Section 5.2: Conservation of Momentum in One Dimension

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1. Given: m_1 = 1350 kg; $\vec{v}_1 = 72$ km/h [S]; m_2 = 1650 kg; $\vec{v}_f = 24$ km/h [S]

Required: $\vec{v}_2$

Analysis: The cars stick together after the collision, so they can be treated as a single object with mass $m_1 + m_2$, velocity $\vec{v}_f$, and momentum $\vec{p}_f$.

$p_f = (m_1 + m_2)v_f$

By conservation of momentum,

$\vec{p}_f = \vec{p}_1 + \vec{p}_2$

and $\vec{p} = m\vec{v}$; therefore,

$$\vec{p}_f = \vec{p}_1 + \vec{p}_2$$

$$(m_1 + m_2)\vec{v}_f = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\vec{v}_2 = \frac{(m_1 + m_2)\vec{v}_f - m_1\vec{v}_1}{m_2}$$

Solution: $\vec{v}_2 = \dfrac{(m_1 + m_2)\vec{v}_f - m_1\vec{v}_1}{m_2}$

$$= \frac{(1350\text{ kg} + 1650\ \cancel{\text{kg}})(24\text{ km/h [S]}) - (1350\ \cancel{\text{kg}})(72\text{ km/h [S]})}{1650\ \cancel{\text{kg}}}$$

$$\vec{v}_2 = -15\text{ km/h [S]}$$

$$\vec{v}_2 = 15\text{ km/h [N]}$$

Statement: The initial velocity of the second car is 15 km/h [N].

2. Given: m_1 = 28 g = 0.028 kg; $\vec{v}_1 = 92$ m/s [forward]; $\vec{v}_2 = 0.039$ m/s [backward]

Required: m_2

Analysis: The momentum of the arrow is the opposite of the momentum of the archer.

$m_1\vec{v}_1 = -m_2\vec{v}_2$

$m_2 = -\dfrac{m_1\vec{v}_1}{\vec{v}_2}$

Solution: $m_2 = -\dfrac{m_1\vec{v}_1}{\vec{v}_2}$

$$= -\frac{(0.028\text{ kg})(92\text{ m/s [forward]})}{(0.039\text{ m/s [backward]})}$$

$$= -\frac{(0.028\text{ kg})(92\ \cancel{\text{m/s}}\ \cancel{\text{[forward]}})}{(-0.039\ \cancel{\text{m/s}}\ \cancel{\text{[forward]}})}$$

$$m_2 = 66\text{ kg}$$

Statement: The mass of the archer and the bow is 6.6×10^1 kg.

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1. The total momentum of a system is conserved if there is no net force applied on the system.

2. Given: mass of student and surfboard, m_1 = 59.6 kg; mass of student, m_2 = 55 kg; velocity of surfboard relative to water, $\vec{v}_1$ = 2.0 m/s [E]; velocity of student relative to surfboard, $\vec{v}_2$ = 1.9 m/s [E]

Required: resultant velocity, $\vec{v}$

Analysis: The momentum of the student and surfboard before the student starts walking is $m_1\vec{v}_1$. The momentum of the system after the student starts walking is the momentum of the student, $m_2\vec{v}_2$, plus the momentum of the student and surfboard, $m_1\vec{v}$.

$$m_1\vec{v}_1 = m_2\vec{v}_2 + m_1\vec{v}$$

$$\vec{v} = \frac{m_1\vec{v}_1 - m_2\vec{v}_2}{m_1}$$

Solution: $\vec{v} = \frac{m_1\vec{v}_1 - m_2\vec{v}_2}{m_1}$

$$= \frac{(59.6\ \cancel{\text{kg}})(2.0\text{ m/s [E]}) - (55\ \cancel{\text{kg}})(1.9\text{ m/s [E]})}{59.6\ \cancel{\text{kg}}}$$

$$\vec{v} = 0.25\text{ m/s [E]}$$

Statement: The final velocity of the surfboard is 0.25 m/s [E].

3. Given: m_1 = 35.6 kg; v_1 = 2.42 m/s; v_2 = 3.25 m/s

Required: m_2

Analysis: Assume the first hockey player is moving left after the collision and the second player is moving right. The momentum before the push is zero, so the total momentum after the push is also zero.

$$m_1\vec{v}_1 + m_2\vec{v}_2 = 0$$

$$m_2 = -\frac{m_1\vec{v}_1}{\vec{v}_2}$$

Solution: $m_2 = -\frac{m_1\vec{v}_1}{\vec{v}_2}$

$$= -\frac{(35.6\text{ kg})(2.42\text{ m/s [left]})}{3.25\text{ m/s [right]}}$$

$$= -\frac{(35.6\text{ kg})(-2.42\ \cancel{\text{m/s}}\ \cancel{\text{[right]}})}{3.25\ \cancel{\text{m/s}}\ \cancel{\text{[right]}}}$$

$$m_2 = 26.5\text{ kg}$$

Statement: The mass of the other player is 26.5 kg.

