Section 9.2: Refraction and Total Internal Reflection

Tutorial 1 Practice, page 449

1. The angle of incidence is 65°. The fact that the experiment takes place in water does not change the angle of incidence.

2. Given: θ_i = 47.5°; θ_R = 34.0°; n_{air} = 1.0003

Required: n_2

Analysis: Index of refraction is a physical property that can be used to identify a substance. Use Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$, to calculate the index of refraction of the medium. Then match it to a substance in Table 1.

Solution: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\begin{aligned} n_2 &= \frac{n_1 \sin\theta_1}{\sin\theta_2} \\ &= \frac{(1.0003)\sin 47.5°}{\sin 34.0°} \\ n_2 &= 1.32 \end{aligned}$$

According to Table 1, the index of refraction lies between that of ice and liquid water but is closer to water.

Statement: The medium is probably water.

3. Given: θ_1 = 35°; θ_R = 25°; n_{air} = 1.0003

Required: n_2

Analysis: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Solution: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\begin{aligned} n_2 &= \frac{n_1 \sin\theta_1}{\sin\theta_2} \\ &= \frac{(1.0003)\sin 35°}{\sin 25°} \\ n_2 &= 1.36 \end{aligned}$$

Statement: The index of refraction of the water is 1.36.

4. Given: n = 2.42; $c = 3.0 \times 10^8$ m/s

Required: v

Analysis: Use the definition of index of refraction, $n = \frac{v}{c}$, to solve for the speed v.

$$n = \frac{c}{v}$$

$$v = \frac{c}{n}$$

Solution: $v = \frac{c}{n}$

$= \frac{3.0 \times 10^8 \text{ m/s}}{2.42}$

$v = 1.2 \times 10^8 \text{ m/s}$

Statement: The speed of light in diamond is 1.2×10^8 m/s.

5. Given: $n = 1.46$; $\lambda_1 = 5.6 \times 10^{-7}$ m

Required: λ_2

Analysis: Use the alternative definition of index of refraction, $n = \frac{\lambda_1}{\lambda_2}$, to solve for λ_2.

$$n = \frac{\lambda_1}{\lambda_2}$$

$$\lambda_2 = \frac{\lambda_1}{n}$$

Solution: $\lambda_2 = \frac{\lambda_1}{n}$

$= \frac{5.6 \times 10^{-7} \text{ m}}{1.46}$

$\lambda_2 = 3.8 \times 10^{-7}$ m

Statement: The wavelength of light in quartz is 3.8×10^{-7} m.

6. Given: $n = 1.45$; $\lambda_1 = 450 \text{ nm} = 4.5 \times 10^{-7}$ m

Required: f_2

Analysis: The frequency of light does not change when light passes from one medium into another. The frequency of the light inside the glass is the same as in vacuum. Rearrange the universal wave equation, $v = f\lambda$, to solve for f.

$$v = f\lambda$$

$$f = \frac{v}{\lambda}$$

Solution: $f = \frac{v}{\lambda}$

$= \frac{3.0 \times 10^8 \cancel{\text{m}}\text{/s}}{4.5 \times 10^{-7} \cancel{\text{m}}}$

$f = 6.7 \times 10^{14}$ Hz

Statement: The frequency of the light is 6.7×10^{14} Hz.

Tutorial 2 Practice, page 452

1. (a) Given: $\theta_1 = 40.0°$; $n_1 = 1.0003$; $n_2 = 1.465$

Required: θ_2

Analysis: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Solution: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2}$$

$$\begin{aligned}\theta_2 &= \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right)\\ &= \sin^{-1}\left(\frac{(1.0003)\sin 40.0°}{1.465}\right)\\ &= 26.033° \text{ (two extra digits carried)}\\ \theta_2 &= 26.0°\end{aligned}$$

Statement: The angle of refraction at the left boundary of the prism is 26.0°.

(b) Given: $\theta_2 = 26.033°$; $n_3 = 1.465$

Required: θ_4

Analysis: From the geometry of the prism, the angle of incidence at the right boundary, θ_3, is $\theta_3 = 60.0° - \theta_2$. Determine θ_2, then use Snell's law, $n_3 \sin\theta_3 = n_4 \sin\theta_4$, to calculate the angle of refraction, θ_4.

Solution: $\theta_3 = 60.0° - \theta_2$

$$\begin{aligned}&= 60.0° - 26.033°\\ \theta_3 &= 33.967° \text{ (two extra digits carried)}\end{aligned}$$

$$n_3 \sin\theta_3 = n_4 \sin\theta_4$$

$$\sin\theta_4 = \frac{n_3 \sin\theta_3}{n_4}$$

$$\begin{aligned}\theta_4 &= \sin^{-1}\left(\frac{n_3 \sin\theta_3}{n_4}\right)\\ &= \sin^{-1}\left(\frac{(1.465)\sin 33.967°}{1.0003}\right)\\ \theta_4 &= 54.9°\end{aligned}$$

Statement: The angle of refraction of the exiting light is 54.9°.

