Section 11.4: Mass–Energy Equivalence

Tutorial 1 Practice, page 602

1. Given: $E_{\text{rest}} = 2.25 \times 10^{16}$ J; $c = 3.0 \times 10^{8}$ m/s

Required: m

Analysis: Rearrange the rest-mass equation by dividing both sides by c^2: $m = \dfrac{E_{\text{rest}}}{c^2}$

Solution:
$$m = \frac{E_{\text{rest}}}{c^2}$$
$$= \frac{2.25 \times 10^{16}\ \text{kg} \cdot \cancel{\text{m}^2/\text{s}^2}}{(3.0 \times 10^{8})^2\ \cancel{\text{m}^2/\text{s}^2}}$$
$$m = 0.25\ \text{kg}$$

Statement: The cellphone's rest mass is 0.25 kg.

2. (a) Given: $m = 1.67 \times 10^{-27}$ kg; $v = 0.800c$; $c = 3.0 \times 10^{8}$ m/s

Required: E_{total} in units of MeV

Analysis: First calculate the rest energy of the proton, $E_{\text{rest}} = mc^2$, and convert it to units of MeV. Then, use the equation $E_{\text{total}} = \dfrac{E_{\text{rest}}}{\sqrt{1 - \dfrac{v^2}{c^2}}}$.

Solution:
$$E_{\text{rest}} = mc^2$$
$$= (1.67 \times 10^{-27}\ \text{kg})(3.0 \times 10^{8}\ \text{m/s})^2 \left(\frac{1\ \text{MeV}}{1.60 \times 10^{-13}\ \text{J}} \right)$$
$$E_{\text{rest}} = 939.38\ \text{MeV (two extra digits carried)}$$
$$E_{\text{total}} = \frac{939.38\ \text{MeV}}{\sqrt{1 - \dfrac{(0.800\cancel{c})^2}{\cancel{c^2}}}}$$
$$E_{\text{total}} = 1.57 \times 10^{3}\ \text{MeV}$$

Statement: The given proton has a total energy of 1.57×10^{3} MeV.

(b) Given: $E_{\text{rest}} = 939.38\ \text{MeV};\ v = 0.800c$

Required: E_{k} in units of MeV

Analysis: Use the equations $E_{\text{total}} = \dfrac{E_{\text{rest}}}{\sqrt{1-\dfrac{v^2}{c^2}}}$ and $E_{\text{k}} = E_{\text{total}} - E_{\text{rest}}$ to obtain

$$E_{\text{k}} = \frac{E_{\text{rest}}}{\sqrt{1-\dfrac{v^2}{c^2}}} - E_{\text{rest}}$$

$$E_{\text{k}} = E_{\text{rest}}\left(\frac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} - 1\right)$$

Use $\dfrac{v}{c} = 0.800 = \dfrac{4}{5}$, making the denominator $\dfrac{3}{5}$.

Solution: $E_{\text{k}} = E_{\text{rest}}\left(\dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} - 1\right)$

$$E_{\text{k}} = (939.38\ \text{MeV})\left(\frac{1}{\left(\dfrac{3}{5}\right)} - 1\right)$$

$$E_{\text{k}} = 6.26\times10^{2}\ \text{MeV}$$

Statement: The proton has a kinetic energy of 6.26×10^2 MeV.

3. (a) Given: $m = 23\ \text{kg};\ c = 3.0\times10^{8}\ \text{m/s}$

Required: E_{rest}

Analysis: Use the rest-mass equation, $E_{\text{rest}} = mc^2$.

Solution: $E_{\text{rest}} = mc^2$

$$= (23\ \text{kg})\,(3.0\times10^{8}\ \text{m/s})^2$$

$$E_{\text{rest}} = 2.1\times10^{18}\ \text{J}$$

Statement: The typical CANDU fuel bundle has a rest-mass energy of 2.1×10^{18} J.

(b) Dividing the result in (a) by the average daily home energy use of 3.6×10^{10} J/day gives 5.8×10^7 days, which is nearly 160 000 years.

4. Given: $m = 2500$ kg; $E_{\text{k}} = 1.5 \times 10^{20}$ J; $c = 3.0 \times 10^{8}$ m/s

Required: v

Analysis: Rearrange the equation for E_{k}.

$$E_{\text{k}} = mc^2 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right)$$

$$\frac{E_{\text{k}}}{mc^2} + 1 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{\frac{E_{\text{k}}}{mc^2} + 1}$$

$$1 - \frac{v^2}{c^2} = \frac{1}{\left(\frac{E_{\text{k}}}{mc^2} + 1 \right)^2}$$

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\left(\frac{E_{\text{k}}}{mc^2} + 1 \right)^2}}$$

Solution:
$$\begin{aligned}
\frac{v}{c} &= \sqrt{1 - \frac{1}{\left(\frac{E_{\text{k}}}{mc^2} + 1 \right)^2}} \\
&= \sqrt{1 - \frac{1}{\left(\frac{1.5 \times 10^{20}}{(2.5 \times 10^{3})(3.0 \times 10^{8})^2} + 1 \right)^2}} \\
&= \sqrt{1 - \frac{1}{\left(\frac{2}{3} + 1 \right)^2}} \\
\frac{v}{c} &= \frac{4}{5} \\
&= 0.80 \\
v &= 0.80c
\end{aligned}$$

Statement: The asteroid has a speed of $0.80c$.