4. Given: m_1 = 0.14 kg; $\vec{v}_1$ = 50 m/s [forward]; m_2 = 80 kg

Required: $\vec{v}_2$

Analysis: The momentum before the throw is zero, so the total momentum after the throw is also zero.

$$m_1\vec{v}_1 + m_2\vec{v}_2 = 0$$

$$\vec{v}_2 = -\frac{m_1\vec{v}_1}{m_2}$$

Solution: $\vec{v}_2 = -\frac{m_1\vec{v}_1}{m_2}$

$$= -\frac{(0.14\ \cancel{\text{kg}})(50\ \text{m/s [forward]})}{80\ \cancel{\text{kg}}}$$

$$= -9\times10^{-2}\ \text{m/s [forward]}$$

$$\vec{v}_2 = 9\times10^{-2}\ \text{m/s [backward]}$$

Statement: The recoil velocity of the pitcher is 9×10^{-2} m/s [backward].

5. Given: m_1 = 4.5 kg; m_2 = 6.2 kg; $\vec{v}_{i_1} = 16$ m/s [E]; $\vec{v}_{i_2} = 0$ m/s; $\vec{v}_{f_2} = 10$ m/s [E]

Required: $\vec{v}_{f_1}$

Analysis: Momentum is conserved.

$$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$$

$$\vec{v}_{f_1} = \frac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} - m_2\vec{v}_{f_2}}{m_1}$$

Solution: $\vec{v}_{f_1} = \frac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} - m_2\vec{v}_{f_2}}{m_1}$

$$= \frac{(4.5\ \cancel{\text{kg}})(16\ \text{m/s [E]}) + (6.2\ \cancel{\text{kg}})(0) - (6.2\ \cancel{\text{kg}})(10\ \text{m/s [E]})}{4.5\ \cancel{\text{kg}}}$$

$$\vec{v}_{f_1} = 2.2\ \text{m/s [E]}$$

Statement: The final velocity of the smaller object is 2.2 m/s [E].

6. Given: $m_1 = m$; $m_2 = 3m$; $\vec{v}_{i_1} = 3v$; $\vec{v}_{i_2} = -v$

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: Use conservation of momentum.

Solution: Assume the lighter mass is initially moving to the right. Consider the conservation of momentum.

$$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$$

$$\cancel{m}(3v) + 3\cancel{m}(-v) = \cancel{m}\vec{v}_{f_1} + 3\cancel{m}\vec{v}_{f_2}$$

$$0 = \vec{v}_{f_1} + 3\vec{v}_{f_2}$$

$$\vec{v}_{f_1} = -3\vec{v}_{f_2}$$

Statement: The final speed of the larger mass will be three times the final speed of the smaller mass, and in the opposite direction.

7. Given: m_1 = 2.5 kg; m_2 = 7.5 kg; $v_{1\,\mathrm{i}} = +6.0$ m/s ; $v_{2\,\mathrm{i}} = -15$ m/s

Required: $\vec{v}_\mathrm{f}$

Analysis: The two objects stick together after the collision, with a mass of $m_1 + m_2$. Momentum is conserved.

$$m_1\vec{v}_{\mathrm{i}_1} + m_2\vec{v}_{\mathrm{i}_2} = (m_1 + m_2)\vec{v}_\mathrm{f}$$

$$\vec{v}_\mathrm{f} = \frac{m_1\vec{v}_{\mathrm{i}_1} + m_2\vec{v}_{\mathrm{i}_2}}{m_1 + m_2}$$

Solution: $$\vec{v}_\mathrm{f} = \frac{m_1\vec{v}_{\mathrm{i}_1} + m_2\vec{v}_{\mathrm{i}_2}}{m_1 + m_2}$$

$$= \frac{(2.5\ \cancel{\mathrm{kg}})(6.0\ \mathrm{m/s}) + (7.5\ \cancel{\mathrm{kg}})(-15\ \mathrm{m/s})}{2.5\ \cancel{\mathrm{kg}} + 7.5\ \cancel{\mathrm{kg}}}$$

$$\vec{v}_\mathrm{f} = -9.8\ \mathrm{m/s}$$

Statement: The final velocity of the two objects is 9.8 m/s [left].

8. Answers may vary. Sample answer: The astronaut can throw the tool bag in a direction away from the International Space Station. Conservation of momentum means she will move in the opposite direction, toward the ISS.