Section 12.2: Photons and the Quantum Theory of Light

Tutorial 1 Practice, page 624

1. Given: $W = 4.60 \times 10^{-19}$ J; $h = 6.63 \times 10^{-34}$ J · s

Required: E_{photon}

Analysis: $E_{\text{photon}} = W$

Solution: $E_{\text{photon}} = W$

$$E_{\text{photon}} = 4.60 \times 10^{-19} \text{ J}$$

Statement: The lowest photon energy that can cause emission of electrons from calcium is 4.60×10^{-19} J, the work function of calcium.

2. Given: $\lambda_0 = 268$ nm $= 2.68 \times 10^{-7}$ m; $h = 6.63 \times 10^{-34}$ J · s; $c = 3.0 \times 10^8$ m/s

Required: the work function, W

Analysis: The lowest photon energy must satisfy the equation $E_{\text{photon}} = W$. Use Einstein's equation for the photon energy, written in terms of the wavelength, and substitute the threshold wavelength λ_0.

$$E_{\text{photon}} = \frac{hc}{\lambda_0}$$

Solution: $E_{\text{photon}} = \dfrac{hc}{\lambda_0}$

$$= \frac{(6.63 \times 10^{-34} \text{ J} \cdot \cancel{\text{s}})(3.0 \times 10^8 \ \cancel{\text{m}}/\cancel{\text{s}})}{(2.68 \times 10^{-7} \ \cancel{\text{m}})}$$

$$E_{\text{photon}} = 7.42 \times 10^{-19} \text{ J}$$

Statement: The minimum photon energy to release an electron from the material is 7.42×10^{-19} J, which is much closer to silver's value (7.43×10^{-19} J) than to lead's (6.81×10^{-19} J). So, the material is silver.

Tutorial 2 Practice, page 626

1. Given: $\lambda = 450 \text{ nm} = 4.50 \times 10^{-7}$ m; $h = 6.63 \times 10^{-34}$ J · s

Required: p_{photon}, the momentum of the photon

Analysis: Use the relation $p_{\text{photon}} = \dfrac{h}{\lambda}$.

Solution: $p_{\text{photon}} = \dfrac{h}{\lambda}$

$$= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{4.50 \times 10^{-7} \text{ m}}$$

$$p_{\text{photon}} = 1.5 \times 10^{-27} \text{ kg} \cdot \text{m/s}$$

Statement: The photon's momentum is 1.5×10^{-27} kg · m/s.

2. Given: $\lambda = 630 \text{ nm} = 6.30 \times 10^{-7} \text{ m}$; $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$; $c = 3.0 \times 10^{8} \text{ m/s}$

Required: E_{photon}, the energy of the photon

Analysis: Use the relation $E_{\text{photon}} = \dfrac{hc}{\lambda_0}$.

Solution:
$$\begin{aligned} E_{\text{photon}} &= \frac{hc}{\lambda_0} \\ &= \frac{(6.63 \times 10^{-34} \text{ J} \cdot \cancel{\text{s}})(3.0 \times 10^{8} \cancel{\text{m}}/\cancel{\text{s}})}{6.30 \times 10^{-7} \text{ m}} \\ E_{\text{photon}} &= 3.2 \times 10^{-19} \text{ J} \end{aligned}$$

Statement: The photon's energy is $3.2 \times 10^{-19} \text{ J}$.

3. Given: $E_{\text{photon}} = 2.2 \times 10^{-14} \text{ J}$; $c = 3.0 \times 10^{8} \text{ m/s}$

Required: p_{photon}, the momentum of the gamma photon

Analysis: $E_{\text{photon}} = hf$ and $p_{\text{photon}} = \dfrac{hf}{c}$, so $p_{\text{photon}} = \dfrac{E_{\text{photon}}}{c}$.

Solution:
$$\begin{aligned} p_{\text{photon}} &= \frac{E_{\text{photon}}}{c} \\ &= \frac{2.2 \times 10^{-14} \text{ J}}{3.0 \times 10^{8} \text{ m/s}} \\ p_{\text{photon}} &= 7.3 \times 10^{-23} \text{ kg} \cdot \text{m/s} \end{aligned}$$

Statement: The momentum of the gamma ray is $7.3 \times 10^{-23} \text{ kg} \cdot \text{m/s}$.

