Section 12.3: Wave Properties of Classical Particles

Tutorial 1 Practice, page 634

1. Given: $p = 1.8 \times 10^{-25}$ kg · m/s; $h = 6.63 \times 10^{-34}$ J · s

Required: λ

Analysis: Use the de Broglie relation, $\lambda = \dfrac{h}{p}$.

Solution:
$$\lambda = \frac{h}{p}$$
$$= \frac{6.63 \times 10^{-34}\ \cancel{\text{kg}} \cdot \text{m}^{\cancel{2}}/\cancel{\text{s}}}{1.8 \times 10^{-25}\ \cancel{\text{kg}} \cdot \cancel{\text{m}}/\cancel{\text{s}}}$$
$$\lambda = 3.7 \times 10^{-9}\ \text{m}$$

Statement: The de Broglie wavelength of the electron is 3.7×10^{-9} m, or 3.7 nm.

2. Given: $m = 1.7 \times 10^{-27}$ kg; $v = 3.4 \times 10^{5}$ m/s; $h = 6.63 \times 10^{-34}$ J · s

Required: λ

Analysis: The speed of the proton is much less than light speed, so we can use the classical momentum $p = mv$. Thus, the de Broglie relation, $\lambda = \dfrac{h}{p}$, becomes $\lambda = \dfrac{h}{mv}$.

Solution:
$$\lambda = \frac{h}{mv}$$
$$= \frac{6.63 \times 10^{-34}\ \cancel{\text{kg}} \cdot \text{m}^{\cancel{2}}/\cancel{\text{s}}}{(1.7 \times 10^{-27}\ \cancel{\text{kg}})(3.4 \times 10^{5}\ \cancel{\text{m}}/\cancel{\text{s}})}$$
$$\lambda = 1.1 \times 10^{-12}\ \text{m}$$

Statement: The proton's de Broglie wavelength is 1.1×10^{-12} m.

3. Given: $m = 140\ \text{g} = 1.4 \times 10^{-1}$ kg; $v = 140$ km/h; $h = 6.63 \times 10^{-34}$ J · s

Required: λ

Analysis: The speed of the proton is much less than light speed, so we can use the classical momentum $p = mv$. Thus, the de Broglie relation, $\lambda = \dfrac{h}{p}$, becomes $\lambda = \dfrac{h}{mv}$.

First, convert kilometres per hour to metres per second.

$$140\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{10^{3}\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3.6 \times 10^{3}\ \text{s}} = 3.89 \times 10^{1}\ \text{m/s (one extra digit carried)}$$

Solution:
$$\lambda = \frac{h}{mv}$$
$$= \frac{6.63 \times 10^{-34}\ \text{J}}{(1.40 \times 10^{-1}\ \text{kg})(3.89 \times 10^{1}\ \text{m/s})}$$
$$\lambda = 1.2 \times 10^{-34}\ \text{m}$$

Statement: The de Broglie wavelength of the baseball is 1.2×10^{-34} m.

4. The de Broglie wavelength of the baseball is 19 orders of magnitude smaller than the diameter of a proton; therefore, we could never expect to see any wave-like behavior of a macroscopic object like a baseball.

Research This: Exploring Quantum Computers, page 638

Answers may vary. Sample answers:
A. Quantum computers differ fundamentally from digital computers in the basic unit of information. For a digital computer, the basic unit is the bit, an element that can be in only one of two states, a "0" and a "1". For a quantum computer, the basic unit is a quantum bit or "qubit." A qubit can be in any superposition of two states, just like the electron trapped in a box (Figure 4 on page 636 of the Student Book) can be in a superposition of state 1 and state 2. Moreover, reading the state of a qubit is much different than reading the state of a bit. The reading of the state destroys the quantum superposition.
B. Several problems presently stand in the way of building practical quantum computers. One is the difficulty of making a computer with many qubits. Another problem is the fragility of the quantum superposition state; it is relatively easy to disturb the system, so that the superposition state gets destroyed. Another difficulty is finding a way to easily read the qubits.
C. A quantum computer's design should allow it to perform very quickly at some computations that are very difficult for digital computers, so some possible applications of quantum computing include the factoring of large numbers, database searching, and the simulation of quantum mechanical systems.

