Section 5.4: Collisions

Tutorial 1 Practice, page 243

1. Given: m_1 = 80.0 g = 0.0800 kg; $\vec{v}_{i_1} = 7.0\text{ m/s [W]}$; m_2 = 60.0 g = 0.0600 kg; $\vec{v}_{i_2} = 0$ m/s

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: Since the second ball is initially at rest in the head-on elastic collision, use the simplified equations, $\vec{v}_{f_1} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}$ and $\vec{v}_{f_2} = \left(\dfrac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$.

Solution:

$$\begin{aligned}\vec{v}_{f_1} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}\\ &= \left(\frac{0.0800\ \cancel{\text{kg}} - 0.0600\ \cancel{\text{kg}}}{0.0800\ \cancel{\text{kg}} + 0.0600\ \cancel{\text{kg}}}\right)(7.0\text{ m/s [W]})\\ \vec{v}_{f_1} &= 1.0\text{ m/s [W]}\end{aligned}$$

$$\begin{aligned}\vec{v}_{f_2} &= \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}\\ &= \left(\frac{2(0.0800\ \cancel{\text{kg}})}{0.0800\ \cancel{\text{kg}} + 0.0600\ \cancel{\text{kg}}}\right)(7.0\text{ m/s [W]})\\ \vec{v}_{f_1} &= 8.0\text{ m/s [W]}\end{aligned}$$

Statement: The final velocity of ball 1 is 1.0 m/s [W], and the final velocity of ball 2 is 8.0 m/s [W].

2. Given: m_1 = 1.5 kg; $\vec{v}_{i_1} = 36.5\text{ cm/s [E]} = 0.365\text{ m/s [E]}$; m_2 = 5 kg;
$\vec{v}_{i_2} = 42.8\text{ cm/s [W]} = -0.428\text{ m/s [E]}$

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: $\vec{v}_{f_1} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\dfrac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$; $\vec{v}_{f_2} = \left(\dfrac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\dfrac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$

Solution:

$$\begin{aligned}\vec{v}_{f_1} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}\\ &= \left(\frac{1.5\ \cancel{\text{kg}} - 5\ \cancel{\text{kg}}}{1.5\ \cancel{\text{kg}} + 5\ \cancel{\text{kg}}}\right)(0.365\text{ m/s [E]}) + \left(\frac{2(5\ \cancel{\text{kg}})}{1.5\ \cancel{\text{kg}} + 5\ \cancel{\text{kg}}}\right)(-0.428\text{ m/s [E]})\\ \vec{v}_{f_1} &= -0.9\text{ m/s}\end{aligned}$$

$$\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

$$= \left(\frac{5\ \cancel{kg} - 1.5\ \cancel{kg}}{1.5\ \cancel{kg} + 5\ \cancel{kg}}\right)(-0.428\text{ m/s [E]}) + \left(\frac{2(1.5\ \cancel{kg})}{1.5\ \cancel{kg} + 5\ \cancel{kg}}\right)(0.365\text{ m/s [E]})$$

$$\vec{v}_{f\,2} = -0.06\text{ m/s [E]}$$

Statement: The final velocity of cart 1 is 90 cm/s [W], and the final velocity of cart 2 is 6 cm/s [W].

Tutorial 2 Practice, page 247

1. (a) Given: m_1 = 1.2 kg; $\vec{v}_{i_1} = 3.0\text{ m/s [right]}$; m_2 = 1.2 kg; $\vec{v}_{i_2} = 3.0\text{ m/s [left]}$; $\vec{v}_{f_2} = 1.5\text{ m/s [right]}$

Required: x

Analysis: Use the conservation of momentum equation to determine the velocity of glider 1 during the collision, when glider 2 is moving at 1.5 m/s [right]. Rearrange this equation to express the final velocity of glider 1 in terms of the other given values.

$$m_1v_{i_1} + m_2v_{i_2} = m_1v_{f_1} + m_2v_{f_2}$$

$$v_{f_1} = \frac{m_1v_{i_1} + m_2v_{i_2} - m_2v_{f_2}}{m_1}$$

Then, apply conservation of mechanical energy to determine the compression of the spring at this particular moment during the collision. Consider right to be positive and left to be negative, and omit the vector notation. Clear fractions first, and then isolate x.

