## Section 5.3: Collisions

### Mini Investigation: Newton’s Cradle, page 234

Answers may vary. Sample answers:

**A.** In Step 2, releasing one end ball caused the far ball on the other end to swing out at the same speed as the original ball, while the middle balls appeared to remain still. Changing the setup did not change the outcome as long as all balls were touching. When the middle ball was removed, momentum was not transferred all the way to the end of the line.

**B.** Yes, the collisions appear to conserve momentum, although the balls slow down after a while.

**C.** Yes, the collisions appear to conserve kinetic energy. The end ball moves at the same speed as the beginning ball, so kinetic energy is conserved (ignoring external forces).

**D.** The device as a whole does not appear to conserve mechanical energy. Energy is lost in the sound of the collisions, and friction between the balls and between the string and the supports.

### Tutorial 1 Practice, page 236

**1. Given:** $m_1 = 3.5$ kg; $\vec{v}_{\mathrm{i}_1} = 5.4 \text{ m/s [right]}$; $m_2 = 4.8$ kg; $\vec{v}_{\mathrm{i}_2} = 0 \text{ m/s}$

**Required:** $\vec{v}_{\mathrm{f}_2}$

**Analysis:** The collision is perfectly elastic, which means that kinetic energy is conserved. Apply conservation of momentum and conservation of kinetic energy to construct and solve a linear-quadratic system of two equations in two unknowns. First, use conservation of momentum to solve for $v_{\mathrm{f}_1}$ in terms of $\vec{v}_{\mathrm{f}_2}$, remembering that $\vec{v}_{\mathrm{i}_2} = 0 \text{ m/s}$. This is a one-dimensional problem, so omit the vector notation for velocities, recognizing that positive values indicate motion to the right and negative values indicate motion to the left.

$$m_1 v_{\mathrm{i}_1} + m_2 v_{\mathrm{i}_2} = m_1 v_{\mathrm{f}_1} + m_2 v_{\mathrm{f}_2}$$

$$m_1 v_{\mathrm{i}_1} = m_1 v_{\mathrm{f}_1} + m_2 v_{\mathrm{f}_2}$$

$$v_{\mathrm{f}_1} = \frac{m_1 v_{\mathrm{i}_1} - m_2 v_{\mathrm{f}_2}}{m_1}$$

Then, substitute the resulting equation into the conservation of kinetic energy equation to solve for the final velocity of ball 2.

**Solution:**

$$\begin{aligned} v_{\mathrm{f}_1} &= \frac{m_1 v_{\mathrm{i}_1} - m_2 v_{\mathrm{f}_2}}{m_1} \\ &= \frac{(3.5 \ \cancel{\text{kg}})(5.4 \text{ m/s}) - (4.8 \ \cancel{\text{kg}})v_{\mathrm{f}_2}}{3.5 \ \cancel{\text{kg}}} \\ v_{\mathrm{f}_1} &= \frac{18.9 \text{ m/s} - 4.8 v_{\mathrm{f}_2}}{3.5} \end{aligned}$$

The conservation of kinetic energy equation can be simplified by multiplying both sides of the equation by 2 and noting that $\vec{v}_{\mathrm{i}_2} = 0 \text{ m/s}$.

$$\frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{1}{2}m_1v_{f_1}^2 + \frac{1}{2}m_2v_{f_2}^2$$

$$m_1v_{i_2}^2 = m_1v_{f_1}^2 + m_2v_{f_2}^2$$

$$(3.5\ \text{kg})(5.4\ \text{m/s})^2 = (3.5\ \text{kg})\frac{(18.9\ \text{m/s} - 4.8v_{f_2})^2}{(3.5)^2} + (4.8\ \text{kg})v_{f_2}^2$$

$$(3.5)(3.5)(5.4\ \text{m/s})^2 = (18.9\ \text{m/s} - 4.8v_{f_2})^2 + (3.5)(4.8)v_{f_2}^2$$

