Section 8.4: Motion of Charged Particles in Magnetic Fields

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1. Given: \( q = 3.2 \times 10^{-19} \text{ C} \); \( m = 6.7 \times 10^{-27} \text{ kg} \); \( B = 2.4 \text{ T} \); \( v = 1.5 \times 10^7 \text{ m/s} \)

Required: \( r \)

Analysis: \( r = \frac{mv}{qB} \)

Solution: \[
\frac{mv}{qB} = \frac{\left(6.7 \times 10^{-27} \text{ kg}\right)\left(1.5 \times 10^7 \text{ m/s}\right)}{\left(3.2 \times 10^{-19} \text{ C}\right)\left(2.4 \frac{\text{kg}}{\text{C} \cdot \text{s}}\right)}
\]
\( r = 0.13 \text{ m} \)

Statement: The radius of the ion’s path is 0.13 m.

2. Given: \( q = 1.60 \times 10^{-19} \text{ C} \); \( m = 1.67 \times 10^{-27} \text{ kg} \); \( B = 1.5 \text{ T} \); \( r = 8.0 \text{ cm} = 0.080 \text{ m} \)

Required: \( v \)

Analysis: \( r = \frac{mv}{qB} \)

\[
v = \frac{rqB}{m}
\]

Solution: \[
\frac{rqB}{m} = \frac{\left(0.080 \text{ m}\right)\left(1.60 \times 10^{-19} \text{ C}\right)\left(1.5 \frac{\text{kg}}{\text{C} \cdot \text{s}}\right)}{\left(1.67 \times 10^{-27} \text{ kg}\right)}
\]
\( v = 1.1 \times 10^7 \text{ m/s} \)

Statement: The speed of the proton is \( 1.1 \times 10^7 \text{ m/s} \).

3. Given: \( q = 1.60 \times 10^{-19} \text{ C} \); \( v = 6.0 \times 10^5 \text{ m/s} \); \( m_1 = 1.67 \times 10^{-27} \text{ kg} \);

\( m_2 = 2(1.67 \times 10^{-27} \text{ kg}) = 3.34 \times 10^{-27} \text{ kg} \); \( \Delta d = 1.5 \text{ mm} = 0.0015 \text{ m} \)

Required: \( B \)

Analysis: \( r = \frac{mv}{qB} \)

In a mass spectrometer, the difference between the entry point and the ion detector is \( 2r \). The greater the mass of an ion, the greater the radius. So, the deuterium ion is detected at \( 2r + 0.0015 \text{ m} \) from the entry point, where \( r \) is the radius of the path of the hydrogen ion: \[
\Delta d = 2r_{\text{deuterium}} - 2r_{\text{hydrogen}}
\]
Solution: \( \Delta d = 2r_{\text{deuterium}} - 2r_{\text{hydrogen}} \)
\[ = \frac{2m_{\text{deuterium}} v}{qB} - \frac{2m_{\text{hydrogen}} v}{qB} \]
\[ = \frac{2v(2m_{\text{hydrogen}} - m_{\text{hydrogen}})}{qB} \]

\[ B = \frac{2vm_{\text{hydrogen}}}{q \Delta d} \]
\[ = \frac{2 \left( \frac{6.0 \times 10^5 \text{ m/s}}{1.67 \times 10^{-27} \text{ kg}} \right) \left( 1.60 \times 10^{-19} \text{ C} \right) \left( 0.0015 \text{ m}^2 / \text{s} \right)}{1.67 \times 10^{-27} \text{ kg}} \]

\[ B = 8.4 \text{ T} \]

**Statement:** The magnitude of the magnetic field is 8.4 T.

4. (a) Since the electric force is up, the balancing magnetic force must be down. By the right-hand rule, the magnetic field should be directed out of the page.

(b) The magnetic force is \( F_M = qvB \) since the angle is 90°. The electric force is \( F_E = \varepsilon q \). These forces are equal when the speed is proper:
\[ F_E = F_M \]
\[ \varepsilon q = qvB \]
\[ \varepsilon = vB \]
\[ v = \frac{\varepsilon}{B} \]

The proper velocity is \( v = \frac{\varepsilon}{B} \).

(c) Since speed only affects the magnetic force, an ion moving too fast will experience a greater magnetic force and be pushed downward. An ion moving too slowly will experience a greater electric force and move upward.

**Mini Investigation: Simulating a Mass Spectrometer, page 401**
Answers may vary. Sample answers:
A. The ball bearings experience a magnetic force and deflect by different amounts, depending on their masses. This effect is similar to what happens in a mass spectrometer.