Solution: $$v_{f_1} = \frac{m_1v_{i_1} + m_2v_{i_2} - m_2v_{f_2}}{m_1}$$

$$= \frac{(\cancel{1.2\text{ kg}})(3.0\text{ m/s}) + (\cancel{1.2\text{ kg}})(-3.0\text{ m/s}) - (\cancel{1.2\text{ kg}})(1.5\text{ m/s})}{\cancel{1.2\text{ kg}}}$$

$$v_{f_1} = -1.5\text{ m/s}$$

$$\cancel{\frac{1}{2}}m_1v_{i_1}^2 + \cancel{\frac{1}{2}}m_2v_{i_2}^2 = \cancel{\frac{1}{2}}(m_1v_{f_1}^2 + m_2v_{f_2}^2) + \cancel{\frac{1}{2}}kx^2$$

$$m_1v_{i_1}^2 + m_2v_{i_2}^2 - (m_1v_{f_1}^2 + m_2v_{f_2}^2) = kx^2$$

$$x = \sqrt{\frac{m_1v_{i_1}^2 + m_2v_{i_2}^2 - (m_1v_{f_1}^2 + m_2v_{f_2}^2)}{k}}$$

$$= \sqrt{\frac{(1.2\text{ kg})(3.0\text{ m/s})^2 + (1.2\text{ kg})(-3.0\text{ m/s})^2 - (1.2\text{ kg})(-1.5\text{ m/s})^2 + (1.2\text{ kg})(1.5\text{ m/s})^2}{6.0\times10^4\text{ N/m}}}$$

$$x = 0.016\text{ m}$$

Statement: The compression of the spring when the second glider is moving at 1.5 m/s [right] is 1.6 cm.

(b) Given: m_1 = 1.2 kg; $\vec{v}_{i_1} = 3.0 \text{ m/s [right]}$; m_2 = 1.2 kg; $\vec{v}_{i_2} = 3.0 \text{ m/s [left]}$;
$\vec{v}_{f_2} = 1.5 \text{ m/s [right]}$

Required: x

Analysis: At the beginning of the collision, as the gliders come together and the spring is being compressed, glider 1 and glider 2 are moving at the same speed, in opposite directions. Immediately after the collision, the gliders will reverse direction, but still have the same speed. At the point of maximum compression of the spring, the two gliders will have the same velocity, $\vec{v}_f$. Use the conservation of momentum equation to determine this velocity.

$$m_1 v_{i_1} + m_2 v_{i_2} = (m_1 + m_2) v_f$$

$$v_{f_1} = \frac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2}$$

Then, apply the conservation of mechanical energy to calculate the maximum compression of the spring.

Solution:
$$\begin{aligned} v_f &= \frac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2} \\ &= \frac{(1.2 \cancel{\text{ kg}})(3.0 \text{ m/s}) + (1.2 \cancel{\text{ kg}})(-3.0 \text{ m/s})}{1.2 \cancel{\text{ kg}} + 1.2 \cancel{\text{ kg}}} \\ v_f &= 0 \text{ m/s} \end{aligned}$$

Now use the law of conservation of mechanical energy to determine the maximum compression of the spring, using the fact that v_f = 0 m/s.

$$\frac{\cancel{1}}{\cancel{2}} m_1 v_{i_1}^2 + \frac{\cancel{1}}{\cancel{2}} m_2 v_{i_2}^2 = \frac{\cancel{1}}{\cancel{2}} (m_1 + m_2) v_f^2 + \frac{\cancel{1}}{\cancel{2}} kx^2$$

$$\begin{aligned} m_1 v_{i_1}^2 + m_2 v_{i_2}^2 &= kx^2 \\ x &= \sqrt{\frac{m_1 v_{i_1}^2 + m_2 v_{i_2}^2}{k}} \\ &= \sqrt{\frac{(1.2 \text{ kg})(3.0 \text{ m/s})^2 + (1.2 \text{ kg})(-3.0 \text{ m/s})^2}{6.0 \times 10^4 \text{ N/m}}} \\ x &= 0.019 \text{ m} \end{aligned}$$

Statement: The maximum compression of the spring is 1.9 cm.