Tutorial 3 Practice, page 629

1. Given: $T = 5100 \text{ K}$

Required: λ

Analysis: Use Wien's law, $\lambda_{\text{max}} = \dfrac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{T}$.

Solution:
$$\begin{aligned} \lambda_{\text{max}} &= \frac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{T} \\ &= \frac{2.90 \times 10^{-3} \text{ m} \cdot \cancel{\text{K}}}{5100 \cancel{\text{K}}} \\ \lambda_{\text{max}} &= 5.7 \times 10^{-7} \text{ m} \end{aligned}$$

Statement: The maximum wavelength of the 5100 K blackbody is $5.7 \times 10^{-7} \text{ m}$, which gives a yellow colour.

2. Given: $\lambda_{\text{max}} = 510 \text{ nm} = 5.10 \times 10^{-7} \text{ m}$
Required: T
Analysis: Use Wien's law, rearranged to obtain T.

$$\lambda_{\text{max}} = \frac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{T}$$

$$T = \frac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{\lambda_{\text{max}}}$$

Solution: $T = \dfrac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{\lambda_{\text{max}}}$

$$= \frac{2.90 \times 10^{-3} \cancel{\text{m}} \cdot \text{K}}{5.10 \times 10^{-7} \cancel{\text{m}}}$$

$$T = 5700 \text{ K}$$

Statement: The temperature of the aqua-coloured blackbody is 5700 K.
3. Given: $T = 37$ ˚C = 310 K
Required: λ_{max}
Analysis: Use Wien's law, $\lambda_{\text{max}} = \dfrac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{T}$.

Solution: $\lambda_{\text{max}} = \dfrac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{T}$

$$= \frac{2.90 \times 10^{-3} \text{ m} \cdot \cancel{\text{K}}}{3.1 \times 10^{2} \cancel{\text{K}}}$$

$$\lambda_{\text{max}} = 9.4 \times 10^{-6} \text{ m}$$

Statement: The wavelength of maximum intensity for a human body is 9.4×10^{-6} m, or 9.4 μm. This is infrared light, which can be imaged with an infrared camera, but not the human eye.

Research This: Exploring Photonics, page 630

Answers may vary. Students will choose a technology or process that uses the particle nature of light. The sample answers for A to C below are for the photomultiplier tube:
A. A photomultiplier tube relies upon the photoelectric effect, and thus relies upon the particle nature of light. The tube also relies upon secondary emission, another process that is similar to the photoelectric effect but occurs for massive, charged particles such as electrons. Photomultiplier tubes consist of a photocathode and a series of electrodes, called dynodes, in an evacuated glass enclosure. The electrodes are each maintained at a more positive potential. A photon that strikes the photoemissive cathode releases an electron from the photocathode (material with very low work function). The released electron gets focused and accelerated by a magnetic electron lens and strikes the first dynode. The collision releases several electrons in a process called secondary emission. The electrons strike the next dynode, releasing even more electrons, and the process cascades along many more dynodes. This cascading effect creates 10^5 to 10^7 electrons for each photon hitting the first cathode, depending on the number of dynodes and the

accelerating voltage. The resulting avalanche of electrons hits the output (anode), where the amplified signal can be measured.
B. The photomultiplier tube counts photons, which would not be possible to interpret without the quantum theory of light.
C. The avalanche photodiode is the solid-state equivalent to the photomultiplier tube.
D. As of 2009, there were over 5000 Canadian companies in the Canadian Photonics Consortium, and these companies employed nearly 300 000 people.

Section 12.2 Questions, page 631

1. Given: W = 5.0 eV = 8.0 × 10^{-19} J; $h = 6.63 \times 10^{-34}\ \text{J} \cdot \text{s}$
Required: f_0, the minimum photon frequency that can eject an electron
Analysis: Use Einstein's relation for the photon energy, $E_{\text{photon}} = hf_0$, rearranged for f_0, and then substitute the photon energy with the work function, $E_{\text{photon}} = W$.

$$E_{\text{photon}} = hf_0$$

$$f_0 = \frac{E_{\text{photon}}}{h}$$

Now, substitute W for E_{photon}.

$$f_0 = \frac{W}{h}$$

Solution:

$$\begin{aligned} f_0 &= \frac{W}{h} \\ &= \frac{8.0 \times 10^{-19}\ \cancel{\text{J}}}{6.63 \times 10^{-34}\ \cancel{\text{J}} \cdot \text{s}} \\ f_0 &= 1.2 \times 10^{15}\ \text{Hz} \end{aligned}$$

Statement: The minimum frequency to eject an electron is 1.2×10^{15} Hz, which is ultraviolet, according to Table 3.

