Section 7.2: Coulomb's Law

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1. Given: $q_1 = 1.00 \times 10^{-4}$ C; $q_2 = 1.00 \times 10^{-5}$ C; $r = 2.00$ m; $k = 8.99 \times 10^9$ N·m²/C²
Required: F_E

Analysis: $F_E = \dfrac{kq_1q_2}{r^2}$

Solution: $F_E = \dfrac{kq_1q_2}{r^2}$

$$= \frac{\left(8.99 \times 10^9 \ \frac{\text{N} \cdot \cancel{\text{m}^2}}{\cancel{\text{C}^2}}\right)(1.00 \times 10^{-4}\ \cancel{\text{C}})(1.00 \times 10^{-5}\ \cancel{\text{C}})}{(2.00\ \cancel{\text{m}})^2}$$

$F_E = 2.25$ N

Statement: The magnitude of the electric force between the two charges is 2.25 N.

2. Given: $q_1 = q$; $q_2 = -2q$; $r_{12} = 1.000$ m; $F_{E13} + F_{E23} = 0$; $k = 8.99 \times 10^9$ N·m²/C²
Required: r_{13}

Analysis: Use $F_E = \dfrac{kq_1q_2}{r^2}$ to develop a quadratic equation to solve for r_{13}.

Solution:

$$F_{E13} + F_{E23} = 0$$

$$F_{E13} = -F_{E23}$$

$$\frac{\cancel{k} q_1 \cancel{q_3}}{r_{13}^2} = -\frac{\cancel{k} q_2 \cancel{q_3}}{r_{23}^2}$$

$$\frac{q_1}{r_{13}^2} = -\frac{q_2}{r_{23}^2}$$

$$\frac{q}{r_{13}^2} = -\frac{-2q}{\left(1.000 + r_{13}\right)^2}$$

$$\left(1 + r_{13}\right)^2 = 2r_{13}^2$$

$$1 + 2r_{13} + r_{13}^2 = 2r_{13}^2$$

$$0 = r_{13}^2 - 2r_{13} - 1$$

Solve the quadratic equation:

$$0 = r_{13}^2 - 2r_{13} - 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r_{13} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$r_{13} = \frac{2 \pm \sqrt{4+4}}{2}$$

$$r_{13} = \frac{2 \pm 2\sqrt{2}}{2}$$

$$r_{13} = 1 \pm \sqrt{2}$$

Only the positive distance is necessary:

$$r_{13} = 1 + \sqrt{2} \text{ m}$$

$$r_{13} = 2.414 \text{ m}$$

Statement: The third charge is 2.414 m to the left of q.

3. Given: q_1 = +2.0 μC = +2.0 × 10^{-6} C; d_1 = 0 m; q_2 = –3.0 μC = –3.0 × 10^{-6} C; d_2 = 40.0 cm = 0.40 m; q_3 = –5.0 μC = –5.0 × 10^{-6} C; d_3 = 120.0 cm = 1.20 m; $k = 8.99 \times 10^9$ N·m^2/C^2

Required: F_{Enet} at q_2

Analysis: $F_{\text{E}} = \dfrac{kq_1q_2}{r^2}$

Solution:

Determine the electric force due to q_1:

$$F_{\text{E12}} = \frac{kq_1q_2}{r^2}$$

$$= \frac{\left(8.99 \times 10^9 \ \frac{\text{N} \cdot \cancel{\text{m}^2}}{\cancel{\text{C}^2}}\right)(2.0 \times 10^{-6} \ \cancel{\text{C}})(-3.0 \times 10^{-6} \ \cancel{\text{C}})}{(0.40 \ \cancel{\text{m}})^2}$$

$$F_{\text{E12}} = -0.3371 \text{ N} \quad \text{(two extra digits carried)}$$

Determine the electric force due to q_3:

$$F_{E23} = \frac{kq_3q_2}{r^2}$$

$$= \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{C}^2}}\right)(-5.0\times10^{-6}\ \cancel{\text{C}})(-3.0\times10^{-6}\ \cancel{\text{C}})}{(1.20\ \cancel{\text{m}} - 0.40\ \cancel{\text{m}})^2}$$

$F_{E23} = 0.2107\text{ N}$ (two extra digits carried)

Determine the net force:

$$\vec{F}_{\text{Enet}} = \vec{F}_{E12} + \vec{F}_{E23}$$

$$= 0.3371\text{ N [left]} + 0.2107\text{ N [left]}$$

$$\vec{F}_{\text{Enet}} = 0.55\text{ N [left]}$$

Statement: The force on the –3.0 μC charge is 0.55 N [left].