Section 12.3 Questions, page 639

1. Given: $m = 9.11 \times 10^{-31}$ kg; $\lambda = 150 \text{ nm} = 1.5 \times 10^{-7}$ m; $h = 6.63 \times 10^{-34}$ J · s
Required: speed of the electron, v
Analysis: Notice that the wavelength here is larger than that in the solution to Sample Problem 1 of Tutorial 1, and in that case the electron's speed is much less than that of light. A larger wavelength means that the speed is slower, so use the classical momentum in the de Broglie relation and solve for v.

$\lambda = \dfrac{h}{mv}$, so $v = \dfrac{h}{m\lambda}$.

Solution:
$$v = \frac{h}{m\lambda}$$
$$= \frac{6.63 \times 10^{-34}\text{ J} \cdot \text{s}}{(9.11 \times 10^{-31}\text{ kg})(1.5 \times 10^{-7}\text{ m})}$$
$$v = 4.9 \times 10^{3}\text{ m/s}$$

Statement: The electron's speed is 4.9×10^3 m/s, a non-relativistic speed.
2. Given: $m_{\text{proton}}/m_{\text{electron}} = 1800$; $\lambda_{\text{proton}} = \lambda_{\text{electron}}$
Required: $E_{\text{proton}}/E_{\text{electron}}$
Analysis: Assume that the two particles are non-relativistic (otherwise, we would need to know if the energy is the total energy or just the kinetic energy). In addition to using the

de Broglie relation, use the classical relation between E and v, as well as that between p and v.

$$\lambda_{\text{electron}} = \frac{h}{p_{\text{electron}}} \text{ and } \lambda_{\text{proton}} = \frac{h}{p_{\text{proton}}}$$

But, as $\lambda_{\text{proton}} = \lambda_{\text{electron}}$, then it follows that $p_{\text{proton}} = p_{\text{electron}}$. The classical kinetic energy can be written in terms of the classical momentum mv.

$$E_{\text{proton}} = \frac{1}{2}m_{\text{proton}}v^2{}_{\text{proton}}$$

$$= \frac{1}{2}\frac{(m_{\text{proton}}v_{\text{proton}})^2}{m_{\text{proton}}}$$

$$E_{\text{proton}} = \frac{1}{2}\frac{p^2{}_{\text{proton}}}{m_{\text{proton}}}$$

The same relation holds for the electron.

$$E_{\text{electron}} = \frac{1}{2}\frac{p^2{}_{\text{electron}}}{m_{\text{electron}}}$$

Thus,

$$\frac{E_{\text{electron}}}{E_{\text{proton}}} = \frac{\frac{1}{2}\frac{p^2{}_{\text{electron}}}{m_{\text{electron}}}}{\frac{1}{2}\frac{p^2{}_{\text{proton}}}{m_{\text{proton}}}}$$

$$\frac{E_{\text{electron}}}{E_{\text{proton}}} = \frac{p^2{}_{\text{electron}}m_{\text{proton}}}{m_{\text{electron}}p^2{}_{\text{proton}}}$$

Solution: $\frac{E_{\text{electron}}}{E_{\text{proton}}} = \frac{p^2{}_{\text{electron}}m_{\text{proton}}}{m_{\text{electron}}p^2{}_{\text{proton}}}$

$$= \frac{m_{\text{proton}}}{m_{\text{electron}}}$$

$$\frac{E_{\text{electron}}}{E_{\text{proton}}} = \frac{1800}{1}$$

Statement: When the proton's wavelength equals that of the electron, then they both have the same momentum. And when they have the same momentum and have non-relativistic speeds, then the ratio of their classical kinetic energies is 1800:1, with the electron having the higher energy because it is lighter.

3. (a) Given: $m = 1000.0$ kg; $v = 100.0$ km/h; $h = 6.63 \times 10^{-34}$ J · s
Required: λ
Analysis: Use the de Broglie relation, assuming non-relativistic speed: $\lambda = \dfrac{h}{mv}$.
First, convert kilometres per hour to metres per second.

$$100.0\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{10^3\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3.6 \times 10^3\ \text{s}} = 2.778 \times 10^1\ \text{m/s (one extra digit carried)}$$

Solution:
$$\begin{aligned}\lambda &= \frac{h}{mv} \\ &= \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{(1000.0\ \text{kg})(2.778 \times 10^1\ \text{m/s})} \\ \lambda &= 2.39 \times 10^{-38}\ \text{m}\end{aligned}$$