$$357.21\ \text{m}^2/\text{s}^2 = 357.21\ \text{m}^2/\text{s}^2 - (181.44\ \text{m/s})v_{f_2} + 23.04v_{f_2}^2 + 16.8v_{f_2}^2$$

$$0 = 39.84v_{f_2}^2 - (181.44\ \text{m/s})v_{f_2}$$

$$0 = v_{f_2}(39.84v_{f_2} - 181.44\ \text{m/s})$$

The factor of $v_{f_2}$ means the equation has a solution $v_{f_2}$ = 0 m/s. This solution describes the system before the collision. The equation has a second solution describing the system after the collision.

$$0 = 39.84v_{f_2} - 181.44\ \text{m/s}$$

$$v_{f_2} = \frac{181.44\ \text{m/s}}{39.84}$$

$$v_{f_2} = 4.6\ \text{m/s}$$

**Statement:** The final velocity of ball 2 is 4.6 m/s [right].

**2. Given:** $\vec{v}_{i_1} = \vec{v}_1$; $\vec{v}_{i_2} = 0$ m/s; $m_1 = m$; $m_2 = m$

**Required:** $v_{f_1}$; $v_{f_2}$

**Analysis:** $m_1v_{i_1} + m_2v_{i_2} = m_1v_{f_1} + m_2v_{f_2}$; $\frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{1}{2}m_1v_{f_1}^2 + \frac{1}{2}m_2v_{f_2}^2$

**Solution:** $m_1v_{i_1} + m_2v_{i_2} = m_1v_{f_1} + m_2v_{f_2}$

$$mv_1 + m(0\ \text{m/s}) = mv_{f_1} + mv_{f_2}$$

$$v_1 = v_{f_1} + v_{f_2}$$

$$v_{f_2} = v_1 - v_{f_1}$$

$$\frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{1}{2}m_1v_{f_1}^2 + \frac{1}{2}m_2v_{f_2}^2$$

$$mv_1^2 + m(0\ \text{m/s})^2 = mv_{f_1}^2 + m(v_1 - v_{f_1})^2$$

$$v_1^2 = v_{f_1}^2 + (v_1 - v_{f_1})^2$$

$$v_1^2 = v_{f_1}^2 + v_1^2 - 2v_1v_{f_1} + v_{f_1}^2$$

$$0 = 2v_{f_1}^2 - 2v_1v_{f_1}$$

$$0 = 2v_{f_1}(v_{f_1} - v_1)$$

$v_{f_1} = 0$ or $v_{f_1} = v_1$

The final speed of the first stone cannot be the same as its initial speed, so $v_{f_1}$ = 0. Substitute $v_{f_1}$ = 0 in the equation for $v_{f_2}$.

$$
\begin{aligned}
v_{f_2} &= v_1 - v_{f_1} \\
&= v_1 - 0 \\
v_{f_2} &= v_1
\end{aligned}
$$

**Statement:** The final speed of the first stone is 0 m/s. The final speed of the second stone is $v_1$.

## Tutorial 2 Practice, page 238

**1. Given:** $m_1$ = 4.0 kg; $m_2$ = 2.0 kg; $\vec{v}_{i_1} = 6.0$ m/s [forward]; $\vec{v}_{i_2} = 0$ m/s

**Required:** $\vec{v}_f$

**Analysis:** Use $\vec{v}_f = \dfrac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}$.

**Solution:**
$$
\begin{aligned}
\vec{v}_f &= \frac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2} \\
&= \frac{(4.0\ \cancel{\text{kg}})(6.0\ \text{m/s [forward]}) + (2.0\ \cancel{\text{kg}})(0\ \text{m/s})}{4.0\ \text{kg} + 2.0\ \cancel{\text{kg}}} \\
\vec{v}_f &= 4.0\ \text{m/s [forward]}
\end{aligned}
$$