2. Given: $m_1 = 4.4 \times 10^2$ kg; $\vec{v}_{i_1} = 3.0 \text{ m/s [E]}$; $m_2 = 4.0 \times 10^2$ kg; $\vec{v}_{i_2} = 3.3 \text{ m/s [W]}$; $\Delta x = 44 \text{ cm} = 0.44 \text{ m}$

Required: k

Analysis: At the point of maximum compression of the spring, the two carts will have the same velocity, $\vec{v}_f$. Use the conservation of momentum equation to determine this velocity.

$$m_1 v_{i_1} + m_2 v_{i_2} = (m_1 + m_2) v_f$$

$$v_{f_1} = \frac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2}$$

Then apply the conservation of mechanical energy to calculate the spring constant.

Solution: $v_f = \dfrac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2}$

$$= \frac{(4.4 \times 10^2 \text{ kg})(3.0 \text{ m/s}) + (4.0 \times 10^2 \text{ kg})(-3.3 \text{ m/s})}{4.4 \times 10^2 \text{ kg} + 4.0 \times 10^2 \text{ kg}}$$

$$v_f = 0 \text{ m/s}$$

Now use the law of conservation of mechanical energy to determine the spring constant, using the fact that $v_f = 0$.

$$\frac{1}{2} m_1 v_{i_1}^2 + \frac{1}{2} m_2 v_{i_2}^2 = \frac{1}{2}(m_1 + m_2) v_f^2 + \frac{1}{2} k (\Delta x)^2$$

$$m_1 v_{i_1}^2 + m_2 v_{i_2}^2 = k(\Delta x)^2$$

$$k = \frac{m_1 v_{i_1}^2 + m_2 v_{i_2}^2}{(\Delta x)^2}$$

$$= \frac{(4.4 \times 10^2 \text{ kg})(3.0 \text{ m/s})^2 + (4.0 \times 10^2 \text{ kg})(-3.3 \text{ m/s})^2}{(0.44 \text{ m})^2}$$

$$k = 4.3 \times 10^4 \text{ N/m}$$

Statement: The spring constant is 4.3×10^4 N/m.

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1. Answers may vary. Sample answer: Elastic collision: No, it is not possible for two moving masses to undergo an elastic head-on collision and both be at rest immediately after the collision. In an elastic collision, kinetic energy is conserved. Therefore, if the objects were moving before the collision, at least one of the objects has to be moving after the collision.

Inelastic collision: Yes, it is possible for two moving masses to undergo an elastic head-on collision and both be at rest immediately after the collision. Kinetic energy is not conserved in an inelastic collision. If the masses have equal but opposite momentum before the collision, then the total momentum is zero. After the collision, they could both be at rest and still conserve momentum.

2. Answers may vary. Sample answer: The two curling stones have the same mass. In an elastic collision, the momentum of the first object can transfer completely to the other object if the objects have the same mass. Thus, the speed of the first object is zero, and the speed of the second object is equal to the initial speed of the first object.

