Section 9.3: Diffraction and Interference of Water Waves

Tutorial 1 Practice, page 461

1. Given: $\lambda = 1.0$ m; $w = 0.5$ m

Required: $\dfrac{\lambda}{w}$

Analysis: Diffraction should be noticeable if $\dfrac{\lambda}{w} \geq 1$, so solve for $\dfrac{\lambda}{w}$.

Solution:

$$\frac{\lambda}{w} = \frac{1.0\text{ m}}{0.5\text{ m}}$$

$$\frac{\lambda}{w} = 2$$

Statement: Yes, the diffraction should be noticeable because $\dfrac{\lambda}{w}$ is greater than 1.

2. Given: λ = 630 nm = 6.3×10^{-7} m

Required: maximum width *w* for noticeable diffraction

Analysis: Use the condition that $\dfrac{\lambda}{w} \geq 1$.

Solution:

$$\frac{\lambda}{w} \geq 1$$

$$\lambda \geq w$$

$$6.3 \times 10^{-7}\text{ m} \geq w$$

Statement: The maximum slit width for significant diffraction to be produced is 6.3×10^{-7} m.

Mini Investigation: Interference from Two Speakers, page 464

A. Answers may vary. Sample answer: The distances to the two speakers should differ by zero or a whole number of wavelengths to get constructive interference.

B. Answers may vary. Sample answers: The distances to the two speakers should differ by a half-whole number of wavelengths to get destructive interference.

C. Answers may vary. Sample answers: If a sound of known frequency and wavelength is played, students can compare their estimates with the known values.

Tutorial 2 Practice, page 468

1. Given: two-source interference; $\lambda = 2.5$ m

Required: *d*, smallest path difference for a node

Analysis: Use $\left|P_nS_1 - P_nS_2\right| = \left(n - \dfrac{1}{2}\right)\lambda$ with $n = 1$.

Solution: $\left|P_nS_1 - P_nS_2\right| = \left(n - \frac{1}{2}\right)\lambda$

$$= \left(1 - \frac{1}{2}\right)(2.5 \text{ m})$$

$$\left|P_1S_1 - P_1S_2\right| = 1.2 \text{ m}$$

Statement: The smallest path difference for a node is 1.2 m.

2. (a) Given: $n = 3$; $P_3S_1 = 35$ cm; $P_3S_2 = 42$ cm

Required: λ

Analysis: $\left|P_nS_1 - P_nS_2\right| = \left(n - \frac{1}{2}\right)\lambda$

$$\lambda = \frac{\left|P_nS_1 - P_nS_2\right|}{n - \frac{1}{2}}$$

Solution: $\lambda = \frac{\left|P_3S_1 - P_3S_2\right|}{n - \frac{1}{2}}$

$$= \frac{\left|35 \text{ cm} - 42 \text{ cm}\right|}{2.5}$$

$$\lambda = 2.8 \text{ cm}$$

Statement: The wavelength of the waves is 2.8 cm.

(b) Given: $f = 10.5$ Hz; $\lambda = 2.8$ cm

Required: v

Analysis: $v = f\lambda$

Solution: $v = f\lambda$

$$= (10.5 \text{ Hz})(2.8 \text{ cm})$$

$$v = 29 \text{ cm/s}$$

Statement: The speed of the waves is 29 cm/s.

3. (a) Given: $n = 2$; $P_2S_1 = 29.5$ cm; $P_2S_2 = 25.0$ cm; $v = 7.5$ cm/s

Required: λ

Analysis: $\left|P_nS_1 - P_nS_2\right| = \left(n - \frac{1}{2}\right)\lambda$

$$\lambda = \frac{\left|P_nS_1 - P_nS_2\right|}{n - \frac{1}{2}}$$

Solution: $\lambda = \dfrac{|P_2S_1 - P_2S_2|}{n - \dfrac{1}{2}}$

$= \dfrac{|29.5\text{ cm} - 25.0\text{ cm}|}{1.5}$

$\lambda = 3.0\text{ cm}$

Statement: The wavelength of the waves is 3.0 cm.

(b) Given: $v = 7.5\text{ cm/s}$; $\lambda = 3.0\text{ cm}$

Required: f

Analysis: $v = f\lambda$

$f = \dfrac{v}{\lambda}$

Solution: $f = \dfrac{v}{\lambda}$

$= \dfrac{7.5\ \dfrac{\cancel{\text{cm}}}{\text{s}}}{3.0\ \cancel{\text{cm}}}$

$f = 2.5\text{ Hz}$

Statement: The frequency at which the sources are vibrating is 2.5 Hz.

Section 9.3 Questions, page 469

1. Diffraction of waves through a slit is maximized when the wavelength is comparable to or somewhat greater than the slit width.

2. Answers may vary. Sample answer: When the waves reach my friend in phase, there is constructive interference and he hears a loud sound. When the phase of one speaker is changed by 180°, the waves reach my friend out of phase and he hears a sound with decreased volume.

3. (a) Given: $\lambda = 6.3 \times 10^{-4}\text{ m}$

Required: maximum width w for noticeable diffraction

Analysis: Use the condition that $\dfrac{\lambda}{w} \geq 1$.

Solution:

$\dfrac{\lambda}{w} \geq 1$

$\lambda \geq w$

$6.3 \times 10^{-4}\text{ m} \geq w$

Statement: The maximum slit width for noticeable diffraction is 6.3×10^{-4} m.