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1. Given: $F_{E1} = 0.080$ N; $r_2 = 3r_1$

Required: F_{E2}

Analysis: Determine how the force changes when the distance is tripled, then substitute for the original force, $F_E = \frac{kq_1q_2}{r^2}$.

Solution:

$$F_{E2} = \frac{kq_1q_2}{r_2^2}$$

$$= \frac{kq_1q_2}{(3r_1)^2}$$

$$= \frac{kq_1q_2}{9r_1^2}$$

$$= \frac{1}{9}\left(\frac{kq_1q_2}{r_1^2}\right)$$

$$= \frac{1}{9}F_{E1}$$

$$= \frac{1}{9}(0.080\text{ N})$$

$$F_{E2} = 8.9\times10^{-3}\text{ N}$$

Statement: The new force is 8.9×10^{-3} N.

2. Given: $F_{E1} = 0.080$ N; $r_2 = 3r_1$; $q_{1B} = 3q_{1A}$
Required: F_{E2}
Analysis: Determine how the force changes when the distance and the charge are tripled, then substitute for the original force; $F_E = \frac{kq_1q_2}{r^2}$.

Solution:
$$
\begin{aligned}
F_{E2} &= \frac{kq_{1B}q_2}{r_2^2} \\
&= \frac{k(3q_{1A})q_2}{(3r_1)^2} \\
&= \frac{3kq_{1A}q_2}{9r_1^2} \\
&= \frac{1}{3}\left(\frac{kq_{1A}q_2}{r_1^2}\right) \\
&= \frac{1}{3}F_{E1} \\
&= \frac{1}{3}(0.080\ \text{N}) \\
F_{E2} &= 2.7\times 10^{-2}\ \text{N}
\end{aligned}
$$

Statement: The new force is 2.7×10^{-2} N.

3. Given: $q_1 = 1.6 \times 10^{-19}$ C; $q_2 = 1.6 \times 10^{-19}$ C; $r = 0.10$ nm $= 1.0 \times 10^{-10}$ m; $k = 8.99 \times 10^9\ \text{N·m}^2/\text{C}^2$
Required: F_E

Analysis: $F_E = \frac{kq_1q_2}{r^2}$

Solution:
$$
\begin{aligned}
F_E &= \frac{kq_1q_2}{r^2} \\
&= \frac{\left(8.99\times 10^9\ \frac{\text{N}\cdot \cancel{\text{m}^2}}{\cancel{\text{C}^2}}\right)(1.6\times 10^{-19}\ \cancel{\text{C}})(1.6\times 10^{-19}\ \cancel{\text{C}})}{(1.0\times 10^{-10}\ \cancel{\text{m}})^2} \\
F_E &= 2.3\times 10^{-8}\ \text{N}
\end{aligned}
$$

Statement: The magnitude of the electric force between the two electrons is 2.3×10^{-8} N.

4. Given: $r_2 = \frac{1}{1.50} r_1$
Required: F_{E2}

Analysis: $F_E = \frac{kq_1q_2}{r^2}$

Solution: $F_{E2} = \dfrac{kq_1q_2}{r_2^2}$

$$= \frac{kq_1q_2}{\left(\dfrac{r_1}{1.50}\right)^2}$$

$$F_{E2} = 2.25\left(\frac{kq_1q_2}{r_1^2}\right)$$

$$F_{E2} = 2.25F_{E1}$$

Statement: The magnitude of the electric force will increase by a factor of 2.25.

5. Given: q_1 = 1.00 μC = 1.00 × 10^{-6} C; q_2 = 1.00 μC = 1.00 × 10^{-6} C; m = 1.00 kg; g = 9.8 m/s^2; $F_E = F_g$; k = 8.99 × 10^9 N·m^2/C^2

Required: F_g; r

Analysis: Rearrange the equation $F_E = \dfrac{kq_1q_2}{r^2}$ to solve for r. Then determine F_g.

$$F_E = \frac{kq_1q_2}{r^2}$$

$$r = \sqrt{\frac{kq_1q_2}{F_E}}$$

$$F_g = mg$$

$$= (1.00\text{ kg})(9.8\text{ m/s}^2)$$

$$F_g = 9.8\text{ N}$$

Solution:

$$r = \sqrt{\frac{kq_1q_2}{F_E}}$$

$$= \sqrt{\frac{\left(8.99\times10^9\ \dfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(1.00\times10^{-6}\text{ C})(1.00\times10^{-6}\text{ C})}{9.8\text{ N}}}$$

$$r = 0.030\text{ m}$$

Statement: The distance between the charges is 0.030 m.