3. Given: m_1 = 1.5 g = 0.0015 kg; m_2 = 3.5 g = 0.0035 kg; $\vec{v}_{i_1}$ = 12 m/s [right];

$\vec{v}_{i_2}$ = 7.5 m/s [left]

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: $\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$; $\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$

Solution: $\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$

$$= \left(\frac{0.0015\ \cancel{\text{kg}} - 0.0035\ \cancel{\text{kg}}}{0.0015\ \cancel{\text{kg}} + 0.0035\ \cancel{\text{kg}}}\right)(12\text{ m/s}) + \left(\frac{2(0.0035\ \cancel{\text{kg}})}{0.0015\ \cancel{\text{kg}} + 0.0035\ \cancel{\text{kg}}}\right)(-7.5\text{ m/s})$$

$\vec{v}_{f_1} = -15$ m/s

$$\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

$$= \left(\frac{0.0035\ \cancel{\text{kg}} - 0.0015\ \cancel{\text{kg}}}{0.0015\ \cancel{\text{kg}} + 0.0035\ \cancel{\text{kg}}}\right)(-7.5\text{ m/s}) + \left(\frac{2(0.0015\ \cancel{\text{kg}})}{0.0015\ \cancel{\text{kg}} + 0.0035\ \cancel{\text{kg}}}\right)(12\text{ m/s})$$

$\vec{v}_{f_2} = 4.2$ m/s

Statement: The velocity of particle 1 after the collision is 15 m/s [left]. The velocity of particle 2 after the collision is 4.2 m/s [right].

4. Given: m_1 = 2.67 kg; m_2 = 5.83 kg; $\vec{v}_{f_1} = 185$ m/s [right]; $\vec{v}_{f_2} = 172$ m/s [right]

Required: $\vec{v}_{i_1}$; $\vec{v}_{i_2}$

Analysis: Consider right to be positive, and let the direction of the chunks' final motion be positive. Use the final velocity equations, $\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$ and

$\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$. Then, solve the resulting linear system.

Solution:

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$$

$$185\ \text{m/s} = \left(\frac{2.67\ \cancel{\text{kg}} - 5.83\ \cancel{\text{kg}}}{2.67\ \cancel{\text{kg}} + 5.83\ \cancel{\text{kg}}}\right)\vec{v}_{i_1} + \left(\frac{2(5.83\ \cancel{\text{kg}})}{2.67\ \cancel{\text{kg}} + 5.83\ \cancel{\text{kg}}}\right)\vec{v}_{i_2}$$

$$185\ \text{m/s} = \left(\frac{-3.16}{8.50}\right)\vec{v}_{i_1} + \left(\frac{11.66}{8.50}\right)\vec{v}_{i_2}$$

$$1572.5\ \text{m/s} = -3.16\vec{v}_{i_1} + 11.66\vec{v}_{i_2}$$

$$\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

$$172\ \text{m/s} = \left(\frac{5.83\ \cancel{\text{kg}} - 2.67\ \cancel{\text{kg}}}{2.67\ \cancel{\text{kg}} + 5.83\ \cancel{\text{kg}}}\right)\vec{v}_{i_2} + \left(\frac{2(2.67\ \cancel{\text{kg}})}{2.67\ \cancel{\text{kg}} + 5.83\ \cancel{\text{kg}}}\right)\vec{v}_{i_1}$$

$$172\ \text{m/s} = \left(\frac{5.83\ \cancel{\text{kg}} - 2.67\ \cancel{\text{kg}}}{8.50}\right)\vec{v}_{i_2} + \left(\frac{2(2.67\ \cancel{\text{kg}})}{8.50}\right)\vec{v}_{i_1}$$

$$1462\ \text{m/s} = 3.16\vec{v}_{i_2} + 5.34\vec{v}_{i_1}$$

Solve the linear system.

$$-3.16\vec{v}_{i_1} + 11.66\vec{v}_{i_2} = 1572.5\ \text{m/s} \quad \text{Equation 1}$$

$$5.34\vec{v}_{i_1} + 3.16\vec{v}_{i_2} = 1462\ \text{m/s} \quad \text{Equation 2}$$

Multiply Equation 1 by $\frac{534}{316}$ and add.

