter 1 Review Question 9 Page 37

Use a systematic trial on a calculator to help you find that $7^9 = 40\,353\,607$.

Chapter 1 Review Question 10 Page 37

Divide by 3: $\frac{-402}{3} = -134$. The three consecutive integers must be -135, -134, and -133.



Question 11 Page 37

a)
$$\frac{2}{5} + \left(-\frac{3}{7}\right) = \frac{14}{35} + \left(-\frac{15}{35}\right)$$

 $= -\frac{1}{35}$
b) $-\frac{2}{9} - \left(-\frac{1}{6}\right) = -\frac{2}{9} + \frac{1}{6}$
 $= -\frac{4}{18} + \frac{3}{18}$
 $= -\frac{1}{18}$
c) $-\frac{2}{3} \times \frac{1}{4} = -\frac{\cancel{2}}{3} \times \frac{1}{\cancel{4}_{2}}$
 $= -\frac{1}{6}$
d) $\frac{7}{12} \div \left(-1\frac{3}{4}\right) = \frac{7}{12} \div \left(-\frac{7}{4}\right)$
 $= \frac{7}{12} \times \left(-\frac{4}{7}\right)$
 $= \frac{\cancel{7}}{\cancel{12}} \times \left(-\frac{\cancel{4}}{\cancel{7}}\right)$
 $= -\frac{1}{3}$

Chapter 1 Review

Question 12 Page 37

Answers will vary. Three possible answers are shown.

$$-\frac{11}{12} + \frac{5}{12} + \left(-\frac{1}{12}\right) = -\frac{7}{12}$$
$$\frac{1}{2} + \left(-\frac{3}{4}\right) + \left(-\frac{1}{3}\right) = -\frac{7}{12}$$
$$\frac{7}{12} + \left(-\frac{2}{3}\right) + \left(-\frac{1}{2}\right) = -\frac{7}{12}$$

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Three different bracelets can be made. One is as shown. The second has one blue bead between the two red beads. The third has 2 blue beads between the two red beads.

You can use diagrams to solve the problem, or make a physical model.

