Section 9.5: Interference of Light Waves: Young’s Double-Slit Experiment

Tutorial 1 Practice, page 482

1. Given: double-slit interference; \( n = 5; \theta_5 = 3.8^\circ; d = 0.042 \text{ mm} = 4.2 \times 10^{-5} \text{ m} \)
   Required: \( \lambda \)

   **Analysis:** Rearrange the equation \( d \sin \theta_n = \left( n - \frac{1}{2} \right) \lambda \) to solve for wavelength;

   \[
   d \sin \theta_n = \left( n - \frac{1}{2} \right) \lambda
   \]

   \[
   \lambda = \frac{d \sin \theta_n}{n - \frac{1}{2}}
   \]

   **Solution:**

   \[
   \lambda = \frac{d \sin \theta_n}{n - \frac{1}{2}}
   \]

   \[
   = \frac{(4.2 \times 10^{-5} \text{ m}) \sin 3.8^\circ}{5 - \frac{1}{2}}
   \]

   \[
   = 6.2 \times 10^{-7} \text{ m}
   \]

   **Statement:** The wavelength of the monochromatic light is \( 6.2 \times 10^{-7} \text{ m} \).

2. Given: double-slit interference; \( d = 0.050 \text{ mm} = 5.0 \times 10^{-5} \text{ m}; \lambda = 650 \text{ nm} = 6.5 \times 10^{-7} \text{ m}; L = 2.6 \text{ m} \)
   Required: \( \Delta x \), at the centre of the interference pattern

   **Analysis:** \( \Delta x = \frac{L \lambda}{d} \)

   **Solution:**

   \[
   \Delta x = \frac{L \lambda}{d}
   \]

   \[
   = \frac{(2.6 \text{ m})(6.5 \times 10^{-7} \text{ m})}{5.0 \times 10^{-5} \text{ m}}
   \]

   \[
   = 3.4 \times 10^{-2} \text{ m}
   \]

   \( \Delta x = 3.4 \text{ cm} \)

   **Statement:** The fringe separation at the centre of the interference pattern is 3.4 cm.

3. Given: double-slit interference; \( n = 3; \lambda = 652 \text{ nm} = 6.52 \times 10^{-7} \text{ m}; d = 6.3 \times 10^{-6} \text{ m} \)
   Required: \( \theta_3 \)

   **Analysis:** Rearrange the equation \( d \sin \theta_n = \left( n - \frac{1}{2} \right) \lambda \) to solve for the angle of the fringe;
\[ d \sin \theta_n = \left( n - \frac{1}{2} \right) \lambda \]

\[ \theta_n = \sin^{-1} \left( \frac{n - \frac{1}{2} \lambda}{d} \right) \]

**Solution:**

\[ \theta_n = \sin^{-1} \left( \frac{n - \frac{1}{2}}{d} \right) \]

\[ = \sin^{-1} \left( \frac{3 - \frac{1}{2}}{6.3 \times 10^{-6} \text{ m}} \right) \]

\[ \theta_3 = 15^\circ \]

**Statement:** The fringe is observed at the angle 15°.

**Mini Investigation: Wavelengths of Light, page 483**

A. Both interference patterns consisted of a horizontal band of light filled with coloured and dark vertical fringes. The fringes were closer together with the green filter than with the red filter.

B. **Table 1:** Number of Lines between Two Sliders

<table>
<thead>
<tr>
<th>Colour of light</th>
<th>Number of lines found between two sliders</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td>9</td>
</tr>
<tr>
<td>green</td>
<td>11</td>
</tr>
</tbody>
</table>

C. **Table 2:** Calculating the Wavelength of Light

<table>
<thead>
<tr>
<th></th>
<th>Red light</th>
<th>Green light</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L ) (m)</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>( d ) (m)</td>
<td>1.76 \times 10^{-4}</td>
<td>1.76 \times 10^{-4}</td>
</tr>
<tr>
<td>( n )</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>( x ) (m)</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>( \Delta x ) (m)</td>
<td>2.5 \times 10^{-3}</td>
<td>2.1 \times 10^{-3}</td>
</tr>
<tr>
<td>( \lambda ) (m)</td>
<td>4.4 \times 10^{-7}</td>
<td>3.9 \times 10^{-7}</td>
</tr>
</tbody>
</table>

Sample calculation:

\[ \frac{\Delta x}{L} = \frac{\lambda}{d} \]

\[ \lambda = \frac{d \Delta x}{L} \]

\[ = \frac{(1.76 \times 10^{-4} \text{ m})(2.5 \times 10^{-3} \text{ m})}{1.0 \text{ m}} \]

\[ \lambda = 4.4 \times 10^{-7} \text{ m} \]

The wavelength is 4.4 \times 10^{-7} m.
**Section 9.5 Questions, page 484**

1. Answers may vary. Sample answer: When all other factors are kept constant, the fringes for the red light, $\lambda = 650$ nm, are more widely spaced than the fringes for the blue light, $\lambda = 470$ nm.

