## Section 7.5: Electric Potential and Electric Potential Energy Due to Point Charges

### Tutorial 1 Practice, page 360

**1. (a) Given:** $q_1 = +6.0 \times 10^{-6}$ C; $q_2 = -3.0 \times 10^{-6}$ C; $q_3 = -3.0 \times 10^{-6}$ C; $r = 3.0$ m; $k = 8.99 \times 10^9$ N·m²/C²

**Required:** $V_1$; $V_2$; $V_3$

**Analysis:** The potential at each midpoint is the sum of the potentials due to the three charges. Use $V = \frac{kq}{r}$ to calculate the potential due to each charge. The distance from a midpoint to an endpoint is 0.5$r$, and the distance from a midpoint to an opposite vertex is $0.5\sqrt{3}r$.

**Solution:** Calculate the potential between $q_1$ and $q_2$, $V_1$:

$$
\begin{aligned}
V_1 &= \frac{kq_1}{0.5r} + \frac{kq_2}{0.5r} + \frac{kq_3}{0.5\sqrt{3}r} \\
&= \frac{k}{r}\left(2q_1 + 2q_2 + \frac{2}{\sqrt{3}}q_3\right) \\
&= \frac{8.99 \times 10^9 \ \frac{\text{J} \cdot \cancel{\text{m}}}{\text{C}^{\cancel{2}}}}{3.0\ \cancel{\text{m}}}\left(2\left(6.0 \times 10^{-6}\ \cancel{\text{C}}\right) + 2\left(-3.0 \times 10^{-6}\ \cancel{\text{C}}\right) + \frac{2}{\sqrt{3}}\left(-3.0 \times 10^{-6}\ \cancel{\text{C}}\right)\right)
\end{aligned}
$$

$V_1 = 7.6 \times 10^3$ J/C

Calculate the potential between $q_1$ and $q_3$, $V_2$:

$$
\begin{aligned}
V_2 &= \frac{kq_1}{0.5r} + \frac{kq_2}{0.5\sqrt{3}r} + \frac{kq_3}{0.5r} \\
&= \frac{k}{r}\left(2q_1 + \frac{2}{\sqrt{3}}q_2 + 2q_2\right) \\
&= \frac{8.99 \times 10^9 \ \frac{\text{J} \cdot \cancel{\text{m}}}{\text{C}^{\cancel{2}}}}{3.0\ \cancel{\text{m}}}\left(2\left(6.0 \times 10^{-6}\ \cancel{\text{C}}\right) + \frac{2}{\sqrt{3}}\left(-3.0 \times 10^{-6}\ \cancel{\text{C}}\right) + 2\left(-3.0 \times 10^{-6}\ \cancel{\text{C}}\right)\right)
\end{aligned}
$$

$V_2 = 7.6 \times 10^3$ J/C

Calculate the potential between $q_2$ and $q_3$, $V_3$:

$$
\begin{aligned}
V_3 &= \frac{kq_1}{0.5\sqrt{3}r} + \frac{kq_2}{0.5r} + \frac{kq_3}{0.5r} \\
&= \frac{k}{r}\left(\frac{2}{\sqrt{3}}q_1 + 2q_2 + 2q_3\right) \\
&= \frac{8.99 \times 10^9 \ \frac{\text{J} \cdot \cancel{\text{m}}}{\text{C}^{\cancel{2}}}}{3.0\ \cancel{\text{m}}}\left(\frac{2}{\sqrt{3}}\left(6.0 \times 10^{-6}\ \cancel{\text{C}}\right) + 2\left(-3.0 \times 10^{-6}\ \cancel{\text{C}}\right) + 2\left(-3.0 \times 10^{-6}\ \cancel{\text{C}}\right)\right)
\end{aligned}
$$

$V_3 = -1.5 \times 10^4$ J/C

 

**Statement:** The potential between $q_1$ and $q_2$, $V_1$, is $7.6 \times 10^3$ J/C. The potential between $q_1$ and $q_3$, $V_2$, is $7.6 \times 10^3$ J/C. The potential between $q_2$ and $q_3$, $V_3$, is $-1.5 \times 10^4$ J/C.

**(b) Given:** $q_1 = +6.0 \times 10^{-6}$ C; $q_2 = -3.0 \times 10^{-6}$ C; $q_3 = -3.0 \times 10^{-6}$ C; $r = 3.0$ m; $k = 8.99 \times 10^9$ N·m²/C²

**Required:** $E_E$

**Analysis:** The total electric potential energy is the sum of the three electric potential energies of a pair of charges. Use $E_E = \dfrac{kq_1q_2}{r}$ to calculate the electric potential energy for each pair of charges.

