## Section 5.1: Momentum and Impulse

Tutorial 1 Practice, page 223

1. Given: $m=160 \mathrm{~g}=0.16 \mathrm{~kg} ; \vec{v}=140 \mathrm{~m} / \mathrm{s}[\mathrm{E}]$

Required: $\vec{p} ; E_{\mathrm{k}}$
Analysis: $\vec{p}=m \vec{v} ; E_{\mathrm{k}}=\frac{1}{2} m v^{2}$
Solution: $\vec{p}=m \vec{v}$

$$
\begin{aligned}
& \qquad \quad=(0.16 \mathrm{~kg})(40.0 \mathrm{~m} / \mathrm{s}[\mathrm{E}]) \\
& \qquad \begin{array}{l}
\vec{p}=6.4 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}[\mathrm{E}] \\
E_{\mathrm{k}}= \\
=\frac{1}{2} m v^{2} \\
=
\end{array} \\
& E_{\mathrm{k}}= \\
& =130 \mathrm{~J}
\end{aligned}
$$

Statement: The momentum of the puck is $6.4 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}[\mathrm{E}]$, and the kinetic energy is 130 J .
2. Given: $m_{1}=6.2 \mathrm{~kg} ; v_{1}=1.6 \mathrm{~m} / \mathrm{s}[\mathrm{E}] ; m_{2}=160 \mathrm{~g}=0.16 \mathrm{~kg} ; v_{2}=40.0 \mathrm{~m} / \mathrm{s}[\mathrm{E}]$

Required: $\vec{p}_{2}-\vec{p}_{1}$
Analysis: $\vec{p}=m \vec{v}$
Solution: $\vec{p}_{1}-\vec{p}_{2}=m_{1} \vec{v}_{1}-m_{2} \vec{v}_{2}$

$$
\begin{aligned}
& =(6.2 \mathrm{~kg})(1.6 \mathrm{~m} / \mathrm{s}[\mathrm{E}])-(0.16 \mathrm{~kg})(40.0 \mathrm{~m} / \mathrm{s}[\mathrm{E}]) \\
\vec{p}_{1}-\vec{p}_{2} & =3.5 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}[\mathrm{E}]
\end{aligned}
$$

Statement: The difference in the momenta is $3.5 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}$ [E].
Tutorial 2 Practice, page 226

1. (a) Given: $\vec{F}=250 \mathrm{~N}$ [forward]; $t=0.0030 \mathrm{~s}$

Required: $\Delta \vec{p}$
Analysis: $\Delta \vec{p}=\vec{F} \Delta t$
Solution: $\Delta \vec{p}=\vec{F} \Delta t$

$$
\begin{aligned}
& =(250 \mathrm{~N}[\text { forward }])(0.0030 \mathrm{~s}) \\
\Delta \vec{p} & =0.75 \mathrm{~N} \cdot \mathrm{~s}[\text { forward }]
\end{aligned}
$$

Statement: The impulse imparted by the hockey stick is $0.75 \mathrm{~N} \cdot \mathrm{~s}$ [forward].
(b) Given: $\vec{F} \Delta t=0.75 \mathrm{~N} \cdot \mathrm{~s} ; m=180 \mathrm{~g}=0.18 \mathrm{~kg} ; \vec{v}_{\mathrm{i}}=0$

Required: $\vec{v}_{f}$

Analysis: $\vec{F} \Delta t=\Delta \vec{p}$

$$
\begin{aligned}
\Delta \vec{p} & =m\left(\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}\right) \\
\vec{F} \Delta t & =m\left(\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}\right) \\
\vec{F} \Delta t & =m \vec{v}_{\mathrm{f}} \\
\vec{v}_{\mathrm{f}} & =\frac{\vec{F} \Delta t}{m}+\vec{v}_{\mathrm{i}}
\end{aligned}
$$

