Section 8.4: Motion of Charged Particles in Magnetic Fields

Tutorial 1 Practice, page 401

1. Given: $q = 3.2 \times 10^{-19}$ C; $m = 6.7 \times 10^{-27}$ kg; $B = 2.4$ T; $v = 1.5 \times 10^7$ m/s

Required: r

Analysis: $r = \dfrac{mv}{qB}$

Solution:
$$r = \frac{mv}{qB}$$
$$= \frac{\left(6.7\times10^{-27}\ \cancel{\text{kg}}\right)\left(1.5\times10^{7}\ \dfrac{\text{m}}{\cancel{\text{s}}}\right)}{\left(3.2\times10^{-19}\ \cancel{\text{C}}\right)\left(2.4\ \dfrac{\cancel{\text{kg}}}{\cancel{\text{C}}\cdot\cancel{\text{s}}}\right)}$$
$$r = 0.13\ \text{m}$$

Statement: The radius of the ion's path is 0.13 m.

2. Given: $q = 1.60 \times 10^{-19}$ C; $m = 1.67 \times 10^{-27}$ kg; $B = 1.5$ T; $r = 8.0$ cm = 0.080 m

Required: v

Analysis: $r = \dfrac{mv}{qB}$
$$v = \frac{rqB}{m}$$

Solution:
$$v = \frac{rqB}{m}$$
$$= \frac{(0.080\ \text{m})\left(1.60\times10^{-19}\ \cancel{\text{C}}\right)\left(1.5\ \dfrac{\cancel{\text{kg}}}{\cancel{\text{C}}\cdot\text{s}}\right)}{\left(1.67\times10^{-27}\ \cancel{\text{kg}}\right)}$$
$$v = 1.1\times10^{7}\ \text{m/s}$$

Statement: The speed of the proton is 1.1×10^7 m/s.

3. Given: $q = 1.60 \times 10^{-19}$ C; $v = 6.0 \times 10^5$ m/s; $m_1 = 1.67 \times 10^{-27}$ kg;
$m_2 = 2(1.67 \times 10^{-27}\ \text{kg}) = 3.34 \times 10^{-27}$ kg; $\Delta d = 1.5$ mm = 0.0015 m

Required: B

Analysis: $r = \dfrac{mv}{qB}$

In a mass spectrometer, the difference between the entry point and the ion detector is $2r$. The greater the mass of an ion, the greater the radius. So, the deuterium ion is detected at $2r + 0.0015$ m from the entry point, where r is the radius of the path of the hydrogen ion:
$$\Delta d = 2r_{\text{deuterium}} - 2r_{\text{hydrogen}}$$

Solution: $\Delta d = 2r_{\text{deuterium}} - 2r_{\text{hydrogen}}$

$$= \frac{2m_{\text{deuterium}}v}{qB} - \frac{2m_{\text{hydrogen}}v}{qB}$$

$$= \frac{2v(2m_{\text{hydrogen}} - m_{\text{hydrogen}})}{qB}$$

$$B = \frac{2vm_{\text{hydrogen}}}{q\Delta d}$$

$$= \frac{2\left(6.0\times10^5\ \frac{\cancel{\text{m}}}{\text{s}}\right)(1.67\times10^{-27}\ \text{kg})}{(1.60\times10^{-19}\ \text{C})(0.0015\ \cancel{\text{m}})}$$

$$B = 8.4\ \text{T}$$

Statement: The magnitude of the magnetic field is 8.4 T.

4. (a) Since the electric force is up, the balancing magnetic force must be down. By the right-hand rule, the magnetic field should be directed out of the page.

(b) The magnetic force is $F_{\text{M}} = qvB$ since the angle is 90°. The electric force is $F_{\text{E}} = \varepsilon q$. These forces are equal when the speed is proper:

$$F_{\text{E}} = F_{\text{M}}$$

$$\varepsilon q = qvB$$

$$\varepsilon = vB$$

$$v = \frac{\varepsilon}{B}$$

The proper velocity is $v = \frac{\varepsilon}{B}$.

(c) Since speed only affects the magnetic force, an ion moving too fast will experience a greater magnetic force and be pushed downward. An ion moving too slowly will experience a greater electric force and move upward.

Mini Investigation: Simulating a Mass Spectrometer, page 401

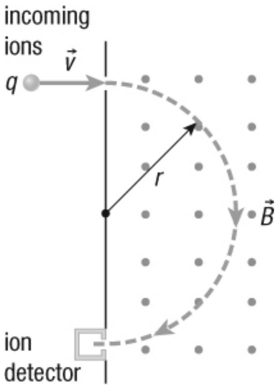
Answers may vary. Sample answers:

A. The ball bearings experience a magnetic force and deflect by different amounts, depending on their masses. This effect is similar to what happens in a mass spectrometer.

