Section 10.2: Single-Slit Diffraction

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1. When light travels from air to a medium that is denser than air, such as water, the light is refracted and the wavelength of the light shortens. According to the equation $\lambda = \frac{w\Delta y}{L}$, λ is proportional to Δy, so when λ is reduced, the central maximum will also be reduced. So the central maximum would be narrower if the equipment were submerged in water.

2. Given: $\lambda = 7.328 \times 10^{-7}$ m; $w = 43$ μm $= 4.3 \times 10^{-5}$ m; $L = 3.0$ m

Required: Δy

Analysis: Rearrange the equation $\lambda = \frac{w\Delta y}{L}$ to solve for the distance between adjacent minima;

$$\Delta y = \frac{L\lambda}{w}$$

Solution:

$$\begin{aligned}\Delta y &= \frac{L\lambda}{w} \\ &= \frac{(3.0\text{ m})(7.328 \times 10^{-7} \times 10^{-7}\ \cancel{\text{m}})}{4.3 \times 10^{-5}\ \cancel{\text{m}}} \\ &= 5.1 \times 10^{-2}\text{ m} \\ \Delta y &= 5.1\text{ cm}\end{aligned}$$

Statement: The separation of adjacent minima is 5.1 cm.

3. Given: $w = 3.00 \times 10^{-6}$ m; $\theta_1 = 25.0°$

Required: λ

Analysis: The angle between the first dark fringes is equal to the angle for the width of the central maximum, which is twice the angle for the first dark fringe, given by $w\sin\theta_n = \lambda$. In this case, $n = 1$ and $2\theta = 25.0°$; $\lambda = w\sin\theta_n$.

Solution:

$$\begin{aligned}2\theta &= 25.0° \\ \theta &= 12.5° \\ \lambda &= w\sin\theta_n \\ &= (3.0 \times 10^{-6}\text{ m})\sin 12.5° \\ \lambda &= 6.49 \times 10^{-7}\text{ m}\end{aligned}$$

Statement: The wavelength of the light is 6.49×10^{-7} m.

4. Given: $\theta_a = 56°$; $\theta_b = 34°$

Required: $\frac{w_a}{w_b}$

Analysis: The formula that relates the angles, the common wavelength, and the slit sizes is $w\sin\theta_n = \lambda$; $w_a \sin\theta_a = \lambda$ and $w_b \sin\theta_b = \lambda$. Divide the equations to determine the ratio $\frac{w_a}{w_b}$.

Solution: $\dfrac{w_{\text{a}} \sin\theta_{\text{a}}}{w_{\text{b}} \sin\theta_{\text{b}}} = \dfrac{\cancel{\lambda}}{\cancel{\lambda}}$

$$\frac{w_{\text{a}}}{w_{\text{b}}} = \frac{\sin\theta_{\text{b}}}{\sin\theta_{\text{a}}}$$

$$= \frac{\sin 34°}{\sin 56°}$$

$$\frac{w_{\text{a}}}{w_{\text{b}}} = 0.67$$

Statement: The ratio of the slit widths, $\dfrac{w_{\text{a}}}{w_{\text{b}}}$, is 0.67.

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1. Given: single-slit diffraction; λ = 794 nm = 7.94×10^{-7} m; L = 1.0 m
n = 9, y_9 = 6.48 cm = 0.0648 m
Required: w

Analysis: $\Delta y = \dfrac{y_9}{n}$; $\Delta y = \dfrac{\lambda L}{w}$

$$w = \frac{\lambda L}{\Delta y}$$

Solution: $\Delta y = \dfrac{y_9}{n}$

$$= \frac{0.0648 \text{ m}}{9}$$

$$\Delta y = 7.2 \times 10^{-3} \text{m}$$

$$w = \frac{\lambda L}{\Delta y}$$

$$= \frac{(7.94 \times 10^{-7} \cancel{\text{m}})(1.0 \text{ m})}{7.2 \times 10^{-3} \cancel{\text{m}}}$$

$$w = 1.1 \times 10^{-3} \text{ m}$$

Statement: The width of the slit is 1.1×10^{-4} m.

2. Given: single-slit diffraction; λ = 600 nm = 6.00×10^{-7} m; θ_1 = 6.9°
Required: w
Analysis: The first dark fringe is located where $w \sin\theta_n = \lambda$. Rearrange the equation $w \sin\theta_n = \lambda$ to solve for slit width; $w = \dfrac{\lambda}{\sin\theta_n}$.

Solution: $w = \dfrac{\lambda}{\sin\theta_1}$

$= \dfrac{6.00 \times 10^{-7}\ \text{m}}{\sin 6.9°}$

$w = 5.0 \times 10^{-6}\ \text{m}$

Statement: The width of the slit is 5.0×10^{-6} m.

