Section 5.5: Collisions in Two Dimensions: Glancing Collisions

Mini Investigation: Glancing Collisions, page 249

Answers may vary. Sample answers:

A. When one puck collides with a second puck at an angle, the speed of the second puck will be less than the initial speed of the first puck, and as the angle of the collision increases, the speed of the second puck will decrease.

B. There is less friction with pucks on an air table than with billiard balls or marbles, which will affect the results. The results for pucks on an air table may be closer to those for an ideal, frictionless system.

Tutorial 1 Practice, page 252

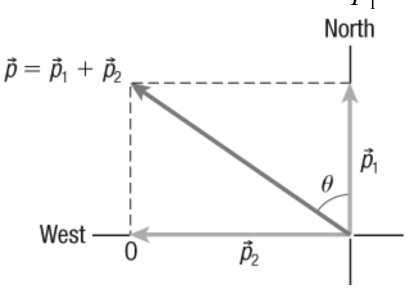
1. Given: Inelastic collision; $m_1 = 1.4 \times 10^4$ kg; $m_2 = 1.5 \times 10^4$ kg; $\vec{v}_{i_1} = 45$ km/h [N];

$\vec{v}_{i_2} = 53$ km/h [W]

Required: $\vec{v}_f$

Analysis: According to the law of conservation of momentum, $\vec{p}_{T_i} = \vec{p}_{T_f}$. Since the initial velocities are at right angles to each other, as shown in the figure below, you can calculate the total velocity and momentum using the Pythagorean theorem and trigonometry:

$p^2 = p_1^2 + p_2^2$, and $\tan\theta = \dfrac{p_2}{p_1}$



First, convert the velocities to metres per second.

$$\vec{v}_1 = 45\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3600\ \text{s}}$$

$\vec{v}_1 = 12.5$ m/s (one extra digit carried)

$$\vec{v}_2 = 53\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3600\ \text{s}}$$

$\vec{v}_2 = 14.72$ m/s (two extra digits carried)

Solution: Engine 1's momentum is

$$\begin{aligned}\vec{p}_1 &= m_1\vec{v}_1 \\ &= (1.4 \times 10^4\ \text{kg})(12.5\ \text{m/s})\ [\text{N}]\end{aligned}$$

$\vec{p}_1 = 1.75 \times 10^5\ \text{kg} \cdot \text{m/s}$ [N] (one extra digit carried)

Engine 2's momentum is

$$\vec{p}_2 = m_2\vec{v}_2$$
$$= (1.5\times10^4\ \text{kg})(14.7\ \text{m/s})\ [\text{W}]$$
$$\vec{p}_2 = 2.208\times10^5\ \text{kg}\cdot\text{m/s}\ [\text{W}]\ \text{(two extra digits carried)}$$

Calculate the magnitude of the total momentum by applying the Pythagorean theorem:

$$p^2 = p_1^2 + p_2^2$$
$$p = \sqrt{p_1^2 + p_2^2}$$
$$= \sqrt{(1.75\times10^5\ \text{kg}\cdot\text{m/s})^2 + (2.208\times10^5\ \text{kg}\cdot\text{m/s})^2}$$
$$p = 2.817\times10^5\ \text{kg}\cdot\text{m/s}\ \text{(two extra digits carried)}$$

Determine the direction by applying the tangent ratio:

$$\tan\theta = \frac{p_2}{p_1}$$
$$\theta = \tan^{-1}\left(\frac{p_2}{p_1}\right)$$
$$= \tan^{-1}\left(\frac{2.208\times10^5\ \cancel{\text{kg}\cdot\text{m/s}}}{1.75\times10^5\ \cancel{\text{kg}\cdot\text{m/s}}}\right)$$
$$\theta = 52^\circ$$

The direction of the two engines is [N 52° W].
By conservation of momentum, the final total momentum of the engines must equal the initial momentum. Since the collision is perfectly inelastic, both engines have the same final velocity:

$$\vec{p}_\text{f} = m_1\vec{v}_\text{f} + m_2\vec{v}_\text{f}$$
$$\vec{p}_\text{f} = (m_1 + m_2)\vec{v}_\text{f}$$
$$\vec{v}_\text{f} = \frac{\vec{p}_\text{f}}{m_1 + m_2}$$
$$= \frac{2.817\times10^5\ \cancel{\text{kg}}\cdot\text{m/s}\ [\text{N } 52^\circ\ \text{W}]}{(1.4\times10^4\ \cancel{\text{kg}} + 1.5\times10^4\ \cancel{\text{kg}})}$$
$$\vec{v}_\text{f} = 9.7\ \text{m/s}\ [\text{N } 52^\circ\ \text{W}]$$

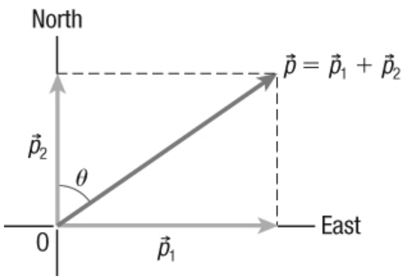
Statement: After the collision, the two engines are travelling together at a velocity of 9.7 m/s [N 52° W].