B. This activity does not quite model the function of a mass spectrometer because the bearings do not experience a uniform magnetic force. The force gets stronger at the bottom of the ramp, and gravity will have a more significant effect in this simulation than on particles in a mass spectrometer.

Section 8.4 Questions, page 404

1. The mass spectrometer makes use of the magnetic force on a moving charged particle. Atoms are converted into ions and then accelerated into a finely focused beam. The force deflects a particle by an amount depending on its mass and its charge. Electric detectors identify how far the ion travelled in the mass spectrometer.



2. Given: $q = 3(1.60 \times 10^{-19}\text{ C}) = 4.80 \times 10^{-19}\text{ C}$; $m_{\text{U238}} = 3.952 \times 10^{-25}\text{ kg}$; $m_{\text{U235}} = 3.903 \times 10^{-25}\text{ kg}$; $B = 9.5\text{ T}$; $\Delta d = 2.2\text{ mm} = 0.0022\text{ m}$

Required: v

Analysis: $r = \dfrac{mv}{qB}$

In a mass spectrometer, the difference between the entry point and the ion detector is $2r$. The greater the mass of an ion, the greater the radius. So, the U-238 ion is detected at $2r + 0.0022$ m from the entry point where r is the radius of the path of the U-235 ion: $\Delta d = 2r_{\text{U238}} - 2r_{\text{U235}}$

$$\begin{aligned}
\Delta d &= 2r_{\text{U238}} - 2r_{\text{U235}} \\
&= \frac{2m_{\text{U238}}v}{qB} - \frac{2m_{\text{U235}}v}{qB} \\
&= \frac{2v\left(m_{\text{U238}} - m_{\text{U235}}\right)}{qB} \\
v &= \frac{qB\Delta d}{2\left(m_{\text{U238}} - m_{\text{U235}}\right)}
\end{aligned}$$

Solution: $v = \dfrac{qB\Delta d}{2\left(m_{\text{U238}} - m_{\text{U235}}\right)}$

$$= \frac{3\left(1.60 \times 10^{-19}\ \cancel{\text{C}}\right)\left(9.5\ \dfrac{\cancel{\text{kg}}}{\cancel{\text{C}} \cdot \text{s}}\right)\left(0.0022\ \text{m}\right)}{2\left(3.952 \times 10^{-25}\ \cancel{\text{kg}} - 3.903 \times 10^{-25}\ \cancel{\text{kg}}\right)}$$

$$v = 1.0 \times 10^{6}\text{ m/s}$$

Statement: The initial speed of the ions is 1.0×10^{6} m/s.

3. Given: $q = -1.60 \times 10^{-19}$ C; $m = 9.11 \times 10^{-31}$ kg; $B = 0.424$ T; $E_k = 2.203 \times 10^{-19}$ J
Required: r
Analysis: Use $E_k = \frac{1}{2}mv^2$ to determine the speed of the electron; then use $r = \frac{mv}{qB}$ to determine the radius of the path.
Solution: Determine the speed of the electron:

$$E_k = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E_k}{m}}$$

$$= \sqrt{\frac{2\left(2.203 \times 10^{-19}\ \cancel{\text{kg}} \cdot \frac{\text{m}}{\text{s}^2} \cdot \text{m}\right)}{\left(9.11 \times 10^{-31}\ \cancel{\text{kg}}\right)}}$$

$v = 6.954 \times 10^5$ m/s (two extra digits carried)
Determine the radius of the path:

$$r = \frac{mv}{qB}$$

$$= \frac{\left(9.11 \times 10^{-31}\ \cancel{\text{kg}}\right)\left(6.954 \times 10^5\ \frac{\text{m}}{\cancel{\text{s}}}\right)}{\left(1.60 \times 10^{-19}\ \cancel{\text{C}}\right)\left(0.424\ \frac{\cancel{\text{kg}}}{\cancel{\text{C}} \cdot \cancel{\text{s}}}\right)}$$

$r = 9.34 \times 10^{-6}$ m
Statement: The radius of the electron's path is 9.34×10^{-6} m.
4. Given: $q = 4 \times 10^{-9}$ C; $\vec{v}_1 = 3 \times 10^3$ m/s [E 45° N]; $\vec{F}_1$ is upward; $\vec{v}_2 = 2 \times 10^4$ m/s [up]; $\vec{F}_2 = 4 \times 10^{-5}$ N [W]
Required: B
Analysis: The upward force in the first situation means that, by the right-hand rule, the direction of the magnetic field must be in the x–y plane, and somewhere within 180° counterclockwise of E 45° N. The westward force in the second situation means that, by the right-hand rule, the direction of the magnetic field must be in the y–z plane. The only possible direction that fits both scenarios is north. Use this information and $F_M = qvB \sin \theta$ to solve for the magnitude of the field.