2. (a) The lower-energy material, cesium, is a better choice than aluminum for the metal piece in the photocell because it will capture all the photons that the aluminum will plus those with energy between 1.95 eV and 4.20 eV. Judging from Table 3, we would probably need a work function less than half the 5.0 eV of the aluminum to capture the lower-frequency visible light.

(b) Given: W = 1.95 eV = 3.12 × 10^{-19} J; $h = 6.63 \times 10^{-34}\ \text{J} \cdot \text{s}$
Required: f_0, the minimum photon frequency that can eject an electron
Analysis: Use Einstein's relation for the photon energy, $E_{\text{photon}} = hf_0$, rearranged for f_0, and then substitute the photon energy with the work function, $E_{\text{photon}} = W$.

$$E_{\text{photon}} = hf_0$$

$$f_0 = \frac{E_{\text{photon}}}{h}$$

Now, substitute W for E_{photon}.

$f_0 = \frac{W}{h}$

Solution: $f_0 = \frac{W}{h}$

$= \frac{3.12 \times 10^{-19} \text{ J}}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}$

$f_0 = 4.71 \times 10^{14} \text{ Hz}$

Statement: The lowest photon frequency that can be measured with the photocell is $4.71 \times 10^{14} \text{ Hz}$.

(c) A frequency of 4.71×10^{14} Hz is near the lower boundary of the visible range of the electromagnetic spectrum (4.3×10^{14} Hz).

3. The higher-intensity red light will not be able to eject electrons according to the photoelectric effect. The ability of light to eject electrons depends only on the frequency, not the intensity. So, if the low-intensity red light could not eject electrons, the higher-intensity red light cannot either.

4. (a) Given: $f = 100 \text{ MHz} = 1.00 \times 10^8 \text{ Hz}$; $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$; $c = 3.0 \times 10^8 \text{ m/s}$

Required: the energy, *E*, and momentum, *p*, of the photon

Analysis: Combine the equations $E = hf$ and $p = \frac{hf}{c}$ to obtain $p = \frac{E}{c}$. Since we can use *E* to calculate *p*, first calculate *E*.

Solution: $E = hf$

$= (6.63 \times 10^{-34} \text{ J} \cdot \text{s})(1.00 \times 10^8 \text{ Hz})$

$E = 6.63 \times 10^{-26}$ J (two extra digits carried)

$p = \frac{E}{c}$

$= \frac{6.63 \times 10^{-26} \text{ J}}{3.0 \times 10^8 \text{ m/s}}$

$p = 2 \times 10^{-34} \text{ kg} \cdot \text{m/s}$

Statement: The energy of an FM radio station with a frequency of 100 MHz is 7×10^{-26} J, and the momentum is $2 \times 10^{-34} \text{ kg} \cdot \text{m/s}$.

(b) Given: $\lambda = 633 \text{ nm} = 6.33 \times 10^{-7} \text{ m}$; $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$; $c = 3.0 \times 10^8 \text{ m/s}$

Required: the energy, *E*, and momentum, *p*, of the photon

Analysis: Combine the equations $E = hf$ and $\lambda = \frac{c}{f}$ to obtain $E = \frac{hc}{\lambda}$, and use $p = \frac{E}{c}$ from part (a). Since we can use *E* to calculate *p*, first calculate *E*.