2. **Given:** double-slit interference; $d = 0.20$ mm $= 2.0 \times 10^{-4}$ m; $L = 3.5$ m; $n = 1$ dark fringe; $x_1 = 4.6$ mm $= 4.6 \times 10^{-3}$ m

**Required:** $\lambda$

**Analysis:** Rearrange the equation $x_n = \left( n - \frac{1}{2} \right) \frac{L \lambda}{d}$ to solve for wavelength;

$$\lambda = \frac{x_n d}{\left( n - \frac{1}{2} \right) L}$$

**Solution:**

$$\lambda = \frac{x_n d}{\left( n - \frac{1}{2} \right) L} = \frac{(4.6 \times 10^{-3} \text{ m})(2.0 \times 10^{-4} \text{ m})}{\left( 1 - \frac{1}{2} \right)(3.5 \text{ m})} = 5.3 \times 10^{-7} \text{ m}$$

$\lambda = 530$ nm

**Statement:** The wavelength of the light is $530$ nm.

3. Answers may vary. Sample answer: An underwater double-slit experiment would have different results than a double-slit experiment in air because, in water, the speed of light is slower and the wavelength is shorter. The spacing between the resulting fringes would be closer together underwater than in air.
4. Given: double-slit interference; \( d = 0.30 \text{ mm} = 3.0 \times 10^{-4} \text{ m} \); \( n = 5 \) dark fringe; 
\( x_n = 12.8 \times 10^{-2} \text{ m} \); \( \lambda = 4.5 \times 10^{-7} \text{ m} \)

Required: \( L \)

Analysis: Rearrange the equation 
\[ x_n = \left(n - \frac{1}{2}\right) \frac{L\lambda}{d} \]

\( L = \frac{x_n d}{\left(n - \frac{1}{2}\right) \lambda} \)

Solution: 
\[ L = \frac{x_n d}{\left(n - \frac{1}{2}\right) \lambda} \]
\[ = \frac{(12.8 \times 10^{-2} \text{ m})(3.0 \times 10^{-4} \text{ m})}{\left(5 - \frac{1}{2}\right)(4.5 \times 10^{-7} \text{ m})} \]
\[ L = 19 \text{ m} \]

Statement: The distance at which the screen is placed is 19 m.

5. (a) Given: double-slit interference; \( d = 0.15 \text{ mm} = 1.5 \times 10^{-4} \text{ m} \); \( L = 2.0 \text{ m} \);
\( \Delta x = 0.56 \text{ cm} = 5.6 \times 10^{-3} \text{ m} \)

Required: \( \lambda \)

Analysis: Rearrange the equation 
\[ \Delta x = \frac{L\lambda}{d} \]

to solve for wavelength

Solution: 
\[ \lambda = \frac{d \Delta x}{L} \]
\[ = \frac{(1.5 \times 10^{-4} \text{ m})(5.6 \times 10^{-3} \text{ m})}{(2.0 \text{ m})} \]
\[ \lambda = 4.2 \times 10^{-7} \text{ m} \]

Statement: The wavelength of the source is \( 4.2 \times 10^{-7} \text{ m} \), or 420 nm.

(b) Given: double-slit interference; \( d = 0.15 \text{ mm} = 1.5 \times 10^{-4} \text{ m} \); \( L = 2.0 \text{ m} \);
\( \lambda = 600 \text{ nm} = 6 \times 10^{-7} \text{ m} \)

Required: \( \Delta x \), of the dark fringes

Analysis: 
\[ \Delta x = \frac{L\lambda}{d} \]
Solution: \[ \Delta x = \frac{L \lambda}{d} \]
\[ = \frac{(2.0 \text{ m})(6 \times 10^{-7} \text{ m})}{(1.5 \times 10^{-4} \text{ m})} \]
\[ \Delta x = 8 \times 10^{-3} \text{ m} \]

Statement: The spacing of the dark fringes is $8 \times 10^{-3}$ m, or 0.8 cm.