2. Sample answer: The light entered and exited the prism on faces that were not parallel. You would only see the exit angle equal to the incident angle if the faces were parallel, as in a sheet of glass.

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3. Given: $\theta_1 = 55°$; $n = 1.60$

Required: θ, the angle of the outgoing ray as measured with the horizontal

Analysis: Calculate the first angle of refraction, θ_2, using Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$. Then determine the second angle of incidence, θ_3, using $\theta_3 = 60.0° - \theta_2$. Use Snell's law to calculate the second angle of refraction, θ_4. Determine the exit angle, θ, with respect to the horizontal.

Solution:

The first angle of refraction is θ_2.

$$\begin{aligned}
n_1 \sin\theta_1 &= n_2 \sin\theta_2 \\
\sin\theta_2 &= \frac{n_1 \sin\theta_1}{n_2} \\
\theta_2 &= \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right) \\
&= \sin^{-1}\left(\frac{(1.0003)\sin 55°}{1.60}\right) \\
\theta_2 &= 30.81° \text{ (two extra digits carried)}
\end{aligned}$$

The second angle of incidence is θ_3.

$$\begin{aligned}
\theta_3 &= 60.0° - \theta_2 \\
&= 60.0° - 30.81° \\
\theta_3 &= 29.19° \text{ (two extra digits carried)}
\end{aligned}$$

The second angle of refraction is θ_4.

$$\begin{aligned}
n_3 \sin\theta_3 &= n_4 \sin\theta_4 \\
\sin\theta_4 &= \frac{n_3 \sin\theta_3}{n_4} \\
\theta_4 &= \sin^{-1}\left(\frac{n_3 \sin\theta_3}{n_4}\right) \\
&= \sin^{-1}\left(\frac{(1.60)\sin 29.19°}{1.0003}\right) \\
\theta_4 &= 51°
\end{aligned}$$

The normal on the right side of the prism is directed at 30° above the horizontal, so the exit angle is $\theta = \theta_4 - 30°$

$$\begin{aligned}
&= 51° - 30° \\
\theta &= 21°
\end{aligned}$$

Statement: The light exits at 21° below the horizontal.

Research This: Using Spectroscopy to Determine Whether Extra-Solar Planets Can Support Life, page 452

A. Answers may vary. Sample answers: Light reflected from other planets can be seen and analyzed on Earth. When the light passes through a spectrometer, it is dispersed (broken up) into its component colours and makes a spectrum similar to the one shown in the text. Scientists use the dark lines in the spectrum to identify the atom or molecule that absorbed the missing colours. This atom or molecule had to be on the planet where the light was reflected.
B. Answers may vary. Sample answers: Astrophysicists and astrobiologists look for oxygen, carbon, nitrogen, and hydrogen. On Earth, these are the main elements involved in biological processes. Finding these elements elsewhere could indicate the right conditions for extraterrestrial life.
C. Answers may vary. Sample answers: Scientists think of light as a wave when using a diffraction grating in a spectrometer. But they also think of light as particle when it is absorbed by or emitted from an atom.

Tutorial 3 Practice, page 457

1. Answers may vary. Sample answer: I will use a liquid with $n = 1.20$ for my comparison.
Given: $n_1 = 1.20$; $n_2 = 1.0003$
Required: θ_c
Analysis: $\theta_c = \sin^{-1}\left(\dfrac{n_2}{n_1}\right)$
Solution: $\theta_c = \sin^{-1}\left(\dfrac{n_2}{n_1}\right)$

$$= \sin^{-1}\left(\frac{1.0003}{1.20}\right)$$

$$\theta_c = 56.4^\circ$$

Statement: If the index of refraction of the liquid is decreased to 1.20, then the critical angle increases to 56.4°. As the index of refraction decreases, the critical angle increases.
2. Given: $n_1 = 1.50$; $n_2 = 1.33$
Required: θ_c
Analysis: $\theta_c = \sin^{-1}\left(\dfrac{n_2}{n_1}\right)$
Solution: $\theta_c = \sin^{-1}\left(\dfrac{n_2}{n_1}\right)$

$$= \sin^{-1}\left(\frac{1.33}{1.50}\right)$$

$$\theta_c = 62.5^\circ$$

Statement: The critical angle for light at the benzene–water boundary is 62.5°.