2. Given: $m_1 = 2\times10^{30}$ kg; $\vec{v}_1 = 2\times10^4$ m/s [E]; $m_2 = 5\times10^{30}$ kg;
$\vec{v}_2 = 3\times10^4$ m/s [at right angle to star 1]

Required: $\vec{v}_\text{f}$

Analysis: Assume that the direction of star 2 is north. According to the law of conservation of momentum, $\vec{p}_{T_i} = \vec{p}_{T_f}$. Since the initial velocities are at right angles to each other, as shown in the figure below, you can calculate the total velocity and momentum using the Pythagorean theorem and trigonometry:

$p^2 = p_1^2 + p_2^2$, and $\tan\theta = \dfrac{p_2}{p_1}$



Solution: Star 1's momentum is

$$\vec{p}_1 = m_1\vec{v}_1$$
$$= (2\times10^{30}\ \text{kg})(2\times10^{4}\ \text{m/s})\ [\text{E}]$$
$$\vec{p}_1 = 4\times10^{34}\ \text{kg}\cdot\text{m/s}\ [\text{E}]$$

Star 2's momentum is

$$\vec{p}_2 = m_2\vec{v}_2$$
$$= (5\times10^{30}\ \text{kg})(3\times10^{4}\ \text{m/s})\ [\text{N}]$$
$$\vec{p}_2 = 1.5\times10^{35}\ \text{kg}\cdot\text{m/s}\ [\text{N}]\ \text{(one extra digit carried)}$$

Calculate the magnitude of the total momentum by applying the Pythagorean theorem:

$$p^2 = p_1^2 + p_2^2$$
$$p = \sqrt{p_1^2 + p_2^2}$$
$$= \sqrt{(4\times10^{34}\ \text{kg}\cdot\text{m/s})^2 + (1.5\times10^{35}\ \text{kg}\cdot\text{m/s})^2}$$
$$p = 1.6\times10^{35}\ \text{kg}\cdot\text{m/s}\ \text{(one extra digit carried)}$$

Determine the direction by applying the tangent ratio:

$$\tan\theta = \frac{p_1}{p_2}$$
$$\theta = \tan^{-1}\left(\frac{p_1}{p_2}\right)$$
$$= \tan^{-1}\left(\frac{4\times10^{34}\ \cancel{\text{kg}\cdot\text{m/s}}}{1.5\times10^{35}\ \cancel{\text{kg}\cdot\text{m/s}}}\right)$$
$$\theta = 14.9^\circ\ \text{(two extra digits carried)}$$

The two stars' direction is [N 10° E].

By conservation of momentum, the final total momentum of the stars must equal the initial momentum. Since the collision is perfectly inelastic, both stars have the same final velocity:

$$\vec{p}_f = m_1\vec{v}_f + m_2\vec{v}_f$$

$$\vec{p}_f = (m_1 + m_2)\vec{v}_f$$

$$\begin{aligned}\vec{v}_f &= \frac{\vec{p}_f}{m_1 + m_2} \\ &= \frac{1.6\times10^{35}\ \text{kg}\cdot\text{m/s [N 10° E]}}{(2\times10^{30}\ \text{kg} + 5\times10^{30}\ \text{kg})}\end{aligned}$$

$$\vec{v}_f = 2\times10^4\ \text{m/s [N 10° E]}$$

Statement: After the collision, the two stars are travelling together at a velocity of 2×10^4 m/s, at 10º to the initial path of the second star.

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1. Given: $m_1 = m_2 = m$; $\vec{v}_{i_1} = 10.0$ m/s [right]; $\vec{v}_{i_2} = 0$ m/s; $\vec{v}_{f_1} = 4.7$ m/s; $\theta = 60.0°$