Solution: $\vec{v}_{\mathrm{f}}=\frac{\vec{F} \Delta t}{m}+\vec{v}_{\mathrm{i}}$

$$
=\frac{0.75 \mathrm{lgg} \cdot \mathrm{~m} / \mathrm{s} \text { [forward] }}{0.18 \mathrm{lgg}}+0 \mathrm{~m} / \mathrm{s}
$$

$$
\vec{v}_{\mathrm{f}}=4.2 \mathrm{~m} / \mathrm{s} \text { [forward] }
$$

Statement: The final velocity is $4.2 \mathrm{~m} / \mathrm{s}$ [forward].
2. Given: average force $=468 \mathrm{~N} ; \Delta t=2.9 \mathrm{~s}$

Required: force-time graph; $\vec{F} \Delta t$
Analysis: The average force is 468 N , so the graph is a straight line at $F=468$. Calculate the area under the curve.
Solution: Draw the graph.


Statement: The impulse of the collision is $1400 \mathrm{~N} \cdot \mathrm{~s}$ [away from the wall].

## Section 5.1 Questions, page 227

1. (a) Given: $m=4.25 \times 10^{2} \mathrm{~kg} ; \vec{v}=6.9 \mathrm{~m} / \mathrm{s}[\mathrm{N}]$

Required: $\vec{p}$
Analysis: $\vec{p}=m \vec{v}$
Solution: $\vec{p}=m \vec{v}$

$$
\begin{aligned}
& =\left(4.25 \times 10^{2} \mathrm{~kg}\right)(6.9 \mathrm{~m} / \mathrm{s}[\mathrm{~N}]) \\
\vec{p} & =2900 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}[\mathrm{~N}]
\end{aligned}
$$

Statement: The momentum of the moose is $2.9 \times 10^{3} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}[\mathrm{N}]$.
(b) Given: $m=9.97 \times 10^{3} \mathrm{~kg}$; $\vec{v}=5 \mathrm{~km} / \mathrm{h}$ [forward]

## Required: $\vec{p}$

Analysis: $\vec{p}=m \vec{v}$
Convert the velocity to metres per second.
$\vec{v}=5 \frac{\mathrm{~km}}{K}[$ forward $] \times \frac{1 K}{3600 \mathrm{~s}} \times \frac{1000 \mathrm{~m}}{1 \mathrm{~km}}$
$\vec{v}=1.4 \mathrm{~m} / \mathrm{s}$ [forward]
Solution: $\vec{p}=m \vec{v}$

$$
\begin{aligned}
& =\left(9.97 \times 10^{3} \mathrm{~kg}\right)(1.4 \mathrm{~m} / \mathrm{s} \text { [forward] }) \\
\vec{p} & =1.4 \times 10^{4} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s} \text { [forward] }
\end{aligned}
$$

Statement: The momentum of the bus is $1.4 \times 10^{4} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}$ [forward].
(c) Given: $m=995 \mathrm{~g}=0.995 \mathrm{~kg} ; \vec{v}=16 \mathrm{~m} / \mathrm{s}[\mathrm{S}]$

Required: $\vec{p}$
Analysis: $\vec{p}=m \vec{v}$
Solution: $\vec{p}=m \vec{v}$

$$
\begin{aligned}
& =(0.995 \mathrm{~kg})(16 \mathrm{~m} / \mathrm{s}[\mathrm{~S}]) \\
\vec{p} & =16 \mathrm{~N} \cdot \mathrm{~s}[\mathrm{~S}]
\end{aligned}
$$

Statement: The momentum of the squirrel is $16 \mathrm{~N} \cdot \mathrm{~s}$ [S].
2. Answers may vary. Sample answer: Impulse is the change in momentum of an object during a certain time. Its units are newton seconds ( $\mathrm{N} \cdot \mathrm{s}$ ).
3. Given: $m=79.3 \mathrm{~kg} ; \vec{p}=2.16 \times 10^{3} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}$ [W]

Required: $\vec{v}$
Analysis: $\vec{p}=m \vec{v}$

$$
\vec{v}=\frac{\vec{p}}{m}
$$

Solution: $\vec{v}=\frac{\vec{p}}{m}$

$$
=\frac{2.16 \times 10^{3} \mathrm{lgg} \cdot \mathrm{~m} / \mathrm{s}[\mathrm{~W}]}{79.3 \mathrm{lg}}
$$

$$
\vec{v}=27.2 \mathrm{~m} / \mathrm{s}[\mathrm{~W}]
$$

Statement: The velocity of the bicycle is $27.2 \mathrm{~m} / \mathrm{s}$ [W].
4. Given: $\vec{v}=9.0 \times 10^{2} \mathrm{~m} / \mathrm{s}[\mathrm{W}] ; \vec{p}=4.5 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}[\mathrm{W}]$