3. Given: $n_1 = 1.40$; $n_2 = 1.0003$

Required: θ_c

Analysis: $\theta_c = \sin^{-1}\left(\dfrac{n_2}{n_1}\right)$

Solution: $\theta_c = \sin^{-1}\left(\dfrac{n_2}{n_1}\right)$

$$= \sin^{-1}\left(\frac{1.0003}{1.40}\right)$$

$$\theta_c = 45.6°$$

Statement: The critical angle for light on the glass–air boundary is 45.6°.

4. Given: $n_d = 2.42$; $n_g = 1.52$; $n_z = 1.92$; $n_{air} = 1.0003$

Required: θ_c; θ_g; θ_z

Analysis: $\theta_c = \sin^{-1}\left(\dfrac{n_{air}}{n_{med}}\right)$

Solution:

Diamond:

$$\theta_{c,d} = \sin^{-1}\left(\frac{n_{air}}{n_d}\right) = \sin^{-1}\left(\frac{1.0003}{2.42}\right)$$

$$\theta_{c,d} = 24.4°$$

Crown glass:

$$\theta_{c,g} = \sin^{-1}\left(\frac{n_{air}}{n_g}\right) = \sin^{-1}\left(\frac{1.0003}{1.52}\right)$$

$$\theta_{c,g} = 41.2°$$

Zircon:

$$\theta_{c,z} = \sin^{-1}\left(\frac{n_{air}}{n_z}\right) = \sin^{-1}\left(\frac{1.0003}{1.92}\right)$$

$$\theta_{c,z} = 31.4°$$

Statement: The critical angle for diamond is 24.4°. The critical angle for zircon is 31.4°. The critical angle for crown glass is 41.2°.

Diamond has a smaller critical angle than crown glass and zircon, so a light ray passing through diamond is more likely to reflect off the surface. If the light passes into the diamond from an angle that is less than the normal angle of 90° (most probable), then the refraction will be more likely to disperse the spectrum than a material such as glass, which has a far lower index of refraction. The diamond appears to glitter.

Additional information: Light rays that pass through a piece of material like diamond may reflect off the surface several times before finally passing out of the material in a different direction than when they entered. This effect gives a viewer the impression that light sources inside the material produced the light, even if the light came from a source outside the material.

Section 9.2 Questions, page 458

1. Answers may vary. Sample answer: When light travels from one medium to another, its direction of propagation changes. This change in direction during refraction makes the light ray appear to "bend."

2. When light is reflected or refracted, its direction changes. The change in angle between the incident ray and the outgoing ray is the angle of deviation.

3. Given: $n = 1.33$; $\lambda_1 = 630 \text{ nm} = 6.3 \times 10^{-7} \text{ m}$

Required: λ_2

Analysis: Rearrange the equation for index of refraction, $n = \dfrac{\lambda_1}{\lambda_2}$, to solve for wavelength.

$$n = \frac{\lambda_1}{\lambda_2}$$

$$\lambda_2 = \frac{\lambda_1}{n}$$

Solution: $\lambda_2 = \dfrac{\lambda_1}{n}$

$$= \frac{6.3 \times 10^{-7} \text{ m}}{1.33}$$

$$\lambda_2 = 4.7 \times 10^{-7} \text{ m}$$

Statement: In water, the laser light has a wavelength of 4.7×10^{-7} m, or 470 nm.

4. Given: $v = 3.0 \times 10^8$ m/s; $c = 3.0 \times 10^8$ m/s

Required: n

Analysis: $n = \dfrac{v}{c}$

Solution: $n = \dfrac{v}{c}$

$$= \frac{3.0 \times 10^8 \cancel{\text{m/s}}}{3.0 \times 10^8 \cancel{\text{m/s}}}$$

$$n = 1.0$$

Statement: The index of refraction of the medium is 1.0.

5. Given: $\theta_1 = 30.0°$; $n_1 = 1.47$; $n_2 = 1.33$; $n_3 = 1.0003$

Required: θ_3

Analysis: One method is to use Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$, to determine the angle of refraction in the water film. This angle is the incident angle for the second refraction into air. Use Snell's law to determine the angle of refraction in air. A second method recognizes that the film of water does not matter because its surfaces are parallel. We could use Snell's law to go directly from glass to air.

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Solution:

First method:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2}$$

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right)$$

$$= \sin^{-1}\left(\frac{(1.47)\sin 30.0°}{1.33}\right)$$

$$\theta_2 = 33.548° \text{ (two extra digits carried)}$$

$$n_2 \sin\theta_2 = n_3 \sin\theta_3$$

$$\sin\theta_3 = \frac{n_2 \sin\theta_2}{n_3}$$

$$\theta_3 = \sin^{-1}\left(\frac{n_2 \sin\theta_2}{n_3}\right)$$

$$= \sin^{-1}\left(\frac{(1.33)\sin 33.548°}{1.0003}\right)$$

$$\theta_3 = 47.3°$$

Second method:

$$n_1 \sin\theta_1 = n_3 \sin\theta_3$$

$$\sin\theta_3 = \frac{n_1 \sin\theta_1}{n_3}$$

$$\theta_3 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_3}\right)$$

$$= \sin^{-1}\left(\frac{(1.47)\sin 30.0°}{1.0003}\right)$$

$$\theta_3 = 47.3°$$

Statement: The angle of refraction of the final outgoing ray is 47.3°.

