Section 10.3: The Diffraction Grating
Tutorial 1 Practice, page 523

1. Slit separation and number of lines are related by the equation $w = \frac{1}{N}$. As N increases, w decreases. The diffraction grating with 10 000 lines/cm has more lines per centimetre than the second diffraction grating, so the separation between adjacent principal maxima in the first grating would have to be smaller.

2. Given: λ = 660 nm $= 6.60 \times 10^{-7}$ m; N = 8500 lines/cm; m = 1

Required: θ, the angular separation between successive maxima

Analysis: Use the equation $w = \frac{1}{N}$ to calculate the slit separation. Then use the equation $m\lambda = w\sin\theta_m$ to locate the maximum for m; $\sin\theta_m = \frac{m\lambda}{w}$.

Solution:

$$w = \frac{1}{N}$$

$$= \frac{1}{8500 \text{ lines/}\cancel{\text{cm}}} \times \frac{1 \text{ m}}{100 \ \cancel{\text{cm}}}$$

$w = 1.176 \times 10^{-6}$ m (two extra digits carried)

$$\sin\theta_m = \frac{m\lambda}{w}$$

$$\sin\theta_1 = \frac{(1)(6.60 \times 10^{-7} \ \cancel{\text{m}})}{1.176 \times 10^{-6} \ \cancel{\text{m}}}$$

$$\theta_1 = 34°$$

Statement: The angular separation of successive maxima is $34°$.

3. Given: λ = 694.3 nm $= 6.943 \times 10^{-7}$ m; m = 3; θ_3 = 22.0°

Required: N

Analysis: Use the equation $m\lambda = w\sin\theta_m$ to calculate the slit separation; $w = \frac{m\lambda}{\sin\theta_m}$. Then use the equation $w = \frac{1}{N}$ to determine the number of grating lines; $N = \frac{1}{w}$.

Solution:

$$w = \frac{m\lambda}{\sin\theta_m}$$

$$= \frac{(3)(6.943 \times 10^{-7} \text{ m})}{\sin 22.0°}$$

$w = 5.5602 \times 10^{-6}$ m (one extra digit carried)

$$N = \frac{1}{w}$$
$$= \frac{1}{5.5602 \times 10^{-6}\ \cancel{\text{m}}} \times \frac{1\ \cancel{\text{m}}}{100\ \text{cm}}$$
$$N = 1798\ \text{lines/cm}$$

Statement: The grating has 1798 lines per centimetre.

Research This: Blu-ray Technology, page 524

A. Answers may vary. Sample answer: The technology is called Blu-ray because it uses a blue laser instead of the red laser used in DVDs.
B. Answers may vary. Sample answer: In Blu-ray technology, the data are placed on the top of a disc coated with a polycarbonate layer. There are pits in the disc. Each pit contains a signal that is interpreted as a zero or a one, much like a computer. The laser reads the pits.
C. Answers may vary. Sample answer: Blu-ray technology can hold more data than CDs and DVDs. The quantity of data is at least five times greater than the quantity that a DVD can store, which means that images can be much more detailed on Blu-ray discs. Blu-ray is also able to play much faster than CDs and DVDs.
D. Answers may vary. Sample answer: The improvements are possible through the use of the blue laser, which has a much smaller wavelength than the red laser used with CD and DVD players. Blu-ray technology can also store information on as many as 20 layers within the disc.
E. Answers may vary. Sample answer: The images are sharpest from Blu-ray discs. Answers will vary based on presentation format, but students should show at least three images to demonstrate the representative features of the three technologies. Hazards associated with using high-power lasers such as the lasers used in Blu-ray technology should be included. For example, if the laser is pointed at a person's eyes, that person's eyesight could be damaged.

Section 10.3 Questions, page 525

1. The surface of a CD has many closely spaced parallel lines, like a diffraction grating. Consequently, when white light reflects from the surface of a CD, we see a rainbow-like pattern because the surface acts like a diffraction grating.
2. Given: $N = 2800$ lines/cm

Analysis: $w = \frac{1}{N}$

Solution:
$$w = \frac{1}{N}$$
$$= \frac{1}{2800\ \text{lines/cm}}$$
$$= 3.6 \times 10^{-4}\ \text{cm}$$
$$w = 3.6 \times 10^{-6}\ \text{m}$$

Statement: The distance between two lines in the diffraction grating is 3.6×10^{-6} m.

3. Given: $N = 10\ 000$ lines/cm; $\theta_1 = 31.2°$; $\theta_2 = 36.4°$; $\theta_3 = 47.5°$

Required: λ_1, λ_2, λ_3

Analysis: Use the equation $w = \dfrac{1}{N}$ to calculate the slit separation. Then use the equation $m\lambda = w\sin\theta_m$ to determine the wavelength for each value of m; $\lambda = \dfrac{w\sin\theta_m}{m}$.