(b) Sample answer: If the slit is wider than 6.3×10^{-4} m, there may still be some diffraction. The wider the slit is, the less diffraction will be noticeable.

4. Given: $d = 1.0$ m; $\lambda = 0.25$ m; $n = 1$

Required: θ_1

Analysis: $d \sin\theta_n = \left(n - \frac{1}{2}\right)\lambda$

$$\sin\theta_n = \frac{\left(n - \frac{1}{2}\right)\lambda}{d}$$

Solution:

$$\sin\theta_n = \frac{\left(n - \frac{1}{2}\right)\lambda}{d}$$

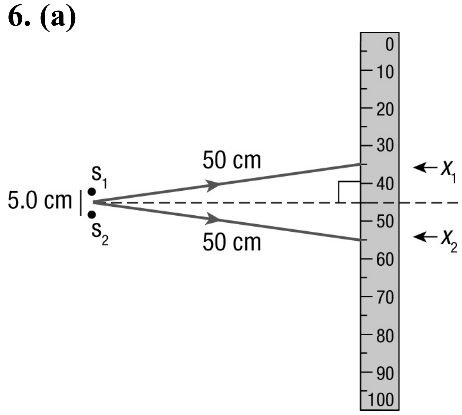
$$\theta_1 = \sin^{-1}\left(\frac{\left(1 - \frac{1}{2}\right)(0.25\ \cancel{\text{m}})}{1.0\ \cancel{\text{m}}}\right)$$

$$\theta_1 = 7.2°$$

Statement: The angle of the first nodal line is 7.2°.

5. Sample answer: For the interference pattern from a two-point source to be stable, the phase between the sources must not change.

6. (a)



(b) Given: $d = 5.0$ cm; $n = 1$; $x_1 = 45\text{ cm} - 35\text{ cm} = 10$ cm; $L = 50$ cm; $f = 6.0$ Hz

Required: λ

Analysis: The distance from the nodal points to the midpoint between the sources is close to the distance L between the line joining the sources and the metre stick. Rearrange $x_n = \left(n - \frac{1}{2}\right)\frac{L\lambda}{d}$ to determine the wavelength, $\lambda = \frac{x_n d}{\left(n - \frac{1}{2}\right)L}$.

Solution: $\lambda = \dfrac{x_n d}{\left(n - \dfrac{1}{2}\right)L}$

$= \dfrac{(10\text{ cm})(5.0\ \cancel{\text{cm}})}{\left(1 - \dfrac{1}{2}\right)(50\ \cancel{\text{cm}})}$

$\lambda = 2\text{ cm}$

Statement: The wavelength of the waves is 2 cm.

(c) Given: $\lambda = 2$ cm; $f = 6.0$ Hz

Required: v

Analysis: $v = f\lambda$

Solution: $v = f\lambda$

$= (6.0\text{ Hz})(2\text{ cm})$

$v = 12\text{ cm/s}$

Statement: The speed of the waves is 12 cm/s.

7. (a) Given: $n = 3$; $x_3 = 35$ cm; $L = 77$ cm; $d = 6.0$ cm; $\theta_3 = 25°$;
distance between 5 crests $= 4.2$ cm

Required: Calculate λ using three different methods.

Analysis: For the first method, use $x_n = \left(n - \dfrac{1}{2}\right)\dfrac{L\lambda}{d}$, with n = 3. For the second method, use $d\sin\theta_n = \left(n - \dfrac{1}{2}\right)\lambda$, with n = 3. For the third method, use the measurement between wave crests to determine the wavelength directly.

Solution:

First method:

$x_n = \left(n - \dfrac{1}{2}\right)\dfrac{L\lambda}{d}$

$\lambda = \dfrac{x_3 d}{\left(3 - \dfrac{1}{2}\right)L}$

$= \dfrac{(35\text{ cm})(6.0\ \cancel{\text{cm}})}{\left(3 - \dfrac{1}{2}\right)(77\ \cancel{\text{cm}})}$

$= 1.09\text{ cm}$ (one extra digit carried)

$\lambda = 1.1\text{ cm}$

Second method:

$$d\sin\theta_n = \left(n - \frac{1}{2}\right)\lambda$$

$$\begin{aligned}\lambda &= \frac{d\sin\theta_n}{n - \frac{1}{2}} \\ &= \frac{(6.0\text{ cm})\sin 25^\circ}{3 - \frac{1}{2}} \\ &= 1.01\text{ cm (one extra digit carried)} \\ \lambda &= 1.0\text{ cm}\end{aligned}$$

Third method:

The distance between five consecutive crests corresponds to four whole wavelengths:

$$4\lambda = 4.2\text{ cm}$$

$$\begin{aligned}\lambda &= \frac{4.2\text{ cm}}{4} \\ &= 1.05\text{ cm (one extra digit carried)} \\ \lambda &= 1.0\text{ cm}\end{aligned}$$

Each of the three methods of analysis resulted in a wavelength between 1.0 cm and 1.1 cm, with an average of 1.0 cm.

Statement: The wavelength is 1.0 cm.

(b) Answers may vary. Sample answer: The three methods are based on data with two significant digits. The three results differed only by one unit in the last digit. I think that these results are consistent and that no particular measurement stands out as being incorrect.