6. Given: $\theta_1 = 30.0°$; $n_1 = 1.44$; $n_2 = 1.0003$

Required: n_2

Analysis: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Solution: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2}$$

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right)$$

$$= \sin^{-1}\left(\frac{(1.44)\sin 30.0°}{1.0003}\right)$$

$$\theta_2 = 46.0°$$

Statement: The angle of refraction is 46.0°.

7. Given: $\theta_1 = 50.0°$; $n_1 = 1.33$; $n_2 = 1.0003$

Required: n_2

Analysis: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Solution: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2}$$

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right)$$

$$= \sin^{-1}\left(\frac{(1.33)\sin 50.0°}{1.0003}\right)$$

$\theta_2 = \sin^{-1}(1.0185)$ There is no solution for θ_2.

Statement: The incident angle of the laser beam in water is greater than the critical angle in water. The laser beam undergoes total internal reflection.

8. (a) Given: $n_1 = 1.65$; $n_2 = 1.33$

Required: θ_c

Analysis: $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

Solution: $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$$= \sin^{-1}\left(\frac{1.33}{1.65}\right)$$

$$\theta_c = 53.7°$$

Statement: The critical angle for light at a glass–water boundary is 53.7°.

(b) The light starts in the medium with the higher index of refraction, which is the glass. There can be no total internal reflection if the light starts in the medium with the lower index of refraction.

9. (a) Given: $\theta_2 = 45°$; $n_1 = 1.0003$; $n_2 = 1.30$

Required: $\theta_i = \theta_1$

Analysis: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Solution:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$\sin\theta_1 = \frac{n_2 \sin\theta_2}{n_1}$$
$$\theta_1 = \sin^{-1}\left(\frac{n_2 \sin\theta_2}{n_1}\right)$$
$$= \sin^{-1}\left(\frac{1.30 \sin 45°}{1.0003}\right)$$
$$\theta_1 = 67°$$

Statement: The angle of incidence is 67°.

(b) Given: $n_1 = 1.30$; $n_2 = 1.0003$

Required: θ_c

Analysis: $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

Solution: $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$$= \sin^{-1}\left(\frac{1.0003}{1.30}\right)$$
$$\theta_c = 50.3°$$

Statement: The critical angle for light at the transparent material–air boundary is 50.3°.

10. (a) Given: $\theta_1 = 30.0°$; $n_1 = 1.33$; $n_2 = 1.63$

Required: $\theta_R = \theta_2$

Analysis: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Solution: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2}$$
$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right)$$
$$= \sin^{-1}\left(\frac{(1.33)\sin 30.0°}{1.63}\right)$$
$$\theta_2 = 24.1°$$

Statement: The angle of refraction is 24.1°.

(b) Light incident on a water–carbon disulfide boundary cannot undergo total internal reflection because the index of refraction of carbon disulfide is greater than the index of refraction of water.
11. Answers may vary. Sample answer: Fibre optics, which use total internal reflection, are used in medicine to view inside various parts of the body. One example of an instrument that uses fibre optics is the endoscope. Doctors use an endoscope to examine a patient's internal tissues and organs.
Additional information: An angioscope is a coated fibre optic cable with a fish-eye lens that can be inserted into a blood vessel to diagnose constrictions, blockages, or weaknesses. A gastroscope is another variation of fibre optic cable that is swallowed and is used to view the esophagus, stomach, and some of the small intestine. Most medical fibre optic scopes also have mechanisms for taking tissue samples or for removing diseased tissue.
12. (a) Answers may vary. Sample answer: Hibernia Atlantic and Emerald Express are two international companies with plans for new fibre optic transatlantic cables.
(b) The biggest advantage of submarine cables is that the time for transmission and reception of the signal is significantly shorter than when using satellite communication. This may not seem like a major advantage for conversations, but most transatlantic communication involves investment trading, when every millisecond counts. The newest cables are aiming for round-trip transit times of 60 ms. The biggest disadvantage of submarine cables is cost. This technology contributes to the escalating prices for communications.
13. Answers may vary. Sample answer: Signal reduction, usually called attenuation, in optical fibres occurs for a number of reasons. One reason is that impurities in the fibre may absorb the signal. More significantly, there are losses due to reflection from the core or cladding, and losses due to splicing of the cables. These losses occur because the signal reflects off these surfaces at an angle that will allow transmission out of the fibre.