**Solution:** Calculate the electric potential energy between $q_1$ and $q_2$, $E_{E1}$:

$$E_{E1} = \frac{kq_1q_2}{r}$$

$$= \frac{\left(8.99\times10^9\ \dfrac{\text{J}\cdot\cancel{\text{m}}}{\cancel{\text{C}^2}}\right)(6.0\times10^{-6}\ \cancel{\text{C}})(-3.0\times10^{-6}\ \cancel{\text{C}})}{3.0\ \cancel{\text{m}}}$$

$E_{E1} = -5.394\times10^{-2}$ J (two extra digits carried)

The electric potential energy between $q_1$ and $q_3$, $E_{E2}$, is equal to $E_{E1}$ because $q_2 = q_3$.

Calculate the electric potential energy between $q_2$ and $q_3$, $E_{E3}$:

$$E_{E3} = \frac{kq_2q_3}{r}$$

$$= \frac{\left(8.99\times10^9\ \dfrac{\text{J}\cdot\cancel{\text{m}}}{\cancel{\text{C}^2}}\right)(-3.0\times10^{-6}\ \cancel{\text{C}})(-3.0\times10^{-6}\ \cancel{\text{C}})}{3.0\ \cancel{\text{m}}}$$

$E_{E3} = 2.697\times10^{-2}$ J (two extra digits carried)

Calculate the total electric potential energy, $E_E$:

$$E_E = E_{E1} + E_{E2} + E_{E3}$$

$$= (-5.394\times10^{-2}\ \text{J}) + (-5.394\times10^{-2}\ \text{J}) + (2.697\times10^{-2}\ \text{J})$$

$$E_E = -8.1\times10^{-2}\ \text{J}$$

**Statement:** The total electric potential energy of the group of charges is $-8.1 \times 10^{-2}$ J.

**2. Given:** $q = 4.5 \times 10^{-6}$ C; $s = 1.5$ m; $k = 8.99 \times 10^9$ N·m²/C²

**Required:** $V$ at the centre of the square

**Analysis:** The potential at the centre is the sum of the potentials due to the four charges. Use $V = \dfrac{kq}{r}$ to calculate the potential due to each charge. The distance from a vertex to the centre is $\dfrac{1}{2}\sqrt{2}s$, or $\dfrac{s}{\sqrt{2}}$.

**Solution:** $V = 4\frac{kq}{r}$

$$= \frac{4kq}{\frac{s}{\sqrt{2}}}$$

$$= \frac{4\sqrt{2}\left(8.99\times10^{9}\ \frac{\text{J}\cdot\cancel{\text{m}}}{\text{C}^{\cancel{2}}}\right)(4.5\times10^{-6}\ \cancel{\text{C}})}{1.5\ \cancel{\text{m}}}$$

$$V = 1.5\times10^{5}\ \text{J/C}$$

**Statement:** The electric potential at the centre of the square is $1.5 \times 10^5$ J/C.

**3. Given:** $q = -1.6 \times 10^{-19}$ C; $r_i = 5.0 \times 10^{-12}$ m; $m = 9.11 \times 10^{-31}$ kg; $k = 8.99 \times 10^9$ N·m²/C²

**Required:** $v_f$

**Analysis:** Determine the initial electric potential energy using the equation $E_{Ei} = \frac{kq_1q_2}{r_i}$. The final kinetic energy of each electron is $\frac{1}{2}mv_f^2$. The initial kinetic energy and final potential energy are both 0. Use the conservation of energy to determine the final speed of each electron.

$$E_{Ei} + E_{ki} = E_{Ef} + E_{kf}$$

$$\frac{kq_1q_2}{r_i} + 0 = 0 + \frac{1}{2}mv_f^2 + \frac{1}{2}mv_f^2$$

$$\frac{kq^2}{r_i} = mv_f^2$$

$$v_f = \sqrt{\frac{kq^2}{mr_i}}$$

**Solution:** $v_f = \sqrt{\frac{kq^2}{mr_i}}$

$$= \sqrt{\frac{\left(8.99\times10^{9}\ \frac{\cancel{\text{kg}}\cdot\frac{\text{m}}{\text{s}^2}\cdot\text{m}^{\cancel{2}}}{\cancel{\text{C}^2}}\right)\left(-1.6\times10^{-19}\ \cancel{\text{C}}\right)^2}{\left(9.11\times10^{-31}\ \cancel{\text{kg}}\right)\left(5.0\times10^{-12}\ \cancel{\text{m}}\right)}}$$

$$v_f = 7.1\times10^{6}\ \text{m/s}$$

**Statement:** The final speed of each electron is $7.1 \times 10^6$ m/s.

**4. Given:** $q = 1.6 \times 10^{-19}$ C; $v_1 = 2.3 \times 10^6$ m/s; $v_2 = 1.2 \times 10^6$ m/s; $m = 1.673 \times 10^{-27}$ kg; $k = 8.99 \times 10^9$ N·m²/C²

