Section 11.2: Time Dilation

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1. Given: $\Delta t_s = 1.00$ s; $v = 0.60c$

Required: $\Delta t_m - \Delta t_s$

Analysis: Use $\dfrac{\Delta t_m}{\Delta t_s} = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$.

Solution:
$$\begin{aligned}
\Delta t_m - \Delta t_s &= \Delta t_s\left(\frac{\Delta t_m}{\Delta t_s} - 1\right) \\
&= \Delta t_s\left(\frac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} - 1\right) \\
&= \Delta t_s\left(\frac{1}{\sqrt{1-\dfrac{(0.60\cancel{c})^2}{\cancel{c^2}}}} - 1\right) \\
&= \Delta t_s\left(\frac{1}{\sqrt{0.64}} - 1\right) \\
&= (1.00 \text{ s})(0.25) \\
\Delta t_m - \Delta t_s &= 0.25 \text{ s}
\end{aligned}$$

Statement: The proper time interval of 1.00 s of the clock appears to increase by 0.25 s when the clock moves with a speed of 0.60*c* relative to the observer.

2. Given: $\Delta t_m = 3.7 \times 10^{-6}$ s; $v = 2.4 \times 10^8$ m/s; $c = 3.0 \times 10^8$ m/s

Required: Δt_s

Analysis:
$$\begin{aligned}
\frac{\Delta t_m}{\Delta t_s} &= \frac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} \\
\Delta t_s &= \Delta t_m\left(\frac{\Delta t_s}{\Delta t_m}\right) \\
&= \Delta t_m\left(\frac{\Delta t_s}{\Delta t_m}\right) \\
\Delta t_s &= \Delta t_m\sqrt{1-\frac{v^2}{c^2}}
\end{aligned}$$

Solution: $\Delta t_{\mathrm{s}} = \Delta t_{\mathrm{m}}\sqrt{1-\frac{v^2}{c^2}}$

$$= \Delta t_{\mathrm{m}}\sqrt{1-\frac{(2.4\times10^{8}\ \cancel{\mathrm{m/s}})^2}{(3.0\times10^{8}\ \cancel{\mathrm{m/s}})^2}}$$

$$= \Delta t_{\mathrm{m}}\sqrt{1-\left(\frac{4}{5}\right)^2}$$

$$= \Delta t_{\mathrm{m}}\sqrt{\frac{25-16}{25}}$$

$$= (3.7\times10^{-6}\ \mathrm{s})\left(\frac{3}{5}\right)$$

$$\Delta t_{\mathrm{s}} = 2.2\times10^{-6}\ \mathrm{s}$$

Statement: At rest, the particle's lifetime is $2.2\times10^{-6}\ \mathrm{s}$, which is less than the lifetime of the same particles in a fast-moving beam.

3. Given: $\Delta t_{\mathrm{s}} = 8.0$ s; $\Delta t_{\mathrm{m}} = 10.0$ s; $c = 3.0\times10^{8}$ m/s

Required: v

Analysis: Use $\frac{\Delta t_{\mathrm{m}}}{\Delta t_{\mathrm{s}}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ to solve for v.

$$\frac{\Delta t_{\mathrm{m}}}{\Delta t_{\mathrm{s}}} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{\Delta t_{\mathrm{s}}}{\Delta t_{\mathrm{m}}} = \sqrt{1-\frac{v^2}{c^2}}$$

$$\left(\frac{\Delta t_{\mathrm{s}}}{\Delta t_{\mathrm{m}}}\right)^2 = 1-\frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1-\left(\frac{\Delta t_{\mathrm{s}}}{\Delta t_{\mathrm{m}}}\right)^2$$

Solution: $\dfrac{v^2}{c^2} = 1 - \left(\dfrac{\Delta t_\text{s}}{\Delta t_\text{m}}\right)^2$

$$= 1 - \left(\frac{4}{5}\right)^2$$

$$= \frac{9}{25}$$

$$\frac{v}{c} = \frac{3}{5}$$

$$v = \frac{3}{5}c$$

$$= \left(\frac{3}{5}\right)(3.0 \times 10^8 \text{ m/s})$$

$$v = 1.8 \times 10^8 \text{ m/s}$$

Statement: The spacecraft is moving at 1.8×10^8 m/s relative to Earth.