Solution:

$$w = \frac{1}{N}$$

$$\begin{aligned} w &= \frac{1}{10\ 000 \text{ lines/cm}} \\ &= 1.0 \times 10^{-4} \text{ cm} \\ w &= 1.0 \times 10^{-6} \text{ m} \end{aligned}$$

For m = 1:

$$\begin{aligned} \lambda &= \frac{w\sin\theta_m}{m} \\ &= \frac{(1.0 \times 10^{-6} \text{ m})\sin 31.2°}{1} \\ &= 5.18 \times 10^{-7} \text{ m} \\ \lambda &= 518 \text{ nm} \end{aligned}$$

For m = 3:

$$\begin{aligned} \lambda &= \frac{w\sin\theta_m}{m} \\ &= \frac{(1.0 \times 10^{-6} \text{ m})\sin 47.5°}{3} \\ &= 2.46 \times 10^{-7} \text{ m} \\ \lambda &= 246 \text{ nm} \end{aligned}$$

For m = 2:

$$\begin{aligned} \lambda &= \frac{w\sin\theta_m}{m} \\ &= \frac{(1.0 \times 10^{-6} \text{ m})\sin 36.4°}{2} \\ &= 2.97 \times 10^{-7} \text{ m} \\ \lambda &= 297 \text{ nm} \end{aligned}$$

Statement: The wavelengths that produce these maxima are 518 nm, 297 nm, and 246 nm.

4. Given: N = 6000 lines/cm; w = 2.0 cm; λ = 450 nm = 4.50×10^{-7} m; m = 1

Required: θ

Analysis: Divide the number of slits, N, by the length of the slit:

$\dfrac{6000 \text{ lines/cm}}{2} = 3000 \text{ lines/cm}$. Then use $w = \dfrac{1}{N}$ to calculate the slit separation, and rearrange the equation $m\lambda = w\sin\theta_m$ to calculate the angle; $\sin\theta_m = \dfrac{m\lambda}{w}$.

Solution:

$w = \frac{1}{N}$

$w = \frac{1}{3000 \text{ lines}/\cancel{\text{cm}}} \times \frac{1 \text{ m}}{100 \ \cancel{\text{cm}}}$

$w = 3.333 \times 10^{-6}$ m (two extra digits carried)

$\sin\theta_m = \frac{m\lambda}{w}$

$= \frac{(1)(4.50 \times 10^{-7} \ \cancel{\text{m}})}{3.333 \times 10^{-6} \ \cancel{\text{m}}}$

$\theta = 7.8°$

Statement: Blue light produces the first intensity maximum at $7.8°$.

5. Given: λ = 600.0 nm = 6.000×10^{-7} m; w = 25 μm = 2.5×10^{-5} m; m = 1

Required: θ

Analysis: $m\lambda = w\sin\theta_m$; $\sin\theta_m = \frac{m\lambda}{w}$

Solution:

$\sin\theta_m = \frac{m\lambda}{w}$

$= \frac{(1)(6.000 \times 10^{-7} \ \cancel{\text{m}})}{2.5 \times 10^{-5} \ \cancel{\text{m}}}$

$\theta = 1.4°$

Statement: The first-order maximum in intensity is at the angle $1.4°$.

6. Given: λ = 780 nm = 7.80×10^{-7} m; L = 10 m; Δy = 0.50 m; m = 1

Required: w

Analysis: Use the sine ratio to calculate θ_1. Then rearrange the equation $m\lambda = w\sin\theta_m$ to calculate the slit separation; $w = \frac{m\lambda}{\sin\theta_m}$.

Solution:

$\sin\theta = \frac{0.50 \text{ m}}{10 \text{ m}}$

$\sin\theta = 0.05$

$w = \frac{m\lambda}{\sin\theta_1}$

$= \frac{(1)(7.80 \times 10^{-7} \text{ m})}{0.05}$

$w = 1.6 \times 10^{-5}$ m

Statement: The spacing between the lines in the diffraction grating is 1.6×10^{-5} m.

7. Given: N = 300 lines/cm; L = 0.84 m; Δy = 3.6 cm = 0.036 m; m = 3
Required: λ
Analysis: Use $w = \frac{1}{N}$ to calculate the slit separation. Use the tan ratio to calculate θ_3. Then rearrange the equation $m\lambda = w\sin\theta_m$ to calculate the wavelength; $\lambda = \frac{w\sin\theta_m}{m}$.
Solution:

$$w = \frac{1}{N}$$
$$= \frac{1}{300 \text{ lines}/\cancel{\text{cm}}} \times \frac{1 \text{ m}}{100 \ \cancel{\text{cm}}}$$
$$w = 3.333 \times 10^{-5} \text{ m} \ \text{(two extra digits carried)}$$
$$\tan\theta = \frac{0.036 \ \cancel{\text{m}}}{0.84 \ \cancel{\text{m}}}$$
$$\theta = 2.454° \qquad \text{(two extra digits carried)}$$
$$\lambda = \frac{w\sin\theta_m}{m}$$
$$= \frac{(3.333 \times 10^{-5})\sin 2.454°}{3}$$
$$= 4.76 \times 10^{-7} \text{ m}$$
$$\lambda = 480 \text{ nm}$$