B. This activity does not quite model the function of a mass spectrometer because the bearings do not experience a uniform magnetic force. The force gets stronger at the bottom of the ramp, and gravity will have a more significant effect in this simulation than on particles in a mass spectrometer.
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1. The mass spectrometer makes use of the magnetic force on a moving charged particle. Atoms are converted into ions and then accelerated into a finely focused beam. The force deflects a particle by an amount depending on its mass and its charge. Electric detectors identify how far the ion travelled in the mass spectrometer.

2. Given: \( q = 3 \times (1.60 \times 10^{-19} \text{ C}) = 4.80 \times 10^{-19} \text{ C}; m_{\text{U}238} = 3.952 \times 10^{-25} \text{ kg}; m_{\text{U}235} = 3.903 \times 10^{-25} \text{ kg}; B = 9.5 \text{ T}; \Delta d = 2.2 \text{ mm} = 0.0022 \text{ m} \)

Required: \( v \)

Analysis: \( r = \frac{mv}{qB} \)

In a mass spectrometer, the difference between the entry point and the ion detector is \( 2r \). The greater the mass of an ion, the greater the radius. So, the U-238 ion is detected at \( 2r + 0.0022 \text{ m} \) from the entry point where \( r \) is the radius of the path of the U-235 ion:

\[
\Delta d = 2r_{\text{U}238} - 2r_{\text{U}235} = \frac{2m_{\text{U}238}v}{qB} - \frac{2m_{\text{U}235}v}{qB} = \frac{2v(m_{\text{U}238} - m_{\text{U}235})}{qB}
\]

\[
v = \frac{qB\Delta d}{2(m_{\text{U}238} - m_{\text{U}235})}
\]

Solution: \( v = \frac{qB\Delta d}{2(m_{\text{U}238} - m_{\text{U}235})} = \frac{3 \times (1.60 \times 10^{-19} \text{ C})}{2 \times (3.952 \times 10^{-25} \text{ kg} - 3.903 \times 10^{-25} \text{ kg})} \times (9.5 \text{ kg} \cdot \text{s}^{-1}) \times (0.0022 \text{ m}) \)

\[
v = 1.0 \times 10^6 \text{ m/s}
\]

Statement: The initial speed of the ions is \( 1.0 \times 10^6 \text{ m/s} \).
3. Given: \( q = -1.60 \times 10^{-19} \text{ C}; m = 9.11 \times 10^{-31} \text{ kg}; B = 0.424 \text{ T}; E_k = 2.203 \times 10^{-19} \text{ J} \)

Required: \( r \)

Analysis: Use \( E_k = \frac{1}{2}mv^2 \) to determine the speed of the electron; then use \( r = \frac{mv}{qB} \) to determine the radius of the path.

Solution: Determine the speed of the electron:

\[
E_k = \frac{1}{2}mv^2
\]

\[
v = \sqrt{\frac{2E_k}{m}} = \sqrt{\frac{2 \times (2.203 \times 10^{-19} \text{ kg} \cdot \text{m}^2/\text{s}^2)}{(9.11 \times 10^{-31} \text{ kg})}}
\]

\[v = 6.954 \times 10^5 \text{ m/s} \text{ (two extra digits carried)}\]

Determine the radius of the path:

\[
r = \frac{mv}{qB} = \frac{(9.11 \times 10^{-31} \text{ kg})(6.954 \times 10^5 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.424 \frac{\text{kg} \cdot \text{m}}{\text{C} \cdot \text{s}})}
\]

\[r = 9.34 \times 10^{-6} \text{ m}\]

Statement: The radius of the electron’s path is \( 9.34 \times 10^{-6} \text{ m} \).

4. Given: \( q = 4 \times 10^{-9} \text{ C}; \bar{v}_1 = 3 \times 10^3 \text{ m/s [E 45° N]} \); \( \vec{F}_1 \) is upward; \( \bar{v}_2 = 2 \times 10^4 \text{ m/s [up]} \); \( \vec{F}_2 = 4 \times 10^{-5} \text{ N [W]} \)

Required: \( B \)

Analysis: The upward force in the first situation means that, by the right-hand rule, the direction of the magnetic field must be in the \( x-y \) plane, and somewhere within \( 180° \) counterclockwise of \( E 45° \text{ N} \). The westward force in the second situation means that, by the right-hand rule, the direction of the magnetic field must be in the \( y-z \) plane. The only possible direction that fits both scenarios is north. Use this information and \( F_M = qvB \sin \theta \) to solve for the magnitude of the field.