$$-5.34\vec{v}_{i_1} + 19.704\vec{v}_{i_2} = 2657.3\ \text{m/s} \quad \text{Equation 1}$$

$$5.34\vec{v}_{i_1} + 3.16\vec{v}_{i_2} = 1462\ \text{m/s} \quad \text{Equation 2}$$

$$22.864\vec{v}_{i_2} = 4119.3$$

$$= 180.2\ \text{m/s [right] (one extra digit carried)}$$

$$\vec{v}_{i_2} = 1.80 \times 10^2\ \text{m/s [right]}$$

Substitute $\vec{v}_{i_2} = 180.2$ into Equation 2.

$$5.34\vec{v}_{i_1} + 3.16\vec{v}_{i_2} = 1462\ \text{m/s}$$

$$5.34\vec{v}_{i_1} + (3.16)(180.2\ \text{m/s}) = 1462\ \text{m/s}$$

$$5.34\vec{v}_{i_1} = 892.568\ \text{m/s}$$

$$\vec{v}_{i_1} = 167\ \text{m/s [right]}$$

Statement: The initial velocity of the more massive chunk is 1.80×10^2 m/s [right], and the initial velocity of the less massive chunk is 167 m/s [right]. (Since all initial and final velocities are in the same direction, the more massive chunk overtakes the less massive one and imparts some of its momentum to it.)

5. (a) Given: $m_1 = 0.84$ kg; $\vec{v}_{i_1} = 4.2$ m/s [right]; $m_2 = 0.48$ kg; $\vec{v}_{i_2} = 2.4$ m/s [left];
$k = 8.0 \times 10^3$ N/m

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: $\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$; $\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$

Solution:
$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$$
$$= \left(\frac{0.84\ \cancel{\text{kg}} - 0.48\ \cancel{\text{kg}}}{0.84\ \cancel{\text{kg}} + 0.48\ \cancel{\text{kg}}}\right)(4.2\text{ m/s}) + \left(\frac{2(0.48\ \cancel{\text{kg}})}{0.84\ \cancel{\text{kg}} + 0.48\ \cancel{\text{kg}}}\right)(-2.4\text{ m/s})$$
$$\vec{v}_{f_1} = -0.60\text{ m/s}$$
$$\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$
$$= \left(\frac{0.48\ \cancel{\text{kg}} - 0.84\ \cancel{\text{kg}}}{0.84\ \cancel{\text{kg}} + 0.48\ \cancel{\text{kg}}}\right)(-2.4\text{ m/s}) + \left(\frac{2(0.84\ \cancel{\text{kg}})}{0.84\ \cancel{\text{kg}} + 0.48\ \cancel{\text{kg}}}\right)(4.2\text{ m/s})$$
$$\vec{v}_{f_2} = 6.0\text{ m/s}$$

Statement: The velocity of cart 1 after the collision is 0.60 m/s [left]. The velocity of cart 2 after the collision is 6.0 m/s [right].

(b) Given: $m_1 = 0.84$ kg; $\vec{v}_{i_1} = 4.2$ m/s [right]; $m_2 = 0.48$ kg; $\vec{v}_{i_2} = 2.4$ m/s [left];
$\vec{v}_{f_1} = 3.0$ m/s [right]; $k = 8.0 \times 10^3$ N/m

Analysis: Use the conservation of momentum to determine the velocity of cart 2 during the collision, when cart 2 is moving 3.0 m/s [right]. Rearrange the conservation of momentum equation to express the final velocity of cart 2 in terms of the other given values.
$$m_1 v_{i_1} + m_2 v_{i_2} = m_1 v_{f_1} + m_2 v_{f_2}$$
$$v_{f_2} = \frac{m_1 v_{i_1} + m_2 v_{i_2} - m_1 v_{f_1}}{m_2}$$
Then, use conservation of mechanical energy to determine the compression of the spring at this particular moment during the collision. Consider right to be positive, and omit the vector notation.