Statement: The wavelength of the light is 480 nm.
8. Given: $N = 3000$ lines/cm; $\lambda = 5.4 \times 10^{-7}$ m
Required: maximum value of m
Analysis: Use $w = \frac{1}{N}$ to calculate the slit separation. Then use the equation $m\lambda = w\sin\theta_m$ to determine the angle in terms of m; $\sin\theta_m = \frac{m\lambda}{w}$.
Solution:

$$w = \frac{1}{N}$$
$$= \frac{1}{3000 \text{ lines}/\cancel{\text{cm}}} \times \frac{1 \text{ m}}{100 \ \cancel{\text{cm}}}$$
$$w = 3.333 \times 10^{-6} \text{ m} \ \text{(two extra digits carried)}$$

For a maximum m, $\sin\theta_m < 1.0$:

$$\sin\theta_m = \frac{m\lambda}{w}$$
$$= \frac{(m)(5.4\times10^{-7}\text{ m})}{3.333\times10^{-6}\text{ m}}$$
$$\sin\theta_m = 0.1620m$$

We need

$$0.1620m < 1.0$$
$$m < \frac{1.0}{0.1620}$$
$$m < 6.17$$
$$m = 6$$

Statement: The maximum order number possible is the 6th order.

9. (a) Given: $w = 0.50$ nm $= 5.0\times10^{-10}$ m; $\lambda = 0.050$ nm $= 5.0\times10^{-11}$ m

Required: $\theta_1, \theta_2, \theta_3$

Analysis: Use the equation $m\lambda = w\sin\theta_m$ to calculate the angles; $\sin\theta_m = \dfrac{m\lambda}{w}$.

Solution:

For $m = 1$:

$$\sin\theta_m = \frac{m\lambda}{w}$$
$$= \frac{(1)(5.0\times10^{-11}\ \cancel{\text{m}})}{5.0\times10^{-10}\ \cancel{\text{m}}}$$
$$\theta = 5.7°$$

For $m = 2$:

$$\sin\theta_m = \frac{m\lambda}{w}$$
$$= \frac{(2)(5.0\times10^{-11}\ \cancel{\text{m}})}{5.0\times10^{-10}\ \cancel{\text{m}}}$$
$$\theta = 12°$$

For $m = 3$:

$$\sin\theta_m = \frac{m\lambda}{w}$$
$$= \frac{(3)(5.0\times10^{-11}\ \cancel{\text{m}})}{5.0\times10^{-10}\ \cancel{\text{m}}}$$
$$\theta = 17°$$

Statement: The angles for the first three maxima are $5.7°, 12°,$ and $17°$.

(b) $w = 5.0\times10^{-10}$ m; $\lambda = 600$ nm $= 6.0\times10^{7}$ m; $m = 1$

Required: θ_1

Analysis: $m\lambda = w\sin\theta_m$; $\sin\theta_m = \dfrac{m\lambda}{w}$

Solution: $\sin\theta_m = \dfrac{m\lambda}{w}$

$$= \frac{(1)(6.0\times10^{-9}\ \cancel{\text{m}})}{5.0\times10^{-10}\ \cancel{\text{m}}}$$

$$= 1.2$$

$$\theta_1 = \text{ no angle possible}$$

Statement: The angle for the first bright fringe does not exist.

(c) The wavelength 600 nm is within the range of visible light, but no fringe angle was possible in part (b). Visible light is not usually diffracted by crystal lattices. It may be possible to get a fringe but only if the wavelength of the light is sufficiently short.

10. Given: $\lambda_\text{A} = 5.00\times10^2$ nm $= 5.00\times10^{-7}$ m; $\theta_\text{A} = 20.0°$; $\theta_\text{B} = 18.0°$; $m = 1$

Required: $n_{\text{atmosphere}}$

Analysis: Use the equation $m\lambda = w\sin\theta_m$ to calculate *w*; $w = \dfrac{m\lambda}{\sin\theta_m}$. Then use the same equation with the value of *w* and the new angle, θ_B, to calculate the wavelength in the planet's atmosphere. Take the ratio of the wavelengths to determine the index of refraction for the planet's atmosphere, $n_{\text{atmosphere}} = \dfrac{\lambda_\text{A}}{\lambda_\text{B}}$.

Solution: $w = \dfrac{m\lambda}{\sin\theta_m}$

$$= \frac{(1)(5.0\times10^{-7}\ \text{m})}{\sin 20.0°}$$

$$w = 1.462\times10^{-6}\ \text{m}\ \text{(one extra digit carried)}$$

$$\lambda_\text{B} = \frac{w\sin\theta_m}{m}$$

$$= \frac{(1.462\times10^{-6}\ \text{m})\sin 18.0°}{1}$$

$$\lambda_\text{B} = 4.518\times10^{-7}\ \text{m}\ \text{(one extra digit carried)}$$

$$n_{\text{atmosphere}} = \frac{\lambda_\text{A}}{\lambda_\text{B}}$$

$$= \frac{5.00\times10^{-7}\ \cancel{\text{m}}}{4.518\times10^{-7}\ \cancel{\text{m}}}$$

$$n_{\text{atmosphere}} = 1.11$$

Statement: The index of refraction of the planet's atmosphere is 1.11.