Required: $\vec{v}_{f_2}$

Analysis: Choose a coordinate system to identify directions: let positive x be to the right and negative x be to the left. Let positive y be up and negative y be down.
Apply conservation of momentum independently in the x-direction and the y-direction to determine the magnitude and direction of the final velocity of ball 2.
Solution: In the y-direction, the total momentum before and after the collision is zero:

$$p_{T_{iy}} = p_{T_{fy}} = 0$$

Therefore, after the collision:

$$mv_{f_{1y}} + mv_{f_{2y}} = 0$$

Divide both sides by m and substitute the vertical component of each velocity vector. The vertical component of the velocity vector for ball 1 is $v_{f_1}\sin\theta$. The vertical component of the velocity vector for ball 2 is $v_{f_2}\sin\phi$

$$v_{f_1}\sin\theta + v_{f_2}\sin\phi = 0$$

$$(-4.7\ \text{m/s})(\sin 60.0°) + v_{f_2}\sin\phi = 0$$

$$v_{f_2}\sin\phi = (4.7\ \text{m/s})(\sin 60.0°)$$

In the x-direction, the total momentum before the collision is equal to the total momentum after the collision. Only ball 1 has momentum in the x-direction before the collision, but both balls have momentum in the x-direction after the collision.

$$\cancel{m}v_{i_{1x}} = \cancel{m}v_{f_{1x}} + \cancel{m}v_{f_{2x}}$$

$$v_{i_{1x}} = v_{f_{1x}} + v_{f_{2x}}$$

The horizontal component of the velocity vector of ball 1 after the collision is v_{f_1} cos 60.0°. The horizontal component of the velocity vector of ball 2 after the collision is $v_{f_2}\cos\phi$. The horizontal component of the velocity vector of ball 1 before the collision is 10.0 m/s.

$$10.0\text{ m/s} = v_{f_1}\cos 60.0° + v_{f_2}\cos\phi$$

$$10.0\text{ m/s} = (4.7\text{ m/s})\left(\frac{1}{2}\right) + v_{f_2}\cos\phi$$

$$v_{f_2}\cos\phi = 7.65\text{ m/s}$$

$$v_{f_2} = \frac{7.65\text{ m/s}}{\cos\phi}$$

Substitute this result into the previous result for v_{f_2}:

$$v_{f_2}\sin\phi = (4.7\text{ m/s})(\sin 60.0°)$$

$$\frac{(7.65\text{ m/s})\sin\phi}{\cos\phi} = (4.7\text{ m/s})(\sin 60.0°)$$

$$\frac{\sin\phi}{\cos\phi} = \frac{(4.7\ \cancel{\text{m/s}})(\sin 60.0°)}{(7.65\ \cancel{\text{m/s}})}$$

$$\tan\phi = 0.532$$

$$\phi = 28°$$

Now, substitute to determine v_{f_2}:

$$v_{f_2} = \frac{7.65\text{ m/s}}{\cos\phi}$$

$$= \frac{7.65\text{ m/s}}{\cos 28°}$$

$$v_{f_2} = 8.7\text{ m/s}$$

Statement: The velocity of ball 2 after the collision is 8.7 m/s, 28° above the horizontal (above the initial path of ball 1).

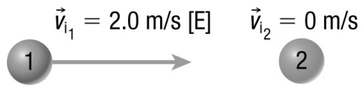
2. Given: $m_1 = 0.16$ kg; $m_2 = 0.17$ kg; $\vec{v}_{i_1} = 2.0\text{ m/s [E]}$; $\vec{v}_{i_2} = 0\text{ m/s}$;

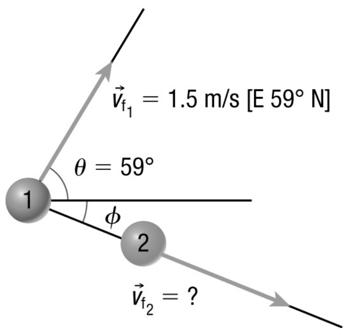
$\vec{v}_{f_1} = 1.5\text{ m/s [N 31° E]} = 1.5\text{ m/s [E 59° N]}$

Required: $\vec{v}_{f_2}$

Analysis: Choose a coordinate system to identify directions: let positive x be to the right and negative x be to the left. Let positive y be up and negative y be down.

Apply conservation of momentum independently in the x-direction and the y-direction to determine the magnitude and direction of the final velocity of puck 2.