Solution: $v_{f_2} = \dfrac{m_1 v_{i_1} + m_2 v_{i_2} - m_1 v_{f_1}}{m_2}$

$$= \frac{(0.84 \text{ kg})(4.2 \text{ m/s}) + (0.48 \text{ kg})(-2.4 \text{ m/s}) - (0.84 \text{ kg})(3.0 \text{ m/s})}{0.48 \text{ kg}}$$

$$v_{f_2} = -0.30 \text{ m/s}$$

Now use the law of conservation of mechanical energy to determine the compression of the spring. Clear fractions first, and then isolate *x*.

$$\frac{1}{2} m_1 v_{i_1}^2 + \frac{1}{2} m_2 v_{i_2}^2 = \frac{1}{2}(m_1 v_{f_1}^2 + m_2 v_{f_2}^2) + \frac{1}{2} kx^2$$

$$m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - (m_1 v_{f_1}^2 + m_2 v_{f_2}^2) = kx^2$$

$$x = \sqrt{\frac{m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - (m_1 v_{f_1}^2 + m_2 v_{f_2}^2)}{k}}$$

$$= \sqrt{\frac{(0.84 \text{ kg})(4.2 \text{ m/s})^2 + (0.48 \text{ kg})(-2.4 \text{ m/s})^2 - (0.84 \text{ kg})(3.0 \text{ m/s})^2 + (0.48 \text{ kg})(-0.30 \text{ m/s})^2}{8.0 \times 10^3 \text{ N/m}}}$$

$$x = 0.035 \text{ m}$$

Statement: The compression of the spring when cart 1 is moving at 3.0 m/s [right] is 3.5×10^{-2} m.

(c) Given: m_1 = 0.84 kg; $\vec{v}_{i_1}$ = 4.2 m/s [right]; m_2 = 0.48 kg; $\vec{v}_{i_2}$ = 2.4 m/s [left]; $k = 8.0 \times 10^3$ N/m

Required: *x*

Analysis: At the point of maximum compression of the spring, the two carts will have the same velocity, $\vec{v}_f$. Use the conservation of momentum equation to determine this velocity.

$$m_1 v_{i_1} + m_2 v_{i_2} = (m_1 + m_2) v_f$$

$$v_{f_1} = \frac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2}$$

Then use the law of conservation of mechanical energy to determine the maximum compression of the spring.

Solution: $v_f = \dfrac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2}$

$$= \frac{(0.84 \text{ kg})(4.2 \text{ m/s}) + (0.48 \text{ kg})(-2.4 \text{ m/s})}{0.84 \text{ kg} + 0.48 \text{ kg}}$$

$$v_f = 1.8 \text{ m/s}$$

$$\frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{1}{2}(m_1 + m_2)v_f^2 + \frac{1}{2}kx^2$$

$$m_1v_{i_1}^2 + m_2v_{i_2}^2 = (m_1 + m_2)v_f^2 + kx^2$$

$$kx^2 = m_1v_{i_1}^2 + m_2v_{i_2}^2 - (m_1 + m_2)v_f^2$$

$$x^2 = \frac{m_1v_{i_1}^2 + m_2v_{i_2}^2 - (m_1 + m_2)v_f^2}{k}$$

$$x = \sqrt{\frac{m_1v_{i_1}^2 + m_2v_{i_2}^2 - (m_1 + m_2)v_f^2}{k}}$$

$$= \sqrt{\frac{(0.84\text{ kg})(4.2\text{ m/s})^2 + (0.48\text{ kg})(-2.4\text{ m/s})^2 - (0.84\text{ kg} + 0.48\text{ kg})(1.8\text{ m/s})^2}{8.0 \times 10^3\text{ N/m}}}$$

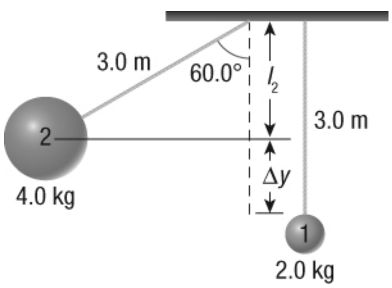
$$x = 0.041\text{ m}$$

Statement: The maximum compression of the spring is 4.1×10^{-2} m.