6. (a) Given: m_1 = 9.11 × 10^{-31} kg; m_2 = 1.67 × 10^{-27} kg; r = 1.0 nm = 1.0 × 10^{-9} m; G = 6.67 × 10^{-11} N·m^2/kg^2

Required: F_g

Analysis: $F_g = \dfrac{Gm_1m_2}{r^2}$

Solution: $F_g = \dfrac{Gm_1m_2}{r^2}$

$$= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(9.11\times10^{-31}\ \cancel{\text{kg}})(1.67\times10^{-27}\ \cancel{\text{kg}})}{(1.0\times10^{-9}\ \cancel{\text{m}})^2}$$

$F_g = 1.0\times10^{-49}$ N

Statement: The magnitude of the gravitational force between the electron and the proton is 1.0×10^{-49} N.

(b) Given: $q_1 = 1.6 \times 10^{-19}$ C; $q_2 = 1.6 \times 10^{-19}$ C; $r = 1.0 \times 10^{-9}$ m; $k = 8.99 \times 10^9$ N·m^2/C^2

Required: F_E

Analysis: $F_E = \dfrac{kq_1q_2}{r^2}$

Solution:

$F_E = \dfrac{kq_1q_2}{r^2}$

$$= \frac{\left(8.99\times10^{9}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{C}^2}}\right)(1.6\times10^{-19}\ \cancel{\text{C}})(1.6\times10^{-19}\ \cancel{\text{C}})}{(1.0\times10^{-9}\ \cancel{\text{m}})^2}$$

$F_E = 2.3\times10^{-10}$ N

Statement: The magnitude of the electric force between the electron and proton is 2.3×10^{-10} N.

(c) If the distance were increased to 1.0 m, there would be no change because the ratios of the forces are independent of the separation distance.

7. Given: $q_1 = q$; $q_2 = 3q$; $r_1 = 50$; $r_2 = -40$; $F_{E13} = F_{E23}$

Required: r_{13}

Analysis: Use $F_E = \dfrac{kq_1q_2}{r^2}$ to develop a quadratic equation to solve for r_{13}. First determine r_{12}:

$r_{12} = 50- (-40) = 90$

Solution:

$$F_{E13} = F_{E23}$$

$$\frac{\cancel{k}q_1\cancel{q_3}}{r_{13}^2} = \frac{\cancel{k}q_2\cancel{q_3}}{r_{23}^2}$$

$$\frac{q_1}{r_{13}^2} = \frac{q_2}{r_{23}^2}$$

$$\frac{\cancel{q}}{r_{13}^2} = \frac{3\cancel{q}}{(90-r_{13})^2}$$

$$\left(90-r_{13}\right)^2=3r_{13}^2$$
$$8100-180r_{13}+r_{13}^2=3r_{13}^2$$
$$0=2r_{13}^2+180r_{13}-8100$$
$$0=r_{13}^2+90r_{13}-4050$$

Solve the quadratic equation:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$0=r_{13}^2+90r_{13}-4050$$
$$r_{13}=\frac{-(90)\pm\sqrt{(90)^2-4(1)(-4050)}}{2(1)}$$
$$=\frac{-90\pm\sqrt{24300}}{2}$$
$$=\frac{-90\pm90\sqrt{3}}{2}$$
$$r_{13}=-45\pm45\sqrt{3}$$

Only the positive distance is necessary:

$$r_{13}=45-45\sqrt{3}$$
$$r_{13}=33$$
$x = –40 + 33$
$x = –7$

Statement: The third charge is at $x = –7$.

8. Given: $q_1 = 2.0 \times 10^{-6}$ C; $q_2 = –1.0 \times 10^{-6}$ C; r_{12} = 10 cm = 0.10 m; $F_{E13} + F_{E23} = 0$

Required: r_{13}

Analysis: Use $F_E=\frac{kq_1q_2}{r^2}$ to develop a quadratic equation to solve for r_{13}.