4. (a) Given: $\Delta t_\text{m} = 30.0$ h; $v = 0.700c$

Required: Δt_s

Analysis: $\dfrac{\Delta t_\text{m}}{\Delta t_\text{s}} = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

$$\Delta t_\text{s} = \Delta t_\text{m}\left(\frac{\Delta t_\text{s}}{\Delta t_\text{m}}\right)$$

Solution: $\Delta t_\text{s} = \Delta t_\text{m}\left(\dfrac{\Delta t_\text{s}}{\Delta t_\text{m}}\right)$

$$= \Delta t_\text{m}\sqrt{1 - \frac{v^2}{c^2}}$$

$$= \Delta t_\text{m}\sqrt{1 - \frac{(0.700\cancel{c})^2}{\cancel{c^2}}}$$

$$= (30.0 \text{ h})(0.71414)$$

$$= 21.424 \text{ h (two extra digits carried)}$$

$$\Delta t_\text{s} = 21.4 \text{ h}$$

Statement: The time between the events, as viewed on Earth, is 21.4 h.

(b) Given: $\Delta t_\text{s} = 21.424$ h; $v = 0.950c$

Required: Δt_m

Analysis: $\dfrac{\Delta t_{\text{m}}}{\Delta t_{\text{s}}} = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$

$$\Delta t_{\text{m}} = \frac{\Delta t_{\text{s}}}{\sqrt{1-\dfrac{v^2}{c^2}}}$$

Solution: $\Delta t_{\text{m}} = \dfrac{\Delta t_{\text{s}}}{\sqrt{1-\dfrac{v^2}{c^2}}}$

$$= \frac{21.424\ \text{h}}{\sqrt{1-\dfrac{(0.950\cancel{c})^2}{\cancel{c^2}}}}$$

$$\Delta t_{\text{m}} = 68.6\ \text{h}$$

This problem can also be done without the steps for part (a) by evaluating the ratio of the ratios $\dfrac{\Delta t_{\text{m}}}{\Delta t_{\text{s}}}$ for both speeds. The Δt_{s} factor is dropped out.

Statement: By going faster, the crew measured the time between events to increase from 30.0 h to 68.6 h.

5. (a) Given: $v = 1.1\times10^4$ m/s; $c = 3.0\times10^8$ m/s

Required: $\dfrac{\Delta t_{\text{m}}}{\Delta t_{\text{s}}}$

Analysis: Use $\dfrac{\Delta t_{\text{m}}}{\Delta t_{\text{s}}} = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$.

Solution: $\dfrac{\Delta t_{\text{m}}}{\Delta t_{\text{s}}} = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$

$$= \frac{1}{\sqrt{1-\dfrac{(1.1\times10^4)^2}{(3.0\times10^8)^2}}}$$

$$= 1 + 0.67\times10^{-9}$$

$$\frac{\Delta t_{\text{m}}}{\Delta t_{\text{s}}} = 1.000\ 000\ 001$$

Statement: The time dilation factor is 1.000 000 001.

(b) For objects going much slower than the speed of light, measuring the effects of time dilation requires extremely high accuracy.

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1. For observer 2 to measure the same time for the light pulse on the light clock that observer 1 measures, she would have to be in the same inertial frame as observer 1. That is, she would have to be moving at the same velocity (same speed and direction) as the railway car.

2. (a) The process will always take longer for the observer who is moving relative to the process.

(b) The observer who is at rest with respect to the process measures the proper time of the process.

3. (a) The clocks do not remain synchronized. As in the case of the Hafele-Keating experiment (atomic clocks on aircraft), the clock that orbits Earth will be observed by a person on Earth to run slow. So when the clock returns to Earth, it will have recorded less elapsed time. That is, the clocks will no longer be synchronized.