6. Given: double-slit interference; $n = 2$ dark fringes; $\theta_2 = 5.4^\circ$

Required: \( \frac{d}{\lambda} \)

Analysis: Rearrange the equation \( d \sin \theta_n = \left( n - \frac{1}{2} \right) \lambda \) to solve for \( \frac{d}{\lambda} \);

\[ d \sin \theta_n = \left( n - \frac{1}{2} \right) \lambda \]

\[ \frac{d}{\lambda} = \sin \theta_n \]

Solution:

\[ \frac{d}{\lambda} = \frac{n - \frac{1}{2}}{\sin \theta_n} \]

\[ = \frac{2 - \frac{1}{2}}{\sin 5.4^\circ} \]

\[ \frac{d}{\lambda} = 16 \]

Statement: The ratio of the separation of the slits to the wavelength is 16:1.

7. (a) Given: double-slit interference; \( d = 0.80 \text{ mm} = 8.0 \times 10^{-4} \text{ m} \); \( L = 49 \text{ cm} = 0.49 \text{ m} \);

\( \Delta x = 0.33 \text{ mm} = 3.3 \times 10^{-4} \text{ m} \)

Required: \( \lambda \)

Analysis: Rearrange the equation \( \Delta x = \frac{L \lambda}{d} \) to solve for wavelength;

\[ \Delta x = \frac{L \lambda}{d} \]

\[ \lambda = \frac{d \Delta x}{L} \]
Solution:
\[ \Delta \lambda = \frac{d \Delta x}{L} \]
\[ = \frac{(8.0 \times 10^{-4} \text{ m})(3.3 \times 10^{-4} \text{ m})}{(0.49 \text{ m})} \]
\[ \lambda = 5.39 \times 10^{-7} \text{ m} \text{ (one extra digit carried)} \]
\[ \lambda = 5.4 \times 10^{-7} \text{ m} \]
Statement: The wavelength of the monochromatic light is 5.4 \times 10^{-7} \text{ m}, or 540 nm.

(b) Given: double-slit interference; \( d = 0.60 \text{ mm} = 6.0 \times 10^{-4} \text{ m} \); \( L = 0.49 \text{ m} \); \( \lambda = 5.39 \times 10^{-7} \text{ m} \)
Required: \( \Delta x \), of the dark fringes

Analysis: \( \Delta x = \frac{L \lambda}{d} \)

Solution:
\[ \Delta x = \frac{L \lambda}{d} \]
\[ = \frac{(0.49 \text{ m})(5.39 \times 10^{-7} \text{ m})}{(6.0 \times 10^{-4} \text{ m})} \]
\[ \Delta x = 4.4 \times 10^{-4} \text{ m} \]
Statement: The spacing of the dark fringes is 4.4 \times 10^{-4} \text{ m}, or 0.44 mm.

8. (a) Given: double-slit interference; \( L = 2.5 \text{ m} \); \( \lambda = 5.1 \times 10^{-7} \text{ m} \); \( \Delta x = 12 \text{ mm} = 1.2 \times 10^{-2} \text{ m} \)
Required: \( d \)

Analysis: Rearrange the equation \( \Delta x = \frac{L \lambda}{d} \) to solve for distance between the slits;
\[ \Delta x = \frac{L \lambda}{d} \]
\[ d = \frac{L \lambda}{\Delta x} \]

Solution:
\[ d = \frac{L \lambda}{\Delta x} \]
\[ = \frac{(2.5 \text{ m})(5.1 \times 10^{-7} \text{ m})}{(1.2 \times 10^{-2} \text{ m})} \]
\[ d = 1.1 \times 10^{-4} \text{ m} \]
Statement: The slit spacing is 1.1 \times 10^{-4} \text{ m}, or 0.11 mm.

(b) Solutions may vary. Sample solution: From \( \Delta x = \frac{L \lambda}{d} \), the fringe separation is inversely proportional to the slit spacing. If the slit spacing is reduced by a factor of three, the fringe separation will increase by a factor of three. The fringe separation will change to \( 3 \times 12 \text{ mm} = 36 \text{ mm}, \text{ or } 3.6 \text{ cm} \).
9. Answers may vary. Student answers should include some of the following points. Newton developed a particle theory of light that explained rectilinear propagation and reflection well. His reputation put the work of others at the time in the shadows. Grimaldi was working on a theory for the diffraction and dispersion of light, and Hooke was looking a wave theory. Later, Newton’s theory that light travelled faster in a medium than in a vacuum was shown experimentally to be incorrect. With this failure, Newton’s explanations of refraction, colour, and dispersion were also refuted. Huygens’ wave ideas and Young’s double-slit interference experiment showed that light is definitely a wave. By 1900, all known properties of light could be explained by a wave model.