Solution: $E = \frac{hc}{\lambda}$

$$= \frac{(6.63\times10^{-34}\ \text{J}\cdot\cancel{\text{s}})(3.0\times10^{8}\ \cancel{\text{m}}/\cancel{\text{s}})}{6.33\times10^{-7}\ \text{m}}$$

$= 3.1422\times10^{-19}$ J (two extra digits carried)

$E = 3.14\times10^{-19}\ \text{J}$

$$p = \frac{E}{c}$$

$$= \frac{3.1422\times10^{-19}\ \text{J}}{3.0\times10^{8}\ \text{m/s}}$$

$p = 1.05\times10^{-27}\ \text{kg}\cdot\text{m/s}$

Statement: The energy of a red light with a wavelength of 633 nm is 3.14×10^{-19} J and the momentum is $1.05\times10^{-27}\ \text{kg}\cdot\text{m/s}$.

(c) Given: $\lambda = 0.070$ nm = $7.0\ \times10^{-11}$ m; $h = 6.63\times10^{-34}\ \text{J}\cdot\text{s}$; $c = 3.0\times10^{8}$ m/s
Required: the energy, *E*, and momentum, *p*, of the photon
Analysis: The same as in part (b) above.

Solution: $E = \frac{hc}{\lambda}$

$$= \frac{(6.63\times10^{-34}\ \text{J}\cdot\cancel{\text{s}})(3.0\times10^{8}\ \cancel{\text{m}}/\cancel{\text{s}})}{7.0\ \times10^{-11}\ \cancel{\text{m}}}$$

$= 2.841\times10^{-15}$ J (two extra digits carried)

$E = 2.8\times10^{-15}\ \text{J}$

$$p = \frac{E}{c}$$

$$= \frac{2.841\times10^{-15}\ \text{J}}{3.0\times10^{8}\ \text{m/s}}$$

$p = 9.5\times10^{-24}\ \text{kg}\cdot\text{m/s}$

Statement: The energy of an X-ray photon of wavelength 0.070 nm is 2.8×10^{-15} J, and the momentum is $9.5\times10^{-24}\ \text{kg}\cdot\text{m/s}$.

5. An X-ray photon has greater energy than an ultraviolet photon because it has a greater frequency.

6. (a) There are 1.60×10^{-19} J in one electron-volt, so we multiply the 13.6 eV by 1.60×10^{-19} J to get 2.176×10^{-18} J (one extra digit carried). There are 2.18×10^{-18} J in 13.6 eV.

(b) Given: $E = 2.176\times10^{-18}$ J; $h = 6.63\times10^{-34}$ J·s; $c = 3.0\times10^{8}$ m/s
Required: f; λ
Analysis: Use Einstein's relation for the energy of a photon, $E = hf$, rearranged to isolate *f*: $f = \frac{E}{h}$. Then, use the relation between wavelength and frequency, $\lambda = \frac{c}{f}$, to calculate λ.
Solution:
$$f = \frac{E}{h}$$
$$= \frac{2.176\times10^{-18}\ \text{J}}{6.63\times10^{-34}\ \text{J}\cdot\text{s}}$$
$$= 3.2821\times10^{15}\ \text{Hz (two extra digits carried)}$$
$$f = 3.28\times10^{15}\ \text{Hz}$$

$$\lambda = \frac{c}{f}$$
$$= \frac{3.0\times10^{8}\ \text{m/s}}{3.2821\times10^{15}\ \text{Hz}}$$
$$\lambda = 9.14\times10^{-8}\ \text{m}$$
Statement: The frequency of the highest-energy photons emitted by a hydrogen atom is $3.28\times10^{15}\ \text{s}^{-1}$, and the wavelength is 9.14×10^{-8} m, which correspond to ultraviolet photons.

7. The working of a solar cell relies on quantum mechanics through the photoelectric effect and the quantum theory of semiconductors. The photoelectric effect removes an electron from a special region within the semiconductor wafer, and the electron flows through a circuit, producing electrical power.

The special region inside the semiconductor is similar to that in an LED. One side has been made to have electron charge carriers, and the other side to have missing electrons, which act as "holes" that carry positive charge according to the quantum theory of semiconductors. At the boundary, the electrons and holes combine, forming the special region called the depletion region. The depletion region does not have many charge carriers. The combining of the electrons and holes creates an electric field and thus a voltage. To generate power, the depletion region needs charges that can flow. These charges are produced by the photovoltaic effect, which is similar to the photoelectric effect; briefly, an incident photo travels into the depletion region and causes an electron to move up to a higher energy level in which it can conduct. The electron then gets pushed by the electric field through the circuit, generating electricity.