(b) Although the clock that orbited will have less elapsed time after returning to Earth, it will thereafter run at the same rate as other clocks that are stationary on Earth. It will not run slow.

(c) As described in part (a), the time elapsed on the clock that orbited will be less. The times will be different.

(d) We are assuming that the clocks are ideal; they run as designed, keeping perfect time. So, the stationary clock did not have the wrong time. Any differences between the stationary and the orbiting clock are due to the nature of time as revealed by special relativity.

(e) The orbiting clock was also assumed to be ideal. Its time is correct, and all differences with the stationary clock were due to the nature of time, not the clock. It did not have the wrong time.

4. Notice that 1) the aircraft is travelling in the opposite direction to that of the ground on the spinning Earth, and 2) the aircraft took exactly one day to travel around the world,

$$8.64\times10^{4}\ \cancel{\text{s}}\times\frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}}\times\frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{h}}}\times\frac{24\ \cancel{\text{h}}}{1\ \text{day}}=1\ \text{day}$$

Therefore, for an observer at rest with respect to the centre of Earth, the plane was stationary and the clock at the airport was moving east (the speed will depend on the latitude, but near the equator, the speed would exceed 400 m/s). Thus, as in the case of the Hafele-Keating experiment, the airport clock would show less elapsed time. The airport clock would have run slower.

5. The accuracy of a GPS system depends on correcting satellite clocks for special relativity because the GPS satellites, which send the signals, are moving rapidly with respect to the GPS receivers, which are stationary on Earth. Thus, according to the receiver, the clocks on the satellites run slow. Even if the time dilation effect is small, differences in elapsed time continue to build. To ensure that that both the receiver and satellite agree on the elapsed time, the GPS system should take into account time dilation from special relativity.

6. (a) Roger, not Mia, moves in Roger's inertial reference frame. Thus, Roger, not Mia, measures Roger's proper time.

(b) Given: $\Delta t_{\text{s}} = 30$ s; $v = 0.85c$

Required: Δt_{m}

Analysis: $\dfrac{\Delta t_{\mathrm{m}}}{\Delta t_{\mathrm{s}}} = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$

$$\Delta t_{\mathrm{m}} = \frac{\Delta t_{\mathrm{s}}}{\sqrt{1-\dfrac{v^2}{c^2}}}$$

Solution: $\Delta t_{\mathrm{m}} = \dfrac{\Delta t_{\mathrm{s}}}{\sqrt{1-\dfrac{v^2}{c^2}}}$

$$= \frac{30\ \mathrm{s}}{\sqrt{1-\dfrac{(0.85\cancel{c})^{\cancel{2}}}{\cancel{c^2}}}}$$

$$\Delta t_{\mathrm{m}} = 57\ \mathrm{s}$$

Statement: Roger observes that over a period of 30 s in his reference frame, Mia's watch has elapsed 57 s.

7. Given: $\Delta t_{\mathrm{s}} = 1.0$ s; $v = 0.95c$

Required: Δt_{m}

Analysis: $\dfrac{\Delta t_{\mathrm{m}}}{\Delta t_{\mathrm{s}}} = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$

$$\Delta t_{\mathrm{m}} = \frac{\Delta t_{\mathrm{s}}}{\sqrt{1-\dfrac{v^2}{c^2}}}$$

Solution: $\Delta t_{\mathrm{m}} = \dfrac{\Delta t_{\mathrm{s}}}{\sqrt{1-\dfrac{v^2}{c^2}}}$

$$= \frac{1.0\ \mathrm{s}}{\sqrt{1-\dfrac{(0.95\cancel{c})^{\cancel{2}}}{\cancel{c^2}}}}$$

$$\Delta t_{\mathrm{m}} = 3.2\ \mathrm{s}$$

Statement: The observer on Earth finds that the signals arrive every 3.2 s.