Solution: In the y-direction, the total momentum before and after the collision is zero:

$$p_{T_{iy}} = p_{T_{fy}} = 0$$

Therefore, after the collision:

$$mv_{f_{1y}} + mv_{f_{2y}} = 0$$

Divide both sides by m and substitute the vertical component of each velocity vector. The vertical component of the velocity vector for puck 1 is $v_{f_1} \sin\theta$. The vertical component of the velocity vector for puck 2 is $v_{f_2} \sin\phi$.

$$v_{f_1} \sin\theta + v_{f_2} \sin\phi = 0$$

$$(1.5 \text{ m/s})(\sin 59°) + v_{f_2} \sin\phi = 0$$

$$v_{f_2} \sin\phi = (-1.5 \text{ m/s})(\sin 59°)$$

In the x-direction, the total momentum before the collision is equal to the total momentum after the collision. Only puck 1 has momentum in the x-direction before the collision, but both pucks have momentum in the x-direction after the collision.

$$\cancel{m}v_{i_{1x}} = \cancel{m}v_{f_{1x}} + \cancel{m}v_{f_{2x}}$$

$$v_{i_{1x}} = v_{f_{1x}} + v_{f_{2x}}$$

The horizontal component of the velocity vector of puck 1 after the collision is $v_{f_1} \cos 59°$. The horizontal component of the velocity vector of puck 2 after the collision is $v_{f_2} \cos\phi$. The horizontal component of the velocity vector of puck 1 before the collision is 2.0 m/s.

$$2.0 \text{ m/s} = v_{f_1} \cos 59° + v_{f_2} \cos\phi$$

$$2.0 \text{ m/s} = (1.5 \text{ m/s})(\cos 59°) + v_{f_2} \cos\phi$$

$$v_{f_2} \cos\phi = 1.23 \text{ m/s}$$

$$v_{f_2} = \frac{1.23 \text{ m/s}}{\cos\phi} \text{ (one extra digit carried)}$$

Substitute this result into the previous result for v_{f_2}:

$$v_{f2}\sin\phi = (-1.5 \text{ m/s})(\sin 59°)$$

$$\frac{(1.23 \text{ m/s})\sin\phi}{\cos\phi} = (-1.5 \text{ m/s})(\sin 59°)$$

$$\frac{\sin\phi}{\cos\phi} = \frac{(-1.5 \cancel{\text{m/s}})(\sin 59°)}{(1.23 \cancel{\text{m/s}})}$$

$$\tan\phi = -1.05$$

$$\phi = -46.4° \text{ (one extra digit carried)}$$

The direction of the final velocity of puck 2 is 46° south of east, or [S 44° E].
Now, substitute to determine v_{f_2}:

$$v_{f_2} = \frac{1.23 \text{ m/s}}{\cos\phi}$$

$$= \frac{1.23 \text{ m/s}}{\cos 46.4°}$$

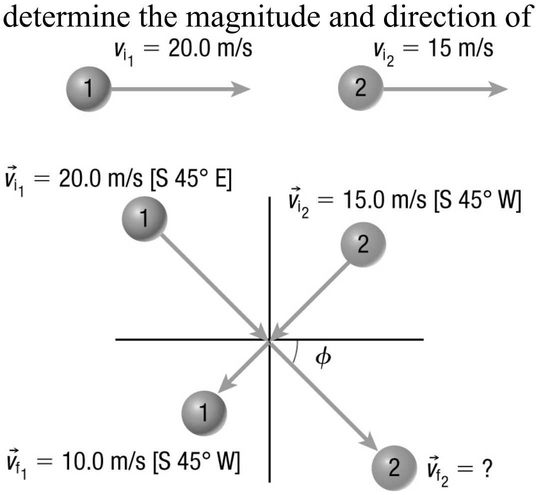
$$v_{f_2} = 1.8 \text{ m/s}$$

Statement: The velocity of puck 2 after the collision is 1.8 m/s [S 44° E].

3. (a) Given: $m_1 = m_2 = m$; $\vec{v}_{i_1} = 20.0$ m/s [S 45° E]; $\vec{v}_{i_2} = 15$ m/s [S 45° W];
$\vec{v}_{f_1} = 10.0$ m/s [S 45° W]

Required: $\vec{v}_{f_2}$

Analysis: Choose a coordinate system to identify directions: let positive x be to the right and negative x be to the left. Let positive y be up and negative y be down.
Apply conservation of momentum independently in the x-direction and the y-direction to determine the magnitude and direction of the final velocity of puck 2.



Solution: In the y-direction, the total momentum before and after the collision is equal:

$$p_{T_{iy}} = p_{T_{fy}}$$

Therefore, after the collision:

$$\not{m}v_{\mathrm{i}_{1y}} + \not{m}v_{\mathrm{i}_{2y}} = \not{m}v_{\mathrm{f}_{1y}} + \not{m}v_{\mathrm{f}_{2y}}$$

$$v_{\mathrm{i}_{1y}} + v_{\mathrm{i}_{2y}} = v_{\mathrm{f}_{1y}} + v_{\mathrm{f}_{2y}}$$