**Statement:** The velocity of the balls is 4.0 m/s [forward] after the collision.

**2. (a) Given:** $m_1$ = 2200 kg; $\vec{v}_{i_1} = 60.0$ km/h [E]; $m_2$ = 1300 kg; $\vec{v}_{i_2} = 30.0$ km/h [E]

**Required:** $\vec{v}_f$

**Analysis:** Convert the velocities to metres per second and use $\vec{v}_f = \dfrac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}$.

$$\vec{v}_{i_1} = 60.0\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\ \text{[E]}$$

$\vec{v}_{i_1} = 16.7$ m/s [E] (one extra digit carried)

$$\vec{v}_{i_2} = 30.0\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\ \text{[E]}$$

$\vec{v}_{i_2} = 8.33$ m/s [E] (one extra digit carried)

**Solution:** $\vec{v}_{\text{f}} = \dfrac{m_1\vec{v}_{\text{i}_1} + m_2\vec{v}_{\text{i}_2}}{m_1 + m_2}$

$$= \frac{(2200\ \cancel{\text{kg}})(16.7\text{ m/s [E]}) + (1300\ \cancel{\text{kg}})(8.33\text{ m/s [E]})}{2200\ \cancel{\text{kg}} + 1300\ \cancel{\text{kg}}}$$

$\vec{v}_{\text{f}} = 13.6$ m/s [E] (one extra digit carried)

**Statement:** The final velocity of the vehicles is 14 m/s [E].

**(b) Given:** $m_1$ = 2200 kg; $m_2$ = 1300 kg; $\vec{v}_{\text{f}}$ = 13.6 m/s [E]

**Required:** $p$

**Analysis:** The momentum before the collision is equal to the momentum after the collision. Use the answer from (a) to determine the momentum after the collision.

$\vec{p} = (m_1 + m_2)\vec{v}_{\text{f}}$

**Solution:** $\vec{p} = (m_1 + m_2)\vec{v}_{\text{f}}$

$= (2200\text{ kg} + 1300\text{ kg})(13.6\text{ m/s [E]})$

$\vec{p} = 4.8 \times 10^4\ \text{kg}\cdot\text{m/s}$

**Statement:** The momentum before and after the collision is $4.8 \times 10^4$ kg·m/s.

**(c) Given:** $m_1$ = 2200 kg; $\vec{v}_{\text{i}_1}$ = 60.0 km/h [E] = 16.7 m/s [E]; $m_2$ = 1300 kg;

$\vec{v}_{\text{i}_2}$ = 30.0 km/h [E] = 8.33 m/s [E]; $\vec{v}_{\text{f}}$ = 13.6 m/s [E]

**Required:** $\Delta E_{\text{k}}$

**Analysis:** $\Delta E_{\text{k}} = E_{\text{k}_\text{f}} - E_{\text{k}_\text{i}}$; $E_{\text{k}} = \dfrac{1}{2}mv^2$

**Solution:**

$$\Delta E_{\text{k}} = E_{\text{k}_\text{f}} - E_{\text{k}_\text{i}}$$

$$= \frac{1}{2}(m_1 + m_2)v_{\text{f}}^2 - \left(\frac{1}{2}m_1 v_{\text{i}_1}^2 + \frac{1}{2}m_2 v_{\text{i}_2}^2\right)$$

$$= \frac{1}{2}(2200\text{ kg} + 1300\text{ kg})(13.6\text{ m/s})^2 - \frac{1}{2}[(2200\text{ kg})(16.7\text{ m/s})^2 + (1300\text{ kg})(8.33\text{ m/s})^2]$$

$$\Delta E_{\text{k}} = -2.8 \times 10^4\ \text{J}$$

**Statement:** The decrease in kinetic energy is $2.8 \times 10^4$ J.

**3. Given:** $m_1$ = 66 kg; $\Delta y$ = 25 m; $m_2$ = 72 kg; $v_{\text{i}_2}$ = 0 m/s

**Required:** $v_{\text{f}}$

**Analysis:** Use conservation of energy to determine the speed of the snowboarder at the bottom of the hill.

$$m_1 g\Delta y = \frac{1}{2}m_1 v_{\text{i}_1}^2$$

Then, use $\vec{v}_{\text{f}} = \dfrac{m_1\vec{v}_{\text{i}_1} + m_2\vec{v}_{\text{i}_2}}{m_1 + m_2}$ to calculate the final velocity of both people after the collision.