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1. Given: $m = 1$ kg; $v = 0.95c$

Required: $m_{\text{relativistic}}$

Analysis: $m_{\text{relativistic}} = \dfrac{m}{\sqrt{1-\dfrac{v^2}{c^2}}}$

Solution:
$$m_{\text{relativistic}} = \frac{m}{\sqrt{1-\dfrac{v^2}{c^2}}}$$
$$= \frac{1\text{ kg}}{\sqrt{1-\dfrac{(0.95\cancel{c})^{\cancel{2}}}{\cancel{c^2}}}}$$
$$m_{\text{relativistic}} = 3.2\text{ kg}$$

Statement: The relativistic mass of the 1 kg object is 3.2 kg.

2. The famous equation $E = mc^2$ tells us that the rest mass of an object and its energy are equivalent, or $E_{\text{rest}} = mc^2$. This equation shows that even at rest, and with no other sources of potential energy, a mass nevertheless has energy. It suggests the possibility, now well established, that mass can be converted to energy and vice versa, and thus the conservation of relativistic energy is really a conservation of mass and energy. In particular, the rest mass m can be converted to another form of energy in the amount of $E_{\text{rest}} = mc^2$.

3. Given: $E_{\text{rest}} = 4.20\times10^6$ J; $c = 3.0\times10^8$ m/s

Required: m

Analysis: $E_{\text{rest}} = mc^2$, so $m = \dfrac{E_{\text{rest}}}{c^2}$.

Solution:
$$m = \frac{E_{\text{rest}}}{c^2}$$
$$= \frac{4.20\times10^6\text{ kg}\cdot\cancel{\text{m}^2/\text{s}^2}}{(3.0\times10^8)^2\ \cancel{\text{m}^2/\text{s}^2}}$$
$$m = 4.67\times10^{-11}\text{ kg}$$

Statement: A rest mass of about 4.67×10^{-11} kg has as much rest energy as 1 kg of TNT explosive.

4. Given: $m = 1.67 \times 10^{-27}$ kg; $c = 3.0 \times 10^{8}$ m/s

Required: E_{rest} for two masses m that annihilate

Analysis: $E_{\text{rest}} = mc^2$. Use the rest mass energy of one proton and multiply by two.

Solution:
$$E_{\text{rest}} = 2mc^2$$
$$= 2(1.67 \times 10^{-27}\text{ kg})(3.0 \times 10^{8}\text{ m/s})^2$$
$$E_{\text{rest}} = 3.01 \times 10^{-10}\text{ J}$$

Statement: When a proton and anti-proton annihilate each other, the energy released is the sum of their rest-mass energies, which equals 3.01×10^{-10} J.

5. (a) Given: $E_{\text{rest}} = 1.28$ MeV; $c = 3.0 \times 10^{8}$ m/s

Required: m

Analysis: $E_{\text{rest}} = mc^2$, so $m = \dfrac{E_{\text{rest}}}{c^2}$. Change the energy units to joules.

Solution:
$$m = \frac{E_{\text{rest}}}{c^2}$$
$$= \frac{1.28\ \cancel{\text{MeV}}}{(3.0 \times 10^{8})^2\text{ m}^2/\text{s}^2}\left(\frac{1.60 \times 10^{-13}\text{ J}}{1\ \cancel{\text{MeV}}}\right)$$
$$m = 2.28 \times 10^{-30}\text{ kg}$$

Statement: The subatomic particle's rest mass is 2.28×10^{-30} kg.

(b) Given: $E_{\text{rest}} = 1.28$ MeV; $E_{\text{total}} = 1.72$ MeV

Required: E_{k}

Analysis: The total energy is the sum of the rest energy and the kinetic energy. Thus, the kinetic energy is the difference between the two given energies.

Solution:
$$E_{\text{k}} = E_{\text{total}} - E_{\text{rest}}$$
$$= 1.72\text{ MeV} - 1.28\text{ MeV}$$
$$E_{\text{k}} = 0.44\text{ MeV}$$

Statement: The relativistic kinetic energy of the subatomic particle is 0.44 MeV.