Substitute the vertical component of each velocity vector. The vertical component of the initial velocity vector for puck 1 is $-v_{\mathrm{i}_1}$ sin 45°. The vertical component of the initial velocity vector for puck 2 is $-v_{\mathrm{i}_2}$ sin 45°. The vertical component of the final velocity vector for puck 1 is $-v_{\mathrm{f}1}\sin 45^{\mathrm{o}}$. The vertical component of the final velocity vector for puck 2 is $-v_{\mathrm{f}_2}\sin\phi$.

$$-v_{\mathrm{i}_1}\sin 45° + (-v_{\mathrm{i}_2}\sin 45°) = -v_{\mathrm{f}_1}\sin 45° - v_{\mathrm{f}_2}\sin\phi$$

$$v_{\mathrm{f}_2}\sin\phi = \sin 45°(v_{\mathrm{i}_1} + v_{\mathrm{i}_2} - v_{\mathrm{f}_1})$$

$$v_{\mathrm{f}_2} = \frac{\sin 45°(v_{\mathrm{i}_1} + v_{\mathrm{i}_2} - v_{\mathrm{f}_1})}{\sin\phi}$$

$$= \frac{\sin 45°(20.0 \text{ m/s} + 15 \text{ m/s} - 10.0 \text{ m/s})}{\sin\phi}$$

$$v_{\mathrm{f}_2} = \frac{(25 \text{ m/s})\sin 45°}{\sin\phi}$$

In the x-direction, the total momentum before the collision is equal to the total momentum after the collision.

$$\not{m}v_{\mathrm{i}_{1x}} + \not{m}v_{\mathrm{i}_{2x}} = \not{m}v_{\mathrm{f}_{1x}} + \not{m}v_{\mathrm{f}_{2x}}$$

$$v_{\mathrm{i}_{1x}} + v_{\mathrm{i}_{2x}} = v_{\mathrm{f}_{1x}} + v_{\mathrm{f}_{2x}}$$

The horizontal component of the initial velocity vector of puck 1 is v_{i_1} cos 45°. The horizontal component of the initial velocity vector of puck 2 is $-v_{\mathrm{i}_2}$ cos 45°. The horizontal component of the final velocity vector of puck 1 is $-v_{\mathrm{f}_1}$ cos 45°. The horizontal component of the final velocity vector of puck 2 is v_{f_2} cos ϕ.

$$v_{\mathrm{i}_1}\cos 45° + (-v_{\mathrm{i}_2}\cos 45°) = -v_{\mathrm{f}_1}\cos 45° + v_{\mathrm{f}_2}\cos\phi$$

$$v_{\mathrm{f}_2} = \frac{(\cos 45°)(v_{\mathrm{i}_1} - v_{\mathrm{i}_2} + v_{\mathrm{f}_1})}{\cos\phi}$$

$$= \frac{(\cos 45°)(20.0 \text{ m/s} - 15 \text{ m/s} + 10.0 \text{ m/s})}{\cos\phi}$$

$$v_{\mathrm{f}_2} = \frac{(15 \text{ m/s})(\cos 45°)}{\cos\phi}$$

Substitute this result into the previous result for v_{f_2}:

$$\frac{(\cos 45°)(15 \text{ m/s})}{\cos\phi} = \frac{(25 \text{ m/s})(\sin 45°)}{\sin\phi}$$

$$\frac{\frac{\sqrt{2}}{2}(15 \text{ m/s})}{\cos\phi} = \frac{(25 \text{ m/s})\frac{\sqrt{2}}{2}}{\sin\phi}$$

$$\frac{\sin\phi}{\cos\phi} = \frac{25 \text{ m/s}}{15 \text{ m/s}}$$

$$\tan\phi = \frac{5}{3}$$

$$\phi = 59°$$

The direction of the final velocity of puck 2 is 59° south of east, or [S 31° E].
Now, substitute to determine v_{f_2}:

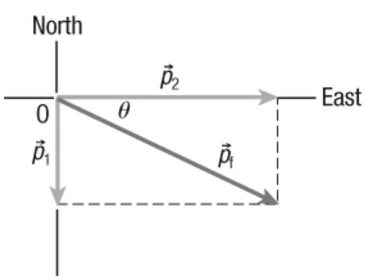
$$v_{f_2} = \frac{(15 \text{ m/s})(\cos 45°)}{\cos 59°}$$

$$v_{f_2} = 21 \text{ m/s}$$

Statement: The velocity of puck 2 after the collision is 21 m/s [S 31° E].
(b) The collision is non-perfectly inelastic, because kinetic energy is not conserved, but the pucks do not move together after the collision.
4. Given: $m_1 = 1.4 \times 10^3$ kg; $m_2 = 2.6 \times 10^4$ kg; $\vec{v}_{i_1} = 32$ km/h [S]; $\vec{v}_{i_2} = 48$ km/h [E]

Required: $\vec{v}_f$

Analysis: The vehicles have the same velocity after the collision. Momentum is conserved in the inelastic collision. Use the Pythagorean theorem and trigonometry to determine $\vec{p}_f$. Then, use $\vec{p}_f = m\vec{v}_f$ to determine $\vec{v}_f$.