**Solution:** $\cancel{m_1} g\Delta y = \frac{1}{2}\cancel{m_1} v_{i_1}^2$

$$v_{i_1} = \sqrt{2g\Delta y}$$

$$= \sqrt{2(9.8\ \text{m/s}^2)(25\ \text{m})}$$

$$v_{i_1} = 22.1\ \text{m/s (one extra digit carried)}$$

$$\vec{v}_f = \frac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}$$

$$= \frac{(66\ \cancel{\text{kg}})(22.1\ \text{m/s}) + (72\ \cancel{\text{kg}})(0\ \text{m/s})}{66\ \cancel{\text{kg}} + 72\ \cancel{\text{kg}}}$$

$$v_f = 11\ \text{m/s}$$

**Statement:** The final speed of each person after the collision is 11 m/s.

## Section 5.3 Questions, page 239

**1.** Answers may vary. Sample answer:

**(a)** Since the boxes stick together after the collision, we know this is an inelastic collision. Momentum is conserved in an inelastic collision. Momentum is always conserved if there are no external forces acting on the system.

**(b)** Kinetic energy is not conserved in an inelastic collision. In an inelastic collision, some kinetic energy is absorbed by one or both objects, causing the kinetic energy after the collision to be less than the kinetic energy before the collision.

**2. Given:** $m_1$ = 85 kg; $m_2$ = 8.0 kg; $\vec{v}_{i_2} = 0$ m/s; $\vec{v}_f = 3.0$ m/s [forward]

**Required:** $v_{i_1}$

**Analysis:** Use $v_f = \frac{m_1v_{i_1} + m_2v_{i_2}}{m_1 + m_2}$, rearranged to isolate $v_{i_1}$.

$$v_f = \frac{m_1v_{i_1} + m_2v_{i_2}}{m_1 + m_2}$$

$$(m_1 + m_2)v_f = m_1v_{i_1} + m_2v_{i_2}$$

$$m_1v_{i_1} = (m_1 + m_2)v_f - m_2v_{i_2}$$

$$v_{i_1} = \frac{(m_1 + m_2)v_f - m_2v_{i_2}}{m_1}$$

**Solution:** $v_{i_1} = \frac{(m_1 + m_2)v_f - m_2v_{i_2}}{m_1}$

$$= \frac{(85\ \cancel{\text{kg}} + 8.0\ \cancel{\text{kg}})(3.0\ \text{m/s}) - (8.0\ \cancel{\text{kg}})(0\ \text{m/s})}{85\ \cancel{\text{kg}}}$$

$$v_{i_1} = 3.3\ \text{m/s}$$

**Statement:** The speed of the skateboarder just before he landed on the skateboard was 3.3 m/s.

**3. Given:** $m_T$ = 3.0 kg; $m_1$ = 2.0 kg; $\vec{v}_i = 0$ m/s; $\vec{v}_{f_1} = 2.5$ m/s [S]

**Required:** $\vec{v}_{f_2}$

**Analysis:** Since the total mass is 3.0 kg, and the mass of the first cart is 2.0 kg, the mass of the second cart, $m_2$, is 1.0 kg. The carts are at rest before they are released, so the initial momentum of the system is zero. The final momentum must equal the initial momentum.