Solution: $F_{E13}+F_{E23}=0$

$$F_{E13}=-F_{E23}$$
$$\frac{\cancel{k}q_1\cancel{q_3}}{r_{13}^2}=-\frac{\cancel{k}q_2\cancel{q_3}}{r_{23}^2}$$
$$\frac{q_1}{r_{13}^2}=-\frac{q_2}{r_{23}^2}$$

$$\frac{2.0\times10^{-6}\text{ C}}{r_{13}^2} = -\frac{-1.0\times10^{-6}\text{ C}}{(0.10+r_{13})^2}$$

$$\frac{2}{r_{13}^2} = \frac{1}{(0.10+r_{13})^2}$$

$$2(0.10+r_{13})^2 = r_{13}^2$$

$$0.01+0.2r_{13}+2r_{13}^2 = r_{13}^2$$

$$r_{13}^2+0.2r_{13}-0.01=0$$

Solve the quadratic equation:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$r_{13}^2+0.2r_{13}-0.01=0$$

$$r_{13}=\frac{-(-0.2)\pm\sqrt{(-0.2)^2-4(1)(-0.01)}}{2(1)}$$

$$r_{13}=\frac{0.2\pm\sqrt{0.04+0.04}}{2}$$

$$r_{13}=\frac{0.2\pm0.2\sqrt{2}}{2}$$

$$r_{13}=0.1\pm\sqrt{2}$$

Only the positive distance is necessary:

$$r_{13}=0.1\text{ m}+0.1\sqrt{2}\text{ m}$$

$$r_{13}=0.24\text{ m}$$

Statement: The third charge is 0.24 m, or 24 cm, beyond the smaller charge.

9. (a) Given: $q = 7.5 \times 10^{-6}$ C; L = 25 cm = 0.25 m; $k = 8.99 \times 10^9$ N·m²/C²

Required: F_{Enet} at q_3

Analysis: Determine the distance between particles using the Pythagorean theorem. Then use $F_{\text{E}}=\frac{kq_1q_2}{r^2}$ to determine the magnitude of the electric force between two particles.

Solution: Determine the distance:

$$r=\sqrt{(0.25\text{ m})^2+(0.25\text{ m})^2}$$

$$=\sqrt{0.125\text{ m}^2}$$

$r = 0.3536$ m (two extra digits carried)

Determine the magnitude of the electric force:

$$F_E = \frac{kq_1q_2}{r^2}$$

$$= \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{C}^2}}\right)(7.5\times10^{-6}\ \cancel{\text{C}})(7.5\times10^{-6}\ \cancel{\text{C}})}{(0.3536\ \cancel{\text{m}})^2}$$

$F_E = 4.044$ N (two extra digits carried)

The x-components of the forces will add to zero, so calculate the y-components of the forces.

$$F_{\text{Enet}} = 2F_E \sin 45°$$
$$= 2(4.044\ \text{N})\sin 45°$$
$$F_{\text{Enet}} = 5.7\ \text{N [down]}$$

Statement: The net force on the charge at the bottom is 5.7 N [down].

(b) Given: $q = 7.5 \times 10^{-6}$ C; $L = 0.25$ m; $k = 8.99 \times 10^9$ N·m^2/C^2; $F_E = 4.044$ N

Required: F_{Enet} at q_2

Analysis: $F_E = \frac{kq_1q_2}{r^2}$

Solution: Determine the magnitude of the force between the two particles on the x-axis:

$$F_E = \frac{kq_1q_2}{r^2}$$

$$= \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{C}^2}}\right)(7.5\times10^{-6}\ \cancel{\text{C}})(7.5\times10^{-6}\ \cancel{\text{C}})}{(0.25\ \cancel{\text{m}} + 0.25\ \cancel{\text{m}})^2}$$

$= 2.023$ N (two extra digits carried)

$F_E = 2.0$ N

Determine the x- and y-components of the diagonal force:

$$F_{Ex} = F_E \sin 45°$$
$$= (4.044\ \text{N})\sin 45°$$
$$F_{Ex} = 2.860\ \text{N}$$

$$F_{Ey} = F_E \cos 45°$$
$$= (4.044\ \text{N})\cos 45°$$
$$F_{Ey} = 2.860\ \text{N}$$

Add the horizontal forces:

2.860 N + 2.023 N = 4.883 N [to the right]

The vertical force is 2.860 N [up].

