Section 9.5: Interference of Light Waves: Young's Double-Slit Experiment

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1. Given: double-slit interference; $n = 5$; $\theta_5 = 3.8°$; $d = 0.042$ mm = 4.2×10^5 m
Required: λ

Analysis: Rearrange the equation $d\sin\theta_n = \left(n - \frac{1}{2}\right)\lambda$ to solve for wavelength;

$$d\sin\theta_n = \left(n - \frac{1}{2}\right)\lambda$$

$$\lambda = \frac{d\sin\theta_n}{n - \frac{1}{2}}$$

Solution:

$$\lambda = \frac{d\sin\theta_n}{n - \frac{1}{2}}$$

$$= \frac{(4.2\times10^{-5}\text{ m})\sin 3.8°}{5 - \frac{1}{2}}$$

$$\lambda = 6.2\times10^{-7}\text{ m}$$

Statement: The wavelength of the monochromatic light is 6.2×10^{-7} m.

2. Given: double-slit interference; $d = 0.050$ mm = 5.0×10^{-5} m; $\lambda = 650$ nm = 6.5×10^{-7} m; $L = 2.6$ m
Required: Δx, at the centre of the interference pattern
Analysis: $\Delta x = \frac{L\lambda}{d}$
Solution:

$$\Delta x = \frac{L\lambda}{d}$$

$$= \frac{(2.6\ \cancel{\text{m}})(6.5\times10^{-7}\text{ m})}{5.0\times10^{-5}\ \cancel{\text{m}}}$$

$$= 3.4\times10^{-2}\text{ m}$$

$$\Delta x = 3.4\text{ cm}$$

Statement: The fringe separation at the centre of the interference pattern is 3.4 cm.

3. Given: double-slit interference; $n = 3$; $\lambda = 652$ nm = 6.52×10^{-7} m; $d = 6.3 \times 10^{-6}$ m
Required: θ_3

Analysis: Rearrange the equation $d\sin\theta_n = \left(n - \frac{1}{2}\right)\lambda$ to solve for the angle of the fringe;

$$d\sin\theta_n = \left(n - \frac{1}{2}\right)\lambda$$

$$\theta_n = \sin^{-1}\left(\frac{\left(n - \frac{1}{2}\right)\lambda}{d}\right)$$

Solution:

$$\theta_n = \sin^{-1}\left(\frac{\left(n - \frac{1}{2}\right)\lambda}{d}\right)$$

$$= \sin^{-1}\left(\frac{\left(3 - \frac{1}{2}\right)(6.52 \times 10^{-7}\ \cancel{m})}{6.3 \times 10^{-6}\ \cancel{m}}\right)$$

$$\theta_3 = 15°$$

Statement: The fringe is observed at the angle 15°.

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A. Both interference patterns consisted of a horizontal band of light filled with coloured and dark vertical fringes. The fringes were closer together with the green filter than with the red filter.

B. Table 1: Number of Lines between Two Sliders

Colour of light	Number of lines found between two sliders
red	9
green	11

C. Table 2: Calculating the Wavelength of Light

	Red light	Green light
L (m)	1.0	1.0
d (m)	1.76×10^{-4}	1.76×10^{-4}
n	6	7
x (m)	0.015	0.015
Δx (m)	2.5×10^{-3}	2.1×10^{-3}
λ (m)	4.4×10^{-7}	3.9×10^{-7}

Sample calculation: $\frac{\Delta x}{L} = \frac{\lambda}{d}$

$$\lambda = \frac{d\Delta x}{L}$$

$$= \frac{(1.76 \times 10^{-4}\ \cancel{m})(2.5 \times 10^{-3}\ \text{m})}{1.0\ \cancel{m}}$$

$$\lambda = 4.4 \times 10^{-7}\ \text{m}$$

The wavelength is 4.4×10^{-7} m.

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1. Answers may vary. Sample answer: When all other factors are kept constant, the fringes for the red light, λ = 650 nm, are more widely spaced than the fringes for the blue light, λ = 470 nm.