Required: $m$
Analysis: $\vec{p}=m \vec{v}$

$$
m=\frac{\vec{p}}{\vec{v}}
$$

Solution: $m=\frac{\vec{p}}{\vec{v}}$

$$
\begin{aligned}
= & \frac{4.5 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}[\mathrm{~W}]}{9.0 \times 10^{2} \mathrm{~m} 1 / \mathrm{s}[\mathrm{~W}]} \\
m & =5.0 \times 10^{-3} \mathrm{~kg}
\end{aligned}
$$

Statement: The mass of the projectile is $5.0 \times 10^{-3} \mathrm{~kg}$ or 5.0 g .
5. Given: $\vec{v}=29.5 \mathrm{~m} / \mathrm{s}$ [forward] ; $\vec{p}=2.31 \times 10^{3} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}$ [forward]

## Required: $m$

Analysis: $\vec{p}=m \vec{v}$

$$
m=\frac{\vec{p}}{\vec{v}}
$$

Solution: $m=\frac{\vec{p}}{\vec{v}}$

$$
=\frac{2.31 \times 10^{3} \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s} \text { [forward] }}{29.5 \mathrm{~m} / \mathrm{s}[\text { forward }]}
$$

$$
m=78.3 \mathrm{~kg}
$$

Statement: The mass of the skier is 78.3 kg .
6. Answers may vary. Sample answer: In lacrosse, when a player follows through when striking a ball, this increases the time interval over which the collision between the ball and lacrosse stick occurs and thus contributes to an increase in the velocity change of the ball. Following through, a hitter can hit the ball in such a way that it is moving faster when it leaves the stick. This can improve performance in lacrosse by making the ball travel farther in a given amount of time and by making it harder for an opposing player to catch up with or react to the ball in time.
7. Both balls will hit the floor with the same speed, and both will rebound with this same speed. Since the basketball has more mass, the same speed means it has more momentum. To reverse its larger momentum requires a larger impulse on the basketball.
8. (a) Given: $\vec{F}=1100.0 \mathrm{~N}$ [forward]; $\Delta t=5.0 \mathrm{~ms}=5.0 \times 10^{-3} \mathrm{~s}$

Required: $\Delta \vec{p}$
Analysis: $\Delta \vec{p}=\vec{F} \Delta t$
Solution: $\Delta \vec{p}=\vec{F} \Delta t$

$$
\begin{aligned}
& =(1100.0 \mathrm{~N}[\text { forward }])\left(5.0 \times 10^{-3} \mathrm{~s}\right) \\
\Delta \vec{p} & =5.5 \mathrm{~N} \cdot \mathrm{~s}[\text { forward }]
\end{aligned}
$$

Statement: The impulse is $5.5 \mathrm{~N} \cdot \mathrm{~s}$ [forward].
(b) Given: $m=0.12 \mathrm{~kg} ; v_{\mathrm{i}}=0 \mathrm{~m} / \mathrm{s} ; \vec{F} \Delta t=5.5 \mathrm{~N} \cdot \mathrm{~s}$ [forward]

Required: $v_{f}$
Analysis: $\vec{F} \Delta t=\Delta \vec{p}$

$$
\begin{aligned}
\Delta \vec{p} & =m\left(\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}\right) \\
\vec{F} \Delta t & =m\left(\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}\right) \\
\vec{v}_{\mathrm{f}} & =\frac{\vec{F} \Delta t}{m}+\vec{v}_{\mathrm{i}}
\end{aligned}
$$