$$F_M = qvB\sin\theta$$

$$B = \frac{F_M}{qv\sin\theta}$$

Solution: $B = \dfrac{F_{\text{M}}}{qv\sin\theta}$

$$= \frac{\left(4\times10^{-5}\ \text{kg}\cdot\dfrac{\cancel{\text{m}}}{\text{s}^2}\right)}{\left(4\times10^{-9}\ \text{C}\right)\left(2\times10^{4}\ \dfrac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)\sin 90^\circ}$$

$$B = 0.5\ \text{T}$$

Statement: The magnetic field is 0.5 T [N].

5. Given: $q = -1.60 \times 10^{-19}$ C; $m = 9.11 \times 10^{-31}$ kg; $\Delta V = 100.0$ V; $B = 0.0400$ T

Required: r

Analysis: Use the law of the conservation of energy, $\Delta E_{\text{E}} + \Delta E_{\text{k}} = 0$, along with the equations $E_{\text{k}} = \dfrac{1}{2}mv^2$ and $\Delta V = \dfrac{E_{\text{E}}}{q}$ to determine the speed of the electron. Then calculate the radius using $r = \dfrac{mv}{qB}$.

Solution: Determine the speed of the electron:

$$\Delta E_{\text{E}} + \Delta E_{\text{k}} = 0$$

$$q\Delta V + \frac{1}{2}mv^2 = 0$$

$$\frac{1}{2}mv^2 = -q\Delta V$$

$$v = \sqrt{\frac{-2q\Delta V}{m}}$$

$$= \sqrt{\frac{-2\left(-1.60\times10^{-19}\ \cancel{\text{C}}\right)\left(100.0\ \cancel{\text{kg}}\cdot\dfrac{\text{m}}{\text{s}^2}\cdot\dfrac{\text{m}}{\cancel{\text{C}}}\right)}{\left(9.11\times10^{-31}\ \cancel{\text{kg}}\right)}}$$

$v = 5.9267\times10^{6}$ m/s (two extra digits carried)

Determine the radius of the path:

$$r = \frac{mv}{qB}$$

$$= \frac{\left(9.11\times10^{-31}\ \cancel{\text{kg}}\right)\left(5.9267\times10^{6}\ \dfrac{\text{m}}{\cancel{\text{s}}}\right)}{\left(1.60\times10^{-19}\ \cancel{\text{C}}\right)\left(0.0400\ \dfrac{\cancel{\text{kg}}}{\cancel{\text{C}}\cdot\cancel{\text{s}}}\right)}$$

$$r = 8.44\times10^{-4}\ \text{m}$$

Statement: The radius of the path described by the electron is 8.44×10^{-4} m = 0.844 mm.

6. (a) Given: $\theta = 90°$; $v = 5.0 \times 10^2$ m/s; $B = 0.050$ T
Required: ε
Analysis: $F_E = F_M$; $F_M = qvB \sin \theta$; $F_E = \varepsilon q$

$$F_E = F_M$$
$$\varepsilon q = qvB\sin\theta$$

Solution:
$$\begin{aligned}\varepsilon &= vB\sin 90° \\ &= \left(5.0\times10^2\ \frac{\text{m}}{\text{s}}\right)\left(0.050\ \frac{\text{kg}}{\text{C}\cdot\text{s}}\right) \\ \varepsilon &= 25\ \text{N/C}\end{aligned}$$

Statement: The strength of the electric field is 25 N/C.

(b) Given: $q = 1.60 \times 10^{-19}$ C; $m = 1.67 \times 10^{-27}$ kg; $B = 0.050$ T; $v = 5.0 \times 10^2$ m/s
Required: r

Analysis: $r = \dfrac{mv}{qB}$

Solution:
$$\begin{aligned}r &= \frac{mv}{qB} \\ &= \frac{(1.67\times10^{-27}\ \cancel{\text{kg}})\left(5.0\times10^2\ \frac{\text{m}}{\cancel{\text{s}}}\right)}{(1.60\times10^{-19}\ \cancel{\text{C}})\left(0.050\ \frac{\cancel{\text{kg}}}{\cancel{\text{C}}\cdot\cancel{\text{s}}}\right)} \\ r &= 1.0\times10^{-4}\ \text{m}\end{aligned}$$

Statement: The radius of the proton's path to point P is 1.0×10^{-4} m.