**Required:** $r_f$

**Analysis:** Calculate the final electric potential energy using the equation $E_{Ei} = \dfrac{kq_1q_2}{r_f}$. The initial kinetic energy of each proton is $\dfrac{1}{2}mv^2$, and the initial potential energy and final kinetic energy are both 0. Use the conservation of energy to determine the final separation of the protons.

$$E_{Ei} + E_{ki} = E_{Ef} + E_{kf}$$

$$0 + \frac{1}{2}m_1v_{i1}^2 + \frac{1}{2}m_2v_{i2}^2 = \frac{kq_1q_2}{r_f} + 0$$

$$mv_{i1}^2 + mv_{i2}^2 = \frac{2kq^2}{r_f}$$

$$r_f = \frac{2kq^2}{mv_{i1}^2 + mv_{i2}^2}$$

**Solution:**

$$r_f = \frac{2kq^2}{mv_{i1}^2 + mv_{i2}^2}$$

$$= \frac{2\left(8.99\times10^{9}\ \frac{\cancel{\text{kg}}\cdot\frac{\text{m}}{\cancel{\text{s}^2}}\cdot\cancel{\text{m}^2}}{\cancel{\text{C}^2}}\right)\left(1.6\times10^{-19}\ \cancel{\text{C}}\right)^2}{\left(1.673\times10^{-27}\ \cancel{\text{kg}}\right)\left(2.3\times10^{6}\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)^2 + \left(1.673\times10^{-27}\ \cancel{\text{kg}}\right)\left(1.2\times10^{6}\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)^2}$$

$$r_f = 4.1\times10^{-14}\ \text{m}$$

**Statement:** The separation of the protons when they are closest to each other is $4.1 \times 10^{-14}$ m.

## Section 7.5 Questions, page 361

**1. (a)** The proton moves to a region of lower potential energy and lower electric potential.
**(b)** The electron moves to a region of lower potential energy and higher electric potential.
**2.** Two particles that are at locations where the electric potential is the same do not necessarily have the same electric potential energy. The potential energy equals the product of the charge and the electric potential. If the particles have different charges, then they have different potential energies.
**3.** No work, or 0 J, is required to move a charge from one spot to another with the same electric potential. The work done equals the change in kinetic energy, and the change in kinetic energy equals the negative change in potential energy if energy is conserved. If the electric potential does not change, then the electric potential energy does not change. Therefore, the kinetic energy does not change, and no work is done.

**4. Given:** $q_1 = 4.5 \times 10^{-5}$ C; $q_2 = 8.5 \times 10^{-5}$ C; $E_E = 40.0$ J; $k = 8.99 \times 10^9$ N·m²/C²
**Required:** $r$

**Analysis:** $E_E = \frac{kq_1q_2}{r}$

$$r = \frac{kq_1q_2}{E_E}$$

**Solution:** $r = \frac{kq_1q_2}{E_E}$

$$= \frac{\left(8.99 \times 10^9 \ \frac{\cancel{N}}{\cancel{C^2}}\right)\left(4.5 \times 10^{-5} \ \cancel{C}\right)\left(8.5 \times 10^{-5} \ \cancel{C}\right)}{40.0 \ \cancel{N} \cdot m}$$

$$= 0.86 \text{ m}$$

$$r = 86 \text{ cm}$$

**Statement:** The distance between the charges is 86 cm.

**5. Given:** $q_1 = 4.5 \times 10^{-5}$ C; $q_2 = 8.5 \times 10^{-5}$ C; $r_i = 2.5$ m; $r_f = 1.5$ m; $k = 8.99 \times 10^9$ N·m²/C²
**Required:** $\Delta E_E$
**Analysis:**