6. (a) Given: m_1 = 2.0 kg; m_2 = 4.0 kg; θ = 60.0°; length of two strings = 3.0 m; $\vec{v}_{i_1} = 0$ m/s

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: Draw a diagram of the situation. Let the $y = 0$ reference point be the vertical position of ball 1 before the collision. Let Δy be the vertical height of ball 2 above ball 1. Let l_2 be the vertical distance of ball 2 from the post.



Use conservation of energy to find the velocity of ball 2 just before the collision, at $y = 0$. Then, use the equations for perfectly elastic collisions to find the velocities of the balls just after the collision.

$E_k = E_g$; $\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$; $\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$

Solution: From the diagram, l_2 = (3.0 m)cos 60° = 1.5 m. Thus, Δy = 1.5 m.
Use conservation of energy.

$$E_k = E_g$$

$$\frac{1}{2}\cancel{m_2}v_{i_2}^2 = \cancel{m_2}g\Delta y$$

$$v_{i_2} = \sqrt{2g\Delta y}$$

$$= \sqrt{2(9.8\text{ m/s}^2)(1.5\text{ m})}$$

$$v_{i_2} = 5.42\text{ m/s (one extra digit carried)}$$

Now, use the equations for perfectly elastic collisions, using the fact that $\vec{v}_{i\,1} = 0$ m/s.

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$$

$$= \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$$

$$= \left(\frac{2(4.0\ \cancel{\text{kg}})}{2.0\ \cancel{\text{kg}} + 4.0\ \cancel{\text{kg}}}\right)(5.42\text{ m/s})$$

$$\vec{v}_{f_1} = 7.23\text{ m/s (one extra digit carried)}$$

$$\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2}$$

$$= \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2}$$

$$= \left(\frac{4.0\ \cancel{\text{kg}} - 2.0\ \cancel{\text{kg}}}{2.0\ \cancel{\text{kg}} + 4.0\ \cancel{\text{kg}}}\right)(5.42\text{ m/s})$$

$$\vec{v}_{f_2} = 1.81\text{ m/s (one extra digit carried)}$$

Statement: The speed of ball 1 is 7.2 m/s, and the speed of ball 2 is 1.8 m/s.
(b) Given: m_1 = 2.0 kg; m_2 = 4.0 kg; $\vec{v}_{f_1}$ = 7.23 m/s ; $\vec{v}_{f_2}$ = 1.81 m/s

Required: $h_{max\,1}$; $h_{max\,2}$
Analysis: Use conservation of energy, $E_g = E_k$.

Solution: $E_{g_1} = E_{k_1}$

$$\cancel{m_1} gh_{\text{max 1}} = \frac{1}{2}\cancel{m_1} v_{f_1}^2$$

$$h_{\text{max 1}} = \frac{v_{f_1}^2}{2g}$$

$$= \frac{(7.23 \text{ m/}\cancel{\text{s}})^{\cancel{2}}}{2(9.8\ \cancel{\text{m}}/\cancel{\text{s}^2})}$$

$$h_{\text{max 1}} = 2.7 \text{ m}$$

$$E_{g_2} = E_{k_2}$$

$$\cancel{m_2} gh_{\text{max 2}} = \frac{1}{2}\cancel{m_2} v_{f_2}^2$$

$$h_{\text{max 2}} = \frac{v_{f_2}^2}{2g}$$

$$= \frac{(1.81 \text{ m/}\cancel{\text{s}})^{\cancel{2}}}{2(9.8\ \cancel{\text{m}}/\cancel{\text{s}^2})}$$

$$h_{\text{max 2}} = 0.17 \text{ m}$$

Statement: After the first collision, the maximum height of ball 1 is 2.7 m, and the maximum height of ball 2 is 0.17 m.