First, convert the speeds to metres per second.

$$v_1 = 32 \frac{\text{km}}{\text{h}} \times \frac{1000 \text{ m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}}$$

$v_1 = 8.889$ m/s (two extra digits carried)

$$v_2 = 48\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3600\ \text{s}}$$

$v_2 = 13.3$ m/s (one extra digit carried)

Solution: $$p_\text{f} = \sqrt{p_1^2 + p_2^2}$$
$$= \sqrt{(m_1 v_1)^2 + (m_2 v_2)^2}$$
$$= \sqrt{[(1.4\times10^3\ \text{kg})(8.89\ \text{m/s})]^2 + [(2.6\times10^4\ \text{kg})(13.33\ \text{m/s})]^2}$$
$$p_\text{f} = 346\,000\ \text{kg}\cdot\text{m/s}\ \text{(two extra digits carried)}$$

$$p_\text{f} = (m_1 + m_2)v_\text{f}$$

$$v_\text{f} = \frac{p_\text{f}}{m_1 + m_2}$$
$$= \frac{346\ 000\ \cancel{\text{kg}}\cdot\text{m/s}}{1.4\times10^3\ \cancel{\text{kg}} + 2.6\times10^4\ \cancel{\text{kg}}}$$
$$v_\text{f} = 13\ \text{m/s}$$

Convert the speed to kilometres per hour:

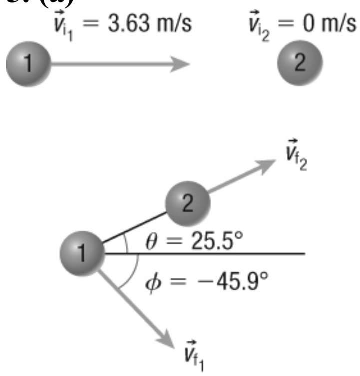
$$v_1 = 13\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \times \frac{1\ \text{km}}{1000\ \cancel{\text{m}}} \times \frac{3600\ \cancel{\text{s}}}{1\ \text{h}}$$
$$v_1 = 47\ \text{km/h}$$

$$\tan\theta = \frac{p_1}{p_2}$$
$$= \frac{m_1 v_1}{m_2 v_2}$$
$$= \frac{(1.4\times10^3\ \cancel{\text{kg}})(8.89\ \cancel{\text{m/s}})}{(2.6\times10^4\ \cancel{\text{kg}})(13.3\ \cancel{\text{m/s}})}$$
$$\tan\theta = 0.0359$$
$$\theta = 2.1°$$

The angle is 2.1° south of east, so it is 90° – 2.1° or 88° east of south.

Statement: The velocity of the vehicles after the collision is 47 km/h [S 88º E].

5. (a)



(b) Given: $\theta = 25.5°$; $\phi = -45.9°$; $\vec{v}_{i_1} = 3.63\text{ m/s}$; $\vec{v}_{i_2} = 0\text{ m/s}$

Required: v_{f_1}; v_{f_2}

Analysis: The momentum before and after the collision is equal. The momentum before the collision consists of the momentum of ball 1 only, which is in the *x*-direction. The momentum after the collision consists of the momentum of both balls, in both the *x*-direction and the *y*-direction. Consider the momentum in the *x*-direction and the *y*-direction separately. Use trigonometry to determine the components.

Solution: Consider momentum in the *x*-direction first.

$$p_{ix} = p_{fx}$$

$$\cancel{m}v_{i_{1x}} = \cancel{m}v_{f_{1x}} + \cancel{m}v_{f_{2x}}$$

$$v_{i_{1x}} = v_{f_{1x}} + v_{f_{2x}}$$

$$3.63\text{ m/s} = v_{f_1}\cos\phi + v_{f_2}\cos\theta$$

$$3.63\text{ m/s} = v_{f_1}\cos(-45.9°) + v_{f_2}\cos 25.5°$$

Now, consider momentum in the *y*-direction. There is no momentum in the *y*-direction before the collision.