Determine the magnitude of the net force:

$$F_{\text{Enet}} = \sqrt{(4.883\ \text{N})^2 + (2.860\ \text{N})^2}$$
$$= \sqrt{32.02\ \text{N}^2}$$

$= 5.659$ N (two extra digits carried)

$F_{\text{Enet}} = 5.7$ N

Determine the direction of the net force:

$$\tan\theta = \frac{2.860\text{ N}}{4.883\text{ N}}$$

$$\theta = \tan^{-1}\left(\frac{2.860\ \cancel{\text{N}}}{4.883\ \cancel{\text{N}}}\right)$$

$$\theta = 30°$$

Statement: The net force on the charge on the right is 5.7 N [E 30° N].

(c) Given: $q = 7.5 \times 10^{-6}$ C; $L = 0.25$ m; $q_e = 1.6 \times 10^{-19}$ C; $k = 8.99 \times 10^9$ N·m^2/C^2

Required: F_{Enet} at origin

Analysis: $F_E = \frac{kq_1q_2}{r^2}$; the two charges on the x-axis have a net force of zero, so the only force is an attractive force from the charge at the bottom.

Solution: $F_E = \frac{kq_1q_2}{r^2}$

$$= \frac{\left(8.99\times10^9\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{C}^2}}\right)(7.5\times10^{-6}\ \cancel{\text{C}})(1.6\times10^{-19}\ \cancel{\text{C}})}{(0.25\ \cancel{\text{m}})^2}$$

$$= 1.726\times10^{-13}\text{ N (two extra digits carried)}$$

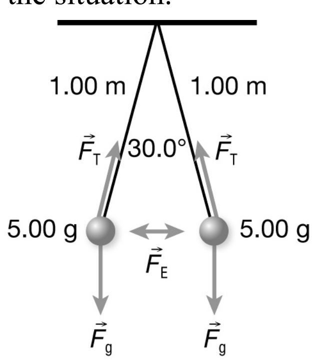
$$F_E = 1.7\times10^{-13}\text{ N}$$

Statement: The net force on the electron is 1.7×10^{-13} N [down].

10. Given: $m = 5.00$ g $= 5.00 \times 10^{-3}$ kg; $L = 1.00$ m; $\theta = 30.0°$; g = 9.8 m/s^2; $k = 8.99 \times 10^9$ N·m^2/C^2

Required: q

Analysis: The electric, gravitational, and tension forces on the pith balls give a net force of zero. Since the electric force is entirely horizontal and the gravitational force is entirely vertical, first determine the gravitational force, $F_g = mg$. Then use trigonometry to determine the tension force and the electric force. Use $F_E = \frac{kq_1q_2}{r^2}$ to determine the charge on each pith ball. Draw a sketch of the situation.



Solution: Determine the gravitational force on one pith ball:

$$F_g = mg$$
$$= (5.00 \times 10^{-3}\ \text{kg})(9.8\ \text{m/s}^2)$$
$$F_g = 0.490\ \text{N}$$

Determine the force of tension on one pith ball:

$$F_g = F_T \cos\left(\frac{30.0°}{2}\right)$$
$$F_T = \frac{F_g}{\cos 15.0°}$$
$$= \frac{0.490\ \text{N}}{\cos 15.0°}$$
$$F_T = 0.5073\ \text{N}$$

Determine the electric force on one pith ball:

$$F_E = F_T \sin\left(\frac{30.0°}{2}\right)$$
$$= (0.5073\ \text{N})\sin 15.0°$$
$$F_E = 1.313 \times 10^{-2}\ \text{N}$$

Determine the distance between pith balls:

$$r = 2L\sin\left(\frac{30.0°}{2}\right)$$
$$= 2(1.00\ \text{m})\sin 15.0°$$
$$r = 0.5176\ \text{m}$$

Determine the charge on the pith balls:

$$F_E = \frac{kq_1q_2}{r^2}$$
$$F_E = \frac{kq^2}{r^2}$$
$$q^2 = \frac{F_E r^2}{k}$$
$$q = \sqrt{\frac{F_E r^2}{k}}$$
$$= \sqrt{\frac{(1.313 \times 10^{-2}\ \cancel{\text{N}})(0.5176\ \cancel{\text{m}})^2}{\left(8.99 \times 10^9\ \frac{\cancel{\text{N}} \cdot \cancel{\text{m}^2}}{\text{C}^2}\right)}}$$
$$q = 6.26 \times 10^{-7}\ \text{C}$$

Statement: The charge on each pith ball is 6.26×10^{-7} C.