(c) Given: $E_{\text{rest}} = 1.28$ MeV; $E_{\text{total}} = 1.72$ MeV; $c = 3.0 \times 10^{8}$ m/s

Required: v

Analysis: Use the expression for the relativistic total energy and solve for v/c.

$$E_{\text{total}} = \frac{E_{\text{rest}}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\left(\frac{E_{\text{total}}}{E_{\text{rest}}}\right)^2 = \frac{1}{1 - \frac{v^2}{c^2}}$$

$$1 - \frac{v^2}{c^2} = \frac{1}{\left(\frac{E_{\text{total}}}{E_{\text{rest}}}\right)^2}$$

$$\frac{v^2}{c^2} = 1 - \frac{1}{\left(\frac{E_{\text{total}}}{E_{\text{rest}}}\right)^2}$$

$$\frac{v}{c} = \sqrt{1 - \frac{1}{\left(\frac{E_{\text{total}}}{E_{\text{rest}}}\right)^2}}$$

Solution: $\frac{v}{c} = \sqrt{1 - \frac{1}{\left(\frac{E_{\text{total}}}{E_{\text{rest}}}\right)^2}}$

$$= \sqrt{1 - \frac{1}{\left(\frac{1.72\ \cancel{\text{MeV}}}{1.28\ \cancel{\text{MeV}}}\right)^2}}$$

$\frac{v}{c} = 0.6680$ (two extra digits carried)

$v = 2.00 \times 10^8\ \text{m/s}$

Statement: The particle's speed in the laboratory is 2.00×10^8 m/s (about 2/3 the speed of light).

6. Given: $m = 7.35 \times 10^{22}$ kg; $v = 1.02 \times 10^3$ m/s; $c = 3.0 \times 10^8$ m/s

Required: mass that, when converted to energy, would produce the observed kinetic energy of the Moon

Analysis: The speed of the Moon is more than 5 orders of magnitude less than the speed of light. Hence, the kinetic energy of the Moon equals the classical kinetic energy to at least 10 decimal places. So, we calculate the classical kinetic energy of the Moon and determine the mass equivalent using $E_{\text{k-classical}}$ = (mass equivalent)c^2.

Solution: $$E_{\text{k-classical}} = (\text{mass equivalent})c^2$$

$$\text{mass equivalent} = \frac{E_{\text{k-classical}}}{c^2}$$

$$= \frac{\frac{1}{2}mv^2}{c^2}$$

$$= \frac{\frac{1}{2}(7.35\times10^{22}\ \text{kg})(1.02\times10^{3}\ \cancel{\text{m}}/\text{s})^2}{(3.0\times10^{8}\ \cancel{\text{m}}/\text{s})^2}$$

$$\text{mass equivalent} = 4.25\times10^{11}\ \text{kg}$$

Statement: A conversion of 4.25×10^{11} kg to energy would be enough to accelerate the Moon from rest to its present orbital speed.

7. Given: $E_{\text{rest T}} = 2809.4$ MeV; $E_{\text{rest P}} = 938.3$ MeV; $E_{\text{rest N}} = 939.6$ MeV

Required: energy released, E_{released}, when two neutrons combine with one proton to form one tritium nucleus

Analysis: Use conservation of mass–energy, E_{total}(before) = E_{total}(after), and the fact that the neutrons and protons are initially at rest. After the nuclear reaction, we have only the rest energy of the tritium plus E_{released}, where E_{released} may include any allowed combination of tritium kinetic energy and radiative energy of gamma radiation.

Solution: $$E_{\text{total}}(\text{before}) = E_{\text{total}}(\text{after})$$

$$E_{\text{rest P}} + 2E_{\text{rest N}} = E_{\text{rest T}} + E_{\text{released}}$$

$$(938.3\ \text{MeV}) + 2(939.6\ \text{MeV}) = (2809.4\ \text{MeV}) + E_{\text{released}}$$

$$E_{\text{released}} = 8.1\ \text{MeV}$$

Statement: The energy released when the proton combines with the two neutrons to form a tritium nucleus is 8.1 MeV (which may contribute to the tritium's kinetic energy and produce gamma radiation).

8. Given: $E_k = E_{\text{rest}}$

Required: v

Analysis: Substitute $E_{\text{rest}} = mc^2$ into $E_{\text{total}} = \dfrac{mc^2}{\sqrt{1-\dfrac{v^2}{c^2}}}$ and then substitute into

$E_k = E_{\text{total}} - E_{\text{rest}}$ to obtain $E_k = \dfrac{E_{\text{rest}}}{\sqrt{1-\dfrac{v^2}{c^2}}} - E_{\text{rest}}$. Set $E_k = E_{\text{rest}}$ and solve for v.

$$E_k = \frac{E_{rest}}{\sqrt{1-\frac{v^2}{c^2}}} - E_{rest}$$

$$E_k = E_{rest}\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

$$\frac{E_k}{E_{rest}} + 1 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Set $E_k = E_{rest}$, so $\frac{E_k}{E_{rest}} = 1$.

$$2 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$1-\frac{v^2}{c^2} = \frac{1}{4}$$

$$\frac{v^2}{c^2} = \frac{3}{4}$$

$$\frac{v}{c} = \frac{\sqrt{3}}{2}$$

$$v = \frac{c\sqrt{3}}{2}$$

Solution: $v = \frac{c\sqrt{3}}{2}$

$$= (3.0\times10^8 \text{ m/s})(0.8660)$$

$$v = 2.60\times10^8 \text{ m/s}$$

Statement: The particle's speed is 2.60×10^8 m/s.