3. Given: single-slit diffraction; λ = 450 nm = 4.50×10^{-7} m; L = 10.0 m; w = 0.15 mm = 1.5×10^{-4} m

Required: y, the distance between the first and third dark fringes

Analysis: $\Delta y = \dfrac{\lambda L}{w}$; $y = 2\Delta y$

Solution: $\Delta y = \dfrac{\lambda L}{w}$

$= \dfrac{(4.50 \times 10^{-7}\ \text{m})(10.0\ \cancel{\text{m}})}{1.50 \times 10^{-4}\ \cancel{\text{m}}}$

$= 3.0 \times 10^{-2}\ \text{m}$

$y = 2\Delta y$

$= 6.0 \times 10^{-2}\ \text{m}$

y = 6.0 cm

Statement: The distance between the first and third dark fringes is 6.0 cm.

4. Given: single-slit diffraction; λ = 550 nm = 5.50×10^{-7} m; L = 2.0 m; y_1 = 5.5 mm = 5.5×10^{-3} m

Required: w

Analysis: Rearrange the equation $y_1 = \dfrac{\lambda L}{w}$ to solve for slit width; $w = \dfrac{\lambda L}{y_1}$

Solution: $w = \dfrac{\lambda L}{y_1}$

$= \dfrac{(5.50 \times 10^{-7}\ \text{m})(2.0\ \cancel{\text{m}})}{5.50 \times 10^{-3}\ \cancel{\text{m}}}$

$= 2.0 \times 10^{-4}\ \text{m}$

w = 0.20 mm

Statement: The width of the slit is 0.20 mm.

5. Given: single-slit diffraction; λ = 630 nm = 6.30×10^{-7} m; L = 3.0 m; w = 0.25 mm = 2.5×10^{-4} m

Required: $2\Delta y$, the width of the central maximum

Analysis: Multiply the equation $\Delta y = \dfrac{\lambda L}{w}$ by 2 to obtain $2\Delta y$.

Solution: $2\Delta y = \frac{2\lambda L}{w}$

$$= \frac{2\left(6.30\times10^{-7}\text{ m}\right)\left(3.0\ \cancel{\text{m}}\right)}{2.50\times10^{-4}\ \cancel{\text{m}}}$$

$$= 1.5\times10^{-2}\text{ m}$$

$2\Delta y$= 1.5 cm

Statement: The width of the central maximum is 1.5 cm.

6. Given: $\Delta y = 0.120\text{ cm} = 1.20\times10^{-3}\text{ m}$; $w = 0.0295\text{ cm} = 2.95\times10^{-4}\text{ m}$; $L = 60.0\text{ cm} = 0.60\text{ m}$

Required: λ

Analysis: Rearrange the equation $\Delta y = \frac{\lambda L}{w}$ to solve for wavelength; $\lambda = \frac{w\Delta y}{L}$

Solution: $\lambda = \frac{w\Delta y}{L}$

$$= \frac{(2.95\times10^{-4}\text{ m})(1.20\times10^{-3}\ \cancel{\text{m}})}{6.0\times10^{-1}\ \cancel{\text{m}}}$$

$$\lambda = 5.90\times10^{-7}\text{ m}$$

Statement: The wavelength of the yellow light is 5.90×10^{-7} m.

7. (a) The distance between successive maxima in single-slit diffraction is given by $\Delta y = \frac{\lambda L}{w}$. If I double the wavelength, λ, then the distance Δy will also double. The angles of the maxima and the minima would be approximately doubled.

(b) If I multiplied both the wavelength, λ, and the slit width, *w*, in the equation $\Delta y = \frac{\lambda L}{w}$ by 2, the 2s will cancel each other out. Therefore, there will be no effect on Δy. The interference pattern will be the same.

8. Blue light has an average wavelength of 475 nm, and green light has an average wavelength of 510 nm. If I replaced the blue light with the green light, then I would be increasing the wavelength. Therefore, spacing of the intensity maxima would be greater.

9. Given: single-slit diffraction

Required: θ_{10}

Analysis: Assume the width of a typical doorway is w = 0.92 m. Assume the visible light has a wavelength of 500 nm. Rearrange the equation $\sin\theta_n = \frac{n\lambda}{w}$ to solve for the angle;

$$\theta_n = \sin^{-1}\left(\frac{n\lambda}{w}\right)$$

Solution: $\theta_n = \sin^{-1}\left(\dfrac{n\lambda}{w}\right)$

$$= \sin^{-1}\left(\frac{(10)\left(5.00\times10^{-7}\ \cancel{\text{m}}\right)}{0.92\ \cancel{\text{m}}}\right)$$

$$= \sin^{-1}\left(5.43\times10^{-6}\right)$$

$$\theta_{10} = 3.1\times10^{-4\circ}$$

Statement: The angle of the tenth minimum for a doorway that is 0.92 m wide is $3.1 \times 10^{-4\circ}$.

10. To improve the resolution of a digital image, I could use more pixels per square centimetre. Or, I could use a wider aperture (size of slit) to increase the resolution. However, the wider aperture would reduce the depth of field (range of the focus).

11. I would be able to resolve the double stars in Mizar with the telescope because, in addition to enlarging the image, the telescope's aperture is wider than the aperture in my eye. The wider aperture increases the resolution, allowing me to see the two stars.

12. In a double-slit interference pattern, there are more intensity maxima than in a single-slit interference pattern. In the double-slit interference pattern, there is less space between fringes because the second slit causes additional destructive interference.