$$p_{iy} = p_{fy}$$

$$0 = \cancel{m}v_{f_{1y}} + \cancel{m}v_{f_{2y}}$$

$$0 = v_{f_{1y}} + v_{f_{2y}}$$

$$0 = v_{f_1}\sin\phi + v_{f_2}\sin\theta$$

$$0 = v_{f_1}\sin(-45.9°) + v_{f_2}\sin 25.5°$$

$$v_{f_2} = \frac{-v_{f_1}\sin(-45.9°)}{\sin 25.5°}$$

$$v_{f_2} = \frac{v_{f_1}\sin 45.9°}{\sin 25.5°}$$

Substitute this expression into the other equation for v_{f_2} and v_{f_1}:

$$3.63 \text{ m/s} = v_{f_1} \cos(-45.9°) + v_{f_2} \cos 25.5°$$

$$3.63 \text{ m/s} = v_{f_1} \cos(-45.9°) + \left(\frac{v_{f_1} \sin 45.9°}{\sin 25.5°}\right)\cos 25.5°$$

$$v_{f_1} = 1.65 \text{ m/s}$$

Substitute again to determine v_{f_2}:

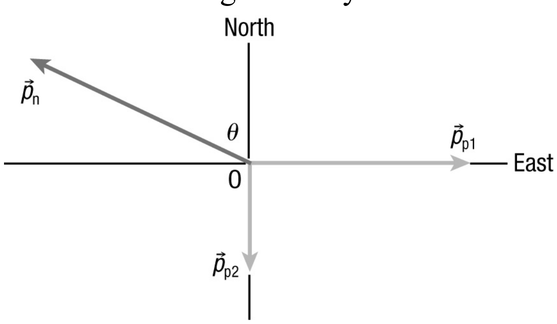
$$\begin{aligned} v_{f_2} &= \frac{v_{f_1} \sin 45.9°}{\sin 25.5°} \\ &= \frac{(1.65 \text{ m/s})\sin 45.9°}{\sin 25.5°} \\ v_{f_2} &= 2.75 \text{ m/s} \end{aligned}$$

Statement: The final speed of ball 1 is 1.65 m/s, and the final speed of ball 2 is 2.75 m/s.

6. (a) Given: $\vec{p}_{p_1} = 7.8 \times 10^{-21}$ kg · m/s [E]; $\vec{p}_{p_2} = 3.5 \times 10^{-21}$ kg · m/s [S]

Required: θ

Analysis: This is like a perfectly inelastic collision, with the particles as the objects that collide, and the nucleus as the objects sticking together, except that the nucleus fires off in the opposite direction. Use trigonometry to determine θ.



Solution: $\tan\theta = \dfrac{p_{p_1}}{p_{p_2}}$

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{p_{p_1}}{p_{p_2}}\right) \\ &= \tan^{-1}\left(\frac{7.8 \times 10^{-21} \cancel{\text{kg} \cdot \text{m/s}}}{3.5 \times 10^{-21} \cancel{\text{kg} \cdot \text{m/s}}}\right) \\ \theta &= 66° \end{aligned}$$

Statement: The direction of the nucleus is [W 24° N].

(b) Given: $\vec{p}_{p_1} = 7.8 \times 10^{-21}$ kg·m/s [E]; $\vec{p}_{p_2} = 3.5 \times 10^{-21}$ kg·m/s [S]

Analysis: Use the Pythagorean theorem to determine the final momentum of the nucleus.

Solution: $p_n^2 = p_{p_1}^2 + p_{p_2}^2$

$$p_n = \sqrt{p_{p_1}^2 + p_{p_2}^2}$$
$$= \sqrt{(7.8 \times 10^{-21} \text{ kg} \cdot \text{m/s})^2 + (3.5 \times 10^{-21} \text{ kg} \cdot \text{m/s})^2}$$
$$p_n = 8.5 \times 10^{-21} \text{ kg} \cdot \text{m/s}$$

Statement: The final momentum is 8.5×10^{-21} kg·m/s [W 24° N].

(c) Given: $p_n = 8.5 \times 10^{-21}$ kg·m/s; $m = 2.3 \times 10^{-26}$ kg

Analysis: $p = mv$

Solution: $p_n = mv_n$

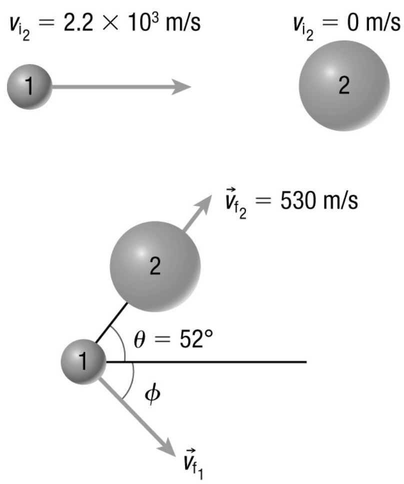
$$v_n = \frac{p_n}{m}$$
$$= \frac{8.5 \times 10^{-21} \cancel{\text{kg}} \cdot \text{m/s}}{2.3 \times 10^{-26} \cancel{\text{kg}}}$$
$$v_n = 3.7 \times 10^5 \text{ m/s}$$

Statement: The final velocity of the nucleus is 3.7×10^5 m/s [W 24° N].