2. Given: double-slit interference; $d = 0.20 \text{ mm} = 2.0\times10^{-4}$ m; L = 3.5 m; n = 1 dark fringe; $x_1 = 4.6 \text{ mm} = 4.6 \times 10^{-3}$ m

Required: λ

Analysis: Rearrange the equation $x_n = \left(n - \frac{1}{2}\right)\frac{L\lambda}{d}$ to solve for wavelength;

$$x_n = \left(n - \frac{1}{2}\right)\frac{L\lambda}{d}$$

$$\lambda = \frac{x_n d}{\left(n - \frac{1}{2}\right)L}$$

Solution:

$$\begin{aligned}\lambda &= \frac{x_n d}{\left(n - \frac{1}{2}\right)L} \\ &= \frac{(4.6\times10^{-3} \cancel{\text{m}})(2.0\times10^{-4} \text{ m})}{\left(1-\frac{1}{2}\right)(3.5 \cancel{\text{m}})} \\ &= 5.3\times10^{-7} \text{ m}\end{aligned}$$

$$\lambda = 530 \text{ nm}$$

Statement: The wavelength of the light is 530 nm.

3. Answers may vary. Sample answer: An underwater double-slit experiment would have different results than a double-slit experiment in air because, in water, the speed of light is slower and the wavelength is shorter. The spacing between the resulting fringes would be closer together underwater than in air.

4. Given: double-slit interference; $d = 0.30 \text{ mm} = 3.0\times10^{-4}\text{ m}$; $n = 5$ dark fringe; $x_5 = 12.8\times10^{-2}\text{ m}$; $\lambda = 4.5\times10^{-7}\text{ m}$

Required: L

Analysis: Rearrange the equation $x_n = \left(n-\frac{1}{2}\right)\frac{L\lambda}{d}$ to solve for distance to the screen;

$$x_n = \left(n-\frac{1}{2}\right)\frac{L\lambda}{d}$$

$$L = \frac{x_n d}{\left(n-\frac{1}{2}\right)\lambda}$$

Solution: $L = \frac{x_n d}{\left(n-\frac{1}{2}\right)\lambda}$

$$= \frac{(12.8\times10^{-2}\ \cancel{\text{m}})(3.0\times10^{-4}\text{ m})}{\left(5-\frac{1}{2}\right)(4.5\times10^{-7}\ \cancel{\text{m}})}$$

$$L = 19\text{ m}$$

Statement: The distance at which the screen is placed is 19 m.

5. (a) Given: double-slit interference; $d = 0.15 \text{ mm} = 1.5\times10^{-4}\text{ m}$; $L = 2.0\text{ m}$; $\Delta x = 0.56 \text{ cm} = 5.6\times10^{-3}\text{ m}$

Required: λ

Analysis: Rearrange the equation $\Delta x = \frac{L\lambda}{d}$ to solve for wavelength

Solution: $\lambda = \frac{d\Delta x}{L}$

$$= \frac{(1.5\times10^{-4}\ \cancel{\text{m}})(5.6\times10^{-3}\text{ m})}{(2.0\ \cancel{\text{m}})}$$

$$\lambda = 4.2\times10^{-7}\text{ m}$$

Statement: The wavelength of the source is 4.2×10^{-7} m, or 420 nm.

(b) Given: double-slit interference; $d = 0.15\text{ mm} = 1.5\times10^{-4}\text{ m}$; $L = 2.0$ m; $\lambda = 600\text{ nm} = 6\times10^{-7}\text{ m}$

Required: Δx, of the dark fringes

Analysis: $\Delta x = \frac{L\lambda}{d}$

Solution: $\Delta x = \frac{L\lambda}{d}$

$$= \frac{(2.0\ \cancel{\text{m}})(6 \times 10^{-7}\ \text{m})}{(1.5 \times 10^{-4}\ \cancel{\text{m}})}$$

$$\Delta x = 8 \times 10^{-3}\ \text{m}$$

Statement: The spacing of the dark fringes is 8×10^{-3} m, or 0.8 cm.