Solution: $v_{\mathrm{f}}=\frac{F \Delta t}{m}+v_{\mathrm{i}}$

$$
\begin{aligned}
& =\frac{5.5 \mathrm{~N} \cdot \mathrm{~s}}{0.12 \mathrm{~kg}}+0 \mathrm{~m} / \mathrm{s} \\
\vec{v}_{\mathrm{f}} & =46 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Statement: The speed of the puck just after it leaves the stick is $46 \mathrm{~m} / \mathrm{s}$.
9. (a) Given: $m=225 \mathrm{~g}=0.225 \mathrm{~kg} ; \Delta y=74 \mathrm{~cm}=0.74 \mathrm{~m}$

Required: $\vec{p}$
Analysis: The potential energy as you drop the phone is equal to the kinetic energy when it hits the ground. Use $E_{\mathrm{k}}=E_{\mathrm{g}}, E_{\mathrm{k}}=\frac{1}{2} m v^{2}$ and $E_{\mathrm{g}}=m g \Delta y$ to obtain $\frac{1}{2} m v^{2}=m g \Delta y$. Isolate $v$ in this equation: $v=\sqrt{2 g \Delta y}$. Calculate the speed, and then use $\vec{p}=m \vec{v}$ to calculate the momentum.
Solution: Calculate the speed when the phone hits the ground.

$$
\begin{aligned}
v & =\sqrt{2 g \Delta y} \\
& =\sqrt{2\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(0.74 \mathrm{~m})} \\
v & =3.81 \mathrm{~m} / \mathrm{s}(\text { one extra digit carried })
\end{aligned}
$$

Calculate the momentum.

$$
\begin{aligned}
\vec{p} & =m \vec{v} \\
& =(0.225 \mathrm{~kg})(3.81 \mathrm{~m} / \mathrm{s} \text { [down] }) \\
\vec{p} & =0.86 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}[\text { down }]
\end{aligned}
$$

Statement: The cellphone's momentum at the moment of impact is $0.86 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}$ [down].
(b) The surface of the impact makes no difference to the momentum. The speed and mass do not change, so the momentum does not change.
10. (a) Given: $m=0.25 \mathrm{~kg} ; \Delta y=1.5 \mathrm{~m} ; \vec{v}_{\mathrm{f}}=4.0 \mathrm{~m} / \mathrm{s}$ [up]

Required: impulse of the ball, $\Delta \vec{p}$
Analysis: The potential energy as you drop the ball is equal to the kinetic energy when it hits the ground. Use $E_{\mathrm{k}}=E_{\mathrm{g}}, E_{\mathrm{k}}=\frac{1}{2} m v^{2}$ and $E_{\mathrm{g}}=m g \Delta y$ to obtain $\frac{1}{2} m v^{2}=m g \Delta y$. Isolate $v$ in this equation: $v=\sqrt{2 g \Delta y}$. Calculate the velocity, and then use $\Delta \vec{p}=m\left(\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}\right)$ to calculate the impulse imparted by the floor to the ball.

Solution: Calculate the velocity of the ball just before it hits the ground.

$$
\begin{aligned}
v & =\sqrt{2 g \Delta y} \\
& =\sqrt{2\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(1.5 \mathrm{~m})}
\end{aligned}
$$

$$
v=5.42 \mathrm{~m} / \mathrm{s} \text { (one extra digit carried) }
$$

The velocity is downward, so $\vec{v}_{\mathrm{i}}=5.42 \mathrm{~m} / \mathrm{s}$ [down].

$$
\begin{aligned}
\Delta \vec{p} & =m\left(\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}\right) \\
& =(0.25 \mathrm{~kg})(4.0 \mathrm{~m} / \mathrm{s}[\text { up }]-5.42 \mathrm{~m} / \mathrm{s}[\text { down }]) \\
& =(0.25 \mathrm{~kg})(4.0 \mathrm{~m} / \mathrm{s}[\text { up }]+5.42 \mathrm{~m} / \mathrm{s}[\text { up }]) \\
& =2.36 \mathrm{~N} \cdot \mathrm{~s}[\text { up }](\text { one extra digit carried }) \\
\Delta \vec{p} & =2.4 \mathrm{~N} \cdot \mathrm{~s}[\text { up }]
\end{aligned}
$$