7. Given: $m_1 = 1.7 \times 10^{-27}$ kg; $v_{i_1} = 2.2$ km/s $= 2.2 \times 10^3$ m/s; $m_2 = 6.6 \times 10^{-27}$ kg; $v_{i_2} = 0$ m/s; $v_{f_2} = 0.53$ km/s = 530 m/s; $\theta = 52°$

Required: $\vec{v}_{f_2}$

Analysis: The momentum before and after the collision is equal. The momentum before the collision consists of the momentum of ball 1 only, which is in the *x*-direction. The momentum after the collision consists of the momentum of both balls, in both the *x*-direction and the *y*-direction. Consider the momentum in the *x*-direction and the *y*-direction separately. Use trigonometry to determine the components.



Solution: Consider momentum in the x-direction first.

$$p_{\mathrm{i}_x} = p_{\mathrm{f}_x}$$

$$m_1 v_{\mathrm{i}_{1x}} = m_1 v_{\mathrm{f}_{1x}} + m_2 v_{\mathrm{f}_{2x}}$$

$$(1.7\times10^{-27}\ \text{kg})(2.2\times10^{3}\ \text{m/s}) = (1.7\times10^{-27}\ \text{kg})v_{\mathrm{f}_{1x}} + (6.6\times10^{-27}\ \text{kg})(530\ \text{m/s})(\cos 52°)$$

$$v_{\mathrm{f}_{1x}} = \frac{(1.7\times10^{-27}\ \cancel{\text{kg}})(2.2\times10^{3}\ \text{m/s}) - (6.6\times10^{-27}\ \cancel{\text{kg}})(530\ \text{m/s})(\cos 52°)}{(1.7\times10^{-27}\ \cancel{\text{kg}})}$$

$$v_{\mathrm{f}_{1x}} = 933.2\ \text{m/s (two extra digits carried)}$$

Now, consider momentum in the y-direction. There is no momentum in the y-direction before the collision.

$$p_{\mathrm{i}_y} = p_{\mathrm{f}_y}$$

$$0 = m_1 v_{\mathrm{f}_{1y}} + m_2 v_{\mathrm{f}_{2y}}$$

$$0 = m_1 v_{\mathrm{f}_{1y}} + m_2 v_{\mathrm{f}_2} \sin\theta$$

$$0 = (1.7\times10^{-27}\ \text{kg})v_{\mathrm{f}_{1y}} + (6.6\times10^{-27}\ \text{kg})(530\ \text{m/s})(\sin 52°)$$

$$v_{\mathrm{f}_{1y}} = \frac{-(6.6\times10^{-27}\ \cancel{\text{kg}})(530\ \text{m/s})(\sin 52°)}{1.7\times10^{-27}\ \cancel{\text{kg}}}$$

$$v_{\mathrm{f}_{1y}} = -1.621\times10^{3}\ \text{m/s (two extra digits carried)}$$

Use the Pythagorean theorem to determine the final speed of the neutron and trigonometry to determine its direction.

$$v_{\mathrm{f}_1}^2 = v_{\mathrm{f}_{1x}}^2 + v_{\mathrm{f}_{1y}}^2$$

$$\begin{aligned} v_{\mathrm{f}_1} &= \sqrt{v_{\mathrm{f}_{1x}}^2 + v_{\mathrm{f}_{1y}}^2} \\ &= \sqrt{(933.2\ \text{m/s})^2 + (1.621\times10^{3}\ \text{m/s})^2} \\ &= 1.9\times10^{3}\ \text{m/s} \end{aligned}$$

$$v_{\mathrm{f}_1} = 1.9\ \text{km/s}$$

$$\tan\theta = \frac{v_{\mathrm{f}_{1y}}}{v_{\mathrm{f}_{1x}}}$$

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{v_{\mathrm{f}_{1y}}}{v_{\mathrm{f}_{1x}}}\right) \\ &= \tan^{-1}\left(\frac{1.621\times10^{3}\ \cancel{\text{m}}/\text{s}}{933.2\ \cancel{\text{m}}/\text{s}}\right) \end{aligned}$$

$$\theta = 60°$$

Statement: The final velocity of the neutron is 1.9 km/s, 60° below its original direction.

8. The statement should be rewritten as: "For a head-on elastic collision between two objects of equal mass, the after-collision velocities of the objects are at an 180° angle to each other."