6. Given: double-slit interference; n = 2 dark fringes; θ_2 = 5.4°

Required: $\frac{d}{\lambda}$

Analysis: Rearrange the equation $d \sin \theta_n = \left(n - \frac{1}{2} \right) \lambda$ to solve for $\frac{d}{\lambda}$;

$$d \sin \theta_n = \left(n - \frac{1}{2} \right) \lambda$$

$$\frac{d}{\lambda} = \frac{n - \frac{1}{2}}{\sin \theta_n}$$

Solution:

$$\frac{d}{\lambda} = \frac{n - \frac{1}{2}}{\sin \theta_n}$$

$$= \frac{2 - \frac{1}{2}}{\sin 5.4^\circ}$$

$$\frac{d}{\lambda} = 16$$

Statement: The ratio of the separation of the slits to the wavelength is 16:1.

7. (a) Given: double-slit interference; $d = 0.80\ \text{mm} = 8.0 \times 10^{-4}\ \text{m}$; L = 49 cm = 0.49 m; $\Delta x = 0.33\ \text{mm} = 3.3 \times 10^{-4}\ \text{m}$

Required: λ

Analysis: Rearrange the equation $\Delta x = \frac{L\lambda}{d}$ to solve for wavelength;

$$\Delta x = \frac{L\lambda}{d}$$

$$\lambda = \frac{d \Delta x}{L}$$

Solution:

$$\Delta\lambda = \frac{d\Delta x}{L}$$

$$= \frac{(8.0\times10^{-4}\ \cancel{\text{m}})(3.3\times10^{-4}\ \text{m})}{(0.49\ \cancel{\text{m}})}$$

$$\lambda = 5.39\times10^{-7}\ \text{m}\ \ \text{(one extra digit carried)}$$

$$\lambda = 5.4\times10^{-7}\ \text{m}$$

Statement: The wavelength of the monochromatic light is 5.4×10^{-7} m, or 540 nm.

(b) Given: double-slit interference; $d = 0.60\ \text{mm} = 6.0\times10^{-4}$ m; $L = 0.49$ m; $\lambda = 5.39 \times 10^{-7}$ m

Required: Δx, of the dark fringes

Analysis: $\Delta x = \frac{L\lambda}{d}$

Solution:

$$\Delta x = \frac{L\lambda}{d}$$

$$= \frac{(0.49\ \cancel{\text{m}})(5.39\times10^{-7}\ \text{m})}{(6.0\times10^{-4}\ \cancel{\text{m}})}$$

$$\Delta x = 4.4\times10^{-4}\ \text{m}$$

Statement: The spacing of the dark fringes is 4.4×10^{-4} m, or 0.44 mm.

8. (a) Given: double-slit interference; $L = 2.5$ m; $\lambda = 5.1\times10^{-7}$ m; $\Delta x = 12\ \text{mm} = 1.2\times10^{-2}$ m

Required: d

Analysis: Rearrange the equation $\Delta x = \frac{L\lambda}{d}$ to solve for distance between the slits;

$$\Delta x = \frac{L\lambda}{d}$$

$$d = \frac{L\lambda}{\Delta x}$$

Solution:

$$d = \frac{L\lambda}{\Delta x}$$

$$= \frac{(2.5\ \cancel{\text{m}})(5.1\times10^{-7}\ \text{m})}{(1.2\times10^{-2}\ \cancel{\text{m}})}$$

$$d = 1.1\times10^{-4}\ \text{m}$$

Statement: The slit spacing is 1.1×10^{-4} m, or 0.11 mm.

(b) Solutions may vary. Sample solution: From $\Delta x = \frac{L\lambda}{d}$, the fringe separation is inversely proportional to the slit spacing. If the slit spacing is reduced by a factor of three, the fringe separation will increase by a factor of three. The fringe separation will change to 3×12 mm = 36 mm, or 3.6 cm.

9. Answers may vary. Student answers should include some of the following points. Newton developed a particle theory of light that explained rectilinear propagation and reflection well. His reputation put the work of others at the time in the shadows. Grimaldi was working on a theory for the diffraction and dispersion of light, and Hooke was looking a wave theory. Later, Newton's theory that light travelled faster in a medium than in a vacuum was shown experimentally to be incorrect. With this failure, Newton's explanations of refraction, colour, and dispersion were also refuted. Huygens' wave ideas and Young's double-slit interference experiment showed that light is definitely a wave. By 1900, all known properties of light could be explained by a wave model.