9. Given: $m = 2500$ kg; $E_k = 2.0\times10^{19}$ J; $c = 3.0\times10^8$ m/s

Required: v

Analysis: Rearrange the equation for E_k.

$$E_k = mc^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right)$$

$$\frac{E_k}{mc^2}+1 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{\frac{E_k}{mc^2}+1}$$

$$1-\frac{v^2}{c^2} = \frac{1}{\left(\frac{E_k}{mc^2}+1\right)^2}$$

$$\frac{v}{c} = \sqrt{1-\frac{1}{\left(\frac{E_k}{mc^2}+1\right)^2}}$$

Solution: $\frac{v}{c} = \sqrt{1-\frac{1}{\left(\frac{E_k}{mc^2}+1\right)^2}}$

$$= \sqrt{1-\frac{1}{\left(\frac{2.0\times10^{19}}{(2.5\times10^3)(3.0\times10^8)^2}+1\right)^2}}$$

$$\frac{v}{c} = 0.40$$

$$v = 0.40c$$

Statement: The spacecraft travels at 0.40 times the speed of light.

10. Given: E_{rest} = 512 keV; the classical kinetic energy $E_{k\text{-class}}$ = 30.0 keV

Required: the relativistic kinetic energy, E_k

Analysis: Rewrite the classical equation for kinetic energy, $E_{k\text{-class}} = \frac{1}{2}mv^2$, as $\frac{2E_{k\text{-class}}}{mc^2} = \frac{v^2}{c^2}$. Then, substitute that into the relativistic kinetic energy equation. Use $E_{rest} = mc^2$ to calculate.

$$E_{\text{k}} = mc^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right)$$

$$E_{\text{k}} = mc^2\left(\frac{1}{\sqrt{1-\frac{2E_{\text{k-class}}}{mc^2}}}-1\right)$$

Solution: $$E_{\text{k}} = mc^2\left(\frac{1}{\sqrt{1-\frac{2E_{\text{k-class}}}{mc^2}}}-1\right)$$

$$= (512\text{ keV})\left(\frac{1}{\sqrt{1-\frac{2(30.0\ \cancel{\text{keV}})}{512\ \cancel{\text{keV}}}}}-1\right)$$

$$E_{\text{k}} = 32.9\text{ keV}$$

Statement: The actual kinetic energy of the electron is 32.9 keV.

11. Given: m_{fuel} = 100.0 mg; $c = 3.0 \times 10^8$ m/s; car mileage = 30 km from 1.0×10^8 J of fuel

Required: number of kilometres travelled on m_{fuel}, d

Analysis: First, using $E_{\text{rest}} = mc^2$, calculate the rest energy in 100.0 mg of fuel, then multiply by the mileage ratio, 3.0×10^{-7} km/J.

Solution: $$E_{\text{rest}} = mc^2$$

$$= (1.000\times10^{-4}\text{ kg})(3.0\times10^{8}\text{ m/s})^2$$

$$E_{\text{rest}} = 9.0\times10^{12}\text{ J}$$

$$d = E_{\text{rest}} \times \frac{30.0\text{ km}}{1.0\times10^{8}\text{ J}}$$

$$= 9.0\times10^{12}\ \cancel{\text{J}} \times \frac{30.0\text{ km}}{1.0\times10^{8}\ \cancel{\text{J}}}$$

$$d = 2.7\times10^{6}\text{ km}$$

Statement: The car could hypothetically drive 2.7×10^6 km using the energy contained in the rest mass of 100.0 mg of fuel.

12. Given: reduction in uranium mass, Δm = 100.000 kg – 99.312 kg = 0.688 kg;
$c = 2.997\ 99 \times 10^8$ m/s

Required: E_{rest} from Δmc^2

Analysis: $E_{\text{rest}} = \Delta mc^2$

Solution:
$$\begin{aligned} E_{\text{rest}} &= \Delta mc^2 \\ &= (0.688\ \text{kg})(2.99799 \times 10^8\ \text{m/s})^2 \\ E_{\text{rest}} &= 6.1837 \times 10^{16}\ \text{J} \end{aligned}$$

Statement: The amount of energy released during the four years was 6.1837×10^{16} J.