\[F_M = qvB \sin \theta\]

\[B = \frac{F_M}{qv \sin \theta}\]
Solution:  \[ B = \frac{F_M}{qv\sin\theta} \]
\[ = \frac{\left( 4 \times 10^{-5} \text{ kg} \cdot \frac{\text{m}^2}{\text{s}^2} \right)}{(4 \times 10^{-9} \text{ C})(2 \times 10^4 \frac{\text{m}}{\text{s}})\sin90^\circ} \]
\[ B = 0.5 \text{ T} \]

**Statement:** The magnetic field is 0.5 T [N].

**5. Given:** \( q = -1.60 \times 10^{-19} \text{ C}; m = 9.11 \times 10^{-31} \text{ kg}; \Delta V = 100.0 \text{ V}; B = 0.0400 \text{ T} \)

**Required:** \( r \)

**Analysis:** Use the law of the conservation of energy, \( \Delta E_E + \Delta E_k = 0 \), along with the equations \( E_k = \frac{1}{2}mv^2 \) and \( \Delta V = \frac{E_E}{q} \) to determine the speed of the electron. Then calculate the radius using \( r = \frac{mv}{qB} \).

**Solution:** Determine the speed of the electron:
\[ \Delta E_E + \Delta E_k = 0 \]
\[ q\Delta V + \frac{1}{2}mv^2 = 0 \]
\[ \frac{1}{2}mv^2 = -q\Delta V \]
\[ v = \sqrt{-\frac{2q\Delta V}{m}} \]
\[ = \sqrt{-2\left(-1.60 \times 10^{-19} \text{ C}\right)\left(100.0 \text{ kg} \cdot \frac{\text{m}}{\text{s}^2} \cdot \frac{\text{m}}{\text{C}}\right)} \]
\[ v = 5.9267 \times 10^6 \text{ m/s} \text{ (two extra digits carried)} \]

Determine the radius of the path:
\[ r = \frac{mv}{qB} \]
\[ = \left(9.11 \times 10^{-31} \text{ kg} \cdot \text{C}^2\right)\left(5.9267 \times 10^6 \frac{\text{m}}{\text{s}}\right) \]
\[ \frac{\left(1.60 \times 10^{-19} \text{ C}\right)\left(0.0400 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \text{C} \cdot \text{m}^2\right)}{0.0400 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2}} \]
\[ r = 8.44 \times 10^{-4} \text{ m} \]

**Statement:** The radius of the path described by the electron is \( 8.44 \times 10^{-4} \text{ m} = 0.844 \text{ mm} \).
6. (a) Given: \( \theta = 90^\circ \); \( v = 5.0 \times 10^2 \text{ m/s} \); \( B = 0.050 \text{ T} \)

Required: \( \varepsilon \)

Analysis: \( F_E = F_M \); \( F_M = qvB \sin \theta \); \( F_E = \varepsilon q \)

\( F_E = F_M \)

\( \varepsilon q = qvB \sin \theta \)

Solution: \( \varepsilon = vB \sin 90^\circ \)

\[
\begin{align*}
\varepsilon &= \left( 5.0 \times 10^2 \text{ m/s} \right) \left( 0.050 \frac{\text{kg}}{\text{C} \cdot \text{s}} \right) \\
\varepsilon &= 25 \text{ N/C}
\end{align*}
\]

Statement: The strength of the electric field is 25 N/C.

(b) Given: \( q = 1.60 \times 10^{-19} \text{ C} \); \( m = 1.67 \times 10^{-27} \text{ kg} \); \( B = 0.050 \text{ T} \); \( v = 5.0 \times 10^2 \text{ m/s} \)

Required: \( r \)

Analysis: \( r = \frac{mv}{qB} \)

Solution: \( r = \frac{mv}{qB} \)

\[
\begin{align*}
&= \left( 1.67 \times 10^{-27} \frac{\text{kg}}{} \right) \left( 5.0 \times 10^2 \frac{\text{m}}{} \right) \\
&= \left( 1.60 \times 10^{-19} \frac{\text{C}}{} \right) \left( 0.050 \frac{\text{kg}}{\text{C} \cdot \text{s}} \right) \\
&= 1.0 \times 10^{-4} \text{ m}
\end{align*}
\]

Statement: The radius of the proton’s path to point P is \( 1.0 \times 10^{-4} \) m.