$\vec{p} = m\vec{v}$

**Solution:** $\vec{p}_f = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$

$$0 = (2.0\ \cancel{\text{kg}})(2.5\text{ m/s [S]}) + (1.0\ \cancel{\text{kg}})\vec{v}_{f_2}$$

$$\vec{v}_{f_2} = -5.0\text{ m/s [S]}$$

$$\vec{v}_{f_2} = 5.0\text{ m/s [N]}$$

**Statement:** The final velocity of the other cart is 5.0 m/s [N].

**4. (a)** Yes. Both momentum and kinetic energy are conserved in a perfectly elastic collision.

**(b) Given:** $m_1$ = 85 kg; $\vec{v}_{i_1} = 6.5$ m/s; $m_2$ = 120 kg; $\vec{v}_{i_2} = 0$ m/s; $\vec{v}_{f_1} = -1.1$ m/s

**Required:** $\vec{v}_{f_2}$

**Analysis:** Use the conservation of momentum equation to solve for $\vec{v}_{f_2}$.

$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$

**Solution:**

$$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$$

$$(85\ \cancel{\text{kg}})(6.5\text{ m/s}) + (120\ \cancel{\text{kg}})(0\text{ m/s}) = (85\ \cancel{\text{kg}})(-1.1\text{ m/s}) + (120\ \cancel{\text{kg}})\vec{v}_{f_2}$$

$$552.5\text{ m/s} = -93.5\text{ m/s} + 120\vec{v}_{f_2}$$

$$\vec{v}_{f_2} = 5.4\text{ m/s}$$

**Statement:** The final velocity of the second person is 5.4 m/s in the direction that the first person was originally travelling.

**5. (a)** It is an inelastic collision because the two skaters stick together after the collision.

**(b) Given:** $m_1$ = 95 kg; $v_{i_1}$ = 5.0 m/s; $m_2$ = 130 kg; $v_{i_2}$ = 0 m/s

**Required:** $v_f$

**Analysis:** Use $v_f = \dfrac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2}$.

**Solution:** $v_f = \dfrac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2}$

$$= \frac{(95\ \cancel{\text{kg}})(5.0\text{ m/s}) + (130\ \cancel{\text{kg}})(0\text{ m/s})}{95\ \cancel{\text{kg}} + 130\ \cancel{\text{kg}}}$$

$$v_f = 2.1\text{ m/s}$$

**Statement:** The final speed of the skaters is 2.1 m/s [initial direction of the first skater].

**6. Given:** $m_1 = m_2 = 1250$ kg; $\vec{v}_{i_1} = 12\text{ m/s [E]}$; $\vec{v}_{i_2} = 12\text{ m/s [W]}$

**Required:** $\vec{v}_f$

**Analysis:** Use $\vec{v}_f = \dfrac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}$.

**Solution:**
$$\begin{aligned}\vec{v}_f &= \frac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}\\ &= \frac{(1250\text{ kg})(12\text{ m/s [E]}) + (1250\text{ kg})(12\text{ m/s [W]})}{1250\text{ kg} + 1250\text{ kg}}\\ &= \frac{(1250\ \cancel{\text{kg}})(12\text{ m/s [E]}) - (1250\ \cancel{\text{kg}})(12\text{ m/s [E]})}{1250\ \cancel{\text{kg}} + 1250\ \cancel{\text{kg}}}\\ \vec{v}_f &= 0\text{ m/s}\end{aligned}$$

**Statement:** The velocity of both cars is 0 m/s after the collision. They come to a complete stop.

**7.** Answers may vary. Sample answer: No, if a moving object collides with a stationary object in a perfectly elastic collision, is it not possible for both objects to be at rest after the collision. Because kinetic energy is conserved in a perfectly elastic collision, and there was kinetic energy before the collision, there must also be kinetic energy after the collision. Therefore, one of the objects must be moving after the collision.