$$\Delta E_E = \frac{kq_1q_2}{r_f} - \frac{kq_1q_2}{r_i}$$

$$\Delta E_E = kq_1q_2\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

**Solution:** $\Delta E_E = kq_1q_2\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$

$$= \left(8.99 \times 10^9 \ \frac{J \cdot \cancel{m}}{\cancel{C^2}}\right)\left(4.5 \times 10^{-5} \ \cancel{C}\right)\left(8.5 \times 10^{-5} \ \cancel{C}\right)\left(\frac{1}{1.5 \ \cancel{m}} - \frac{1}{2.5 \ \cancel{m}}\right)$$

$$\Delta E_E = 9.2 \text{ J}$$

**Statement:** The electric potential energy increases by +9.2 J.

**6. Given:** $q_1 = 3.5 \times 10^{-6}$ C; $q_2 = 7.5 \times 10^{-6}$ C; $r_i \to \infty$; $r_f = 2.5$ m; $k = 8.99 \times 10^9$ N·m²/C²
**Required:** $W$
**Analysis:**

$$W = \frac{kq_1q_2}{r_f} - \frac{kq_1q_2}{r_i}$$

$$= \frac{kq_1q_2}{r_f} - 0$$

$$W = \frac{kq_1q_2}{r_f}$$

**Solution:** $W = \frac{kq_1q_2}{r_f}$

$$= \frac{\left(8.99\times10^9\ \frac{\text{J}\cdot\cancel{\text{m}}}{\cancel{\text{C}^2}}\right)\left(3.5\times10^{-6}\ \cancel{\text{C}}\right)\left(7.5\times10^{-6}\ \cancel{\text{C}}\right)}{2.5\ \cancel{\text{m}}}$$

$W = 9.4\times10^{-2}$ J

**Statement:** The work required to bring the point charges together is $9.4 \times 10^{-2}$ J.

**7. Given:** $q_1 = -1.6 \times 10^{-19}$ C; $q_2 = 1.6 \times 10^{-19}$ C; $r_i = 5.00 \times 10^{-11}$ m; $r_f \to \infty$; $k = 8.99 \times 10^9$ N·m²/C²

**Required:** $W$

**Analysis:**

$$W = \frac{kq_1q_2}{r_f} - \frac{kq_1q_2}{r_i}$$

$$= 0 - \frac{kq_1q_2}{r_i}$$

$$W = -\frac{kq_1q_2}{r_i}$$

**Solution:** $W = -\frac{kq_1q_2}{r_i}$

$$= -\frac{\left(8.99\times10^9\ \frac{\text{J}\cdot\cancel{\text{m}}}{\cancel{\text{C}^2}}\right)\left(-1.6\times10^{-19}\ \cancel{\text{C}}\right)\left(1.6\times10^{-19}\ \cancel{\text{C}}\right)}{5.00\times10^{-11}\ \cancel{\text{m}}}$$

$W = 4.6\times10^{-18}$ J

**Statement:** The work required to separate the electron and the proton is $4.6 \times 10^{-18}$ J.

**8. (a) Given:** $r_f = 15$ cm = 0.15 m; $V = -8.5 \times 10^4$ V; $k = 8.99 \times 10^9$ N·m²/C²

**Required:** $q$

**Analysis:** $V = \frac{kq}{r}$; $q = \frac{Vr}{k}$

**Solution:** $q = \frac{Vr}{k}$

$$= \frac{\left(-8.5\times10^4\ \frac{\cancel{\text{J}}}{\cancel{\text{C}}}\right)\left(0.15\ \cancel{\text{m}}\right)}{\left(8.99\times10^9\ \frac{\cancel{\text{J}}\cdot\cancel{\text{m}}}{\text{C}^{\cancel{2}}}\right)}$$

$q = -1.4\times10^{-6}$ C

**Statement:** The charge on the sphere is $-1.4 \times 10^{-6}$ C.

**(b) Given:** $r_f = 0.15$ m; $V = -8.5 \times 10^4$ V
**Required:** $\varepsilon$
**Analysis:**

$$\varepsilon = \frac{kq}{r^2}$$

$$\varepsilon = \frac{V}{r}$$

**Solution:** $\varepsilon = \frac{V}{r}$

$$= \frac{\left(-8.5 \times 10^4 \ \frac{\text{N} \cdot \cancel{\text{m}}}{\text{C}}\right)}{\left(0.15 \ \cancel{\text{m}}\right)}$$

$$\varepsilon = -5.7 \times 10^5 \text{ N/C}$$

**Statement:** The magnitude of the electric field near the surface of the sphere is $-5.7 \times 10^5$ N/C.
**(c)** By convention, the electric field points radially inward from the surface of a negatively charged sphere.