Statement: The de Broglie wavelength of the car travelling at 100.0 km/h is 2.39×10^{-38} m.
(b) Given: $m = 1000.0$ kg; $v = 10.0 \times 10^3$ km/h = 1.0×10^4 km/h; $h = 6.63 \times 10^{-34}$ J · s
Required: λ
Analysis: The car's speed, though fast, is still non-relativistic (~2800 m/s), so we can use the same relation as in (a), $\lambda = \dfrac{h}{mv}$. First, convert kilometres per hour to metres per second.

$$1.0 \times 10^4\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{10^3\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3.6 \times 10^3\ \text{s}} = 2.778 \times 10^3\ \text{m/s (one extra digit carried)}$$

Solution:
$$\begin{aligned}\lambda &= \frac{h}{mv} \\ &= \frac{6.63 \times 10^{-34}\ \text{J}}{(1000.0\ \text{kg})(2.778 \times 10^3\ \text{m/s})} \\ \lambda &= 2.39 \times 10^{-40}\ \text{m}\end{aligned}$$

Statement: The de Broglie wavelength of the car travelling at 1.0×10^4 km/h is 2.39×10^{-40} m.
(c) The de Broglie wavelength of the car at rest is undefined. As the speed decreases, the wavelength increases, and at zero speed, the wavelength blows up. (This result seems impossible, because we always see parked cars as solid objects and not spread out. However, consider how small the speed needs to be for the wavelength of the car to exceed 1 μm. The observer would have to establish that the speed of the car was less than about 6×10^{-31} m/s. Such a determination would be impossible, so we do not see parked cars spread out like a wave.)
4. In classical physics, particles occupy a definite position in space, and we can calculate exactly how a particle's position changes with time. Moreover, we can determine both the particle's position and the particle's velocity at each instant of time with arbitrary precision. In quantum mechanics, we do not know what happens to the particle between

measurements. Moreover, a measurement cannot determine the particle's position and velocity with arbitrary precision. Instead, quantum mechanics gives us the probabilities for obtaining various outcomes of the measurement.
5. Answers may vary. Sample answers:
(a) An example of experimental evidence for wave-like properties of matter is the Davisson–Germer experiment with electrons diffracting from a crystal. Other experiments have shown diffraction of larger particles.
(b) An example of experimental evidence for particle-like properties of electromagnetic radiation is the early experiments by Heinrich Hertz on the photoelectric effect. (Other experiments include that of the photovoltaic effect (e.g., solar cells)).
6. I think wave functions are real. Wave functions cannot be observed directly, so one might conclude that they are not real. However, we can say the same thing about atoms, and yet atoms seem to be quite real; we can touch objects and we can feel the wind. Similarly, we can infer the existence of wave functions through their influence on measurements. For example, the probability distribution of electrons striking the wall behind a pair of slits is a result of the electron's wave function.
7. Presently, all interpretations of quantum mechanics are consistent with the same observable results that we measure and experience. Yet quantum mechanics describes things that we cannot observe directly, such as the wave function. This indeterminacy of various aspects of quantum mechanics makes it possible for several views to be consistent with what we observe. Thus, different interpretations of quantum mechanics exist.

I think the Copenhagen interpretation is most likely because I am comfortable with the idea that there are things we simply cannot know. I do not like the pilot-wave interpretation, as it seems to imply that future events are predetermined. Future events might be predetermined, but I am not comfortable with the idea. Similarly, I do not like the many-worlds interpretation because it is hard for me to picture the universe continually splitting in two. The collapse interpretation is not so objectionable, but I prefer the Copenhagen interpretation.
8. According to the Heisenberg uncertainty principle, one cannot take exact measurements of an electron (or any other object) when it is at rest. If the electron is at rest, then $\Delta p = 0$ and Δx, the uncertainty in the electron's position, blows up. Thus, we could not determine where the electron was.
9. Willard Boyle earned a PhD in physics from McGill University. He worked at Bell Labs in New Jersey, then left for a job providing NASA with technological support for the Apollo space program, and then returned to Bell Labs in 1964, where he worked on developing electronic devices, including the charge-coupled device. Charge-coupled devices (CCDs) are designed around the photoelectric effect and the quantum mechanics of semiconductors. Other physical aspects of their operation are the motion of charges under an applied voltage, and for CCDs used for imaging, their operation depends on optics. They were originally designed to be used in several applications, including use as a memory device and shift register, but their most common application is for imaging. They were immediately useful in astronomy because, as imaging sensors, they could detect far fainter objects than those detected using film. They are now used in nearly all digital cameras.