**8. (a) Given:** $m_1 = 1.3 \times 10^4$ kg; $\vec{v}_{i_1} = 9.0\times10^1\text{ km/h [N]}$; $m_2 = 1.1 \times 10^3$ kg;
$\vec{v}_{i_2} = 3.0\times10^1\text{ km/h [N]}$

**Required:** $\vec{v}_f$

**Analysis:** Convert the velocities to metres per second and then use $\vec{v}_f = \dfrac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}$.

$$\vec{v}_{i_1} = 9.0\times10^1\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}}\times\frac{1000\text{ m}}{1\ \cancel{\text{km}}}\times\frac{1\ \cancel{\text{h}}}{3600\text{ s}}\ \text{[N]}$$
$$\vec{v}_{i_1} = 25\text{ m/s [N]}$$
$$\vec{v}_{i_2} = 3.0\times10^1\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}}\times\frac{1000\text{ m}}{1\ \cancel{\text{km}}}\times\frac{1\ \cancel{\text{h}}}{3600\text{ s}}\ \text{[N]}$$
$$\vec{v}_{i_2} = 8.33\text{ m/s [N]}$$

**Solution:**
$$\begin{aligned}\vec{v}_f &= \frac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}\\ &= \frac{(1.3\times10^4\ \cancel{\text{kg}})(2.5\times10^1\text{ m/s [N]}) + (1.1\times10^3\ \cancel{\text{kg}})(8.33\text{ m/s [N]})}{1.3\times10^4\ \cancel{\text{kg}} + 1.1\times10^3\ \cancel{\text{kg}}}\\ \vec{v}_f &= 23.7\text{ m/s [N]}\end{aligned}$$

Convert back to kilometres per hour.

$$\vec{v}_f = 23.7\ \frac{\cancel{m}}{\cancel{s}} \times \frac{1\text{ km}}{1000\ \cancel{m}} \times \frac{3600\ \cancel{s}}{1\text{ h}}$$

$$\vec{v}_f = 85\text{ km/h}$$

**Statement:** The velocity of the vehicles after the collision is 85 km/h [N].

**(b) Given:** $m_1 = 1.3 \times 10^4$ kg; $\vec{v}_{i_1} = 25$ m/s [N]; $m_2 = 1.1 \times 10^3$ kg; $\vec{v}_{i_2} = 8.33$ m/s [N];

$\vec{v}_f = 23.7$ m/s [N]

**Required:** $E_{k_i}$; $E_{k_f}$

**Analysis:** $E_k = \frac{1}{2}mv^2$

**Solution:**

$$\begin{aligned} E_{k_i} &= \frac{1}{2}(m_1 v_{i_1}^2 + m_2 v_{i_2}^2) \\ &= \frac{1}{2}[(1.3\times10^4\text{ kg})(25\text{ m/s})^2 + (1.1\times10^3\text{ kg})(8.33\text{ m/s})^2] \\ E_{k_i} &= 4.1\times10^6\text{ J} \end{aligned}$$

$$\begin{aligned} E_{k_f} &= \frac{1}{2}(m_1 + m_2)v_f^2 \\ &= \frac{1}{2}(1.3\times10^4\text{ kg} + 1.1\times10^3\text{ kg})(23.7\text{ m/s})^2 \\ E_{k_f} &= 3.96\times10^6\text{ J (one extra digit carried)} \end{aligned}$$

**Statement:** The total kinetic energy before the collision was $4.1 \times 10^6$ J, and the total kinetic energy after the collision was $4.0 \times 10^6$ J.

**(c) Given:** $E_{k_i} = 4.1 \times 10^6$ J; $E_{k_f} = 3.96 \times 10^6$ J

**Required:** $\Delta E_k$

**Analysis:** $\Delta E_k = E_{k_f} - E_{k_i}$

**Solution:** $\Delta E_k = E_{k_f} - E_{k_i}$

$$\begin{aligned} &= (3.96\times10^6\text{ J}) - (4.1\times10^6\text{ J}) \\ \Delta E_k &= -1.4\times10^6\text{ J} \end{aligned}$$

**Statement:** The decrease in kinetic energy during the collision is $1.4 \times 10^5$ J.