Statement: The impulse imparted by the floor to the ball is $2.4 \mathrm{~N} \cdot \mathrm{~s}$ [up].
(b) Given: $\Delta \vec{p}=2.36 \mathrm{~N} \cdot \mathrm{~s}$ [up]; $\vec{F}=18 \mathrm{~N}$ [up]

Required: $\Delta t$
Analysis: $\vec{F} \Delta t=\Delta \vec{p}$, so $\Delta t=\frac{\Delta \vec{p}}{\vec{F}}$.
Solution: $\Delta t=\frac{\Delta \vec{p}}{\vec{F}}$

$$
=\frac{2.36 \not X \cdot \mathrm{~s}}{18 \not X}
$$

$$
\Delta t=0.13 \mathrm{~s}
$$

Statement: The ball is in contact with the floor for 0.13 s .
11. (a) Given: $m=0.030 \mathrm{~kg} ; \vec{v}=88 \mathrm{~m} / \mathrm{s}$ [forward]

Required: $\Delta \vec{p}$
Analysis: The initial velocity of the arrow is $0 \mathrm{~m} / \mathrm{s}$. Use $\Delta \vec{p}=m \Delta \vec{v}$.
Solution: $\Delta \vec{p}=m \Delta \vec{v}$

$$
\begin{aligned}
& =(0.030 \mathrm{~kg})(88 \mathrm{~m} / \mathrm{s} \text { [forward] }) \\
& =2.64 \mathrm{~N} \cdot \mathrm{~s} \text { [forward] (one extra digit carried) } \\
\Delta \vec{p} & =2.6 \mathrm{~N} \cdot \mathrm{~s} \text { [forward] }
\end{aligned}
$$

Statement: The impulse is $2.6 \mathrm{~N} \cdot \mathrm{~s}$ [forward].
(b) Given: $\Delta t=0.015 \mathrm{~s} ; \Delta \vec{p}=2.64 \mathrm{~N} \cdot \mathrm{~s}$ [forward]

Required: $\vec{F}$
Analysis: $\vec{F} \Delta t=\Delta \vec{p}$, so $\vec{F}=\frac{\Delta \vec{p}}{\Delta t}$.

Solution: $\vec{F}=\frac{\Delta \vec{p}}{\Delta t}$

$$
\begin{aligned}
& =\frac{2.64 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s} \text { [forward] }}{0.015 \mathrm{~s}} \\
\vec{F} & =1.8 \times 10^{2} \mathrm{~N}[\text { forward }]
\end{aligned}
$$

Statement: The average force of the bowstring on the arrow is $1.8 \times 10^{2} \mathrm{~N}$ [forward].
12. (a) Given: $\vec{v}_{\mathrm{i}}=63 \mathrm{~m} / \mathrm{s}[\mathrm{W}] ; m=0.057 \mathrm{~kg} ; \vec{v}_{\mathrm{f}}=41 \mathrm{~m} / \mathrm{s}[\mathrm{E}]$

Required: $\Delta \vec{p}$
Analysis: $\Delta \vec{p}=m\left(\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}\right)$
Solution: $\Delta \vec{p}=m\left(\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}\right)$

$$
\begin{aligned}
& =(0.057 \mathrm{~kg})(41 \mathrm{~m} / \mathrm{s}[\mathrm{E}]-63 \mathrm{~m} / \mathrm{s}[\mathrm{~W}]) \\
& =(0.057 \mathrm{~kg})(41 \mathrm{~m} / \mathrm{s}[\mathrm{E}]+63 \mathrm{~m} / \mathrm{s}[\mathrm{E}]) \\
\Delta \vec{p} & =5.93 \mathrm{~N} \cdot \mathrm{~s}[\mathrm{E}](\text { one extra digit carried })
\end{aligned}
$$

Statement: The magnitude of the impulse is $5.9 \mathrm{~N} \cdot \mathrm{~s}$.
(b) Given: $\Delta \vec{p}=5.93 \mathrm{~N} \cdot \mathrm{~s}[\mathrm{E}] ; \Delta t=0.023 \mathrm{~s}$

Required: $\vec{F}$
Analysis: $\vec{F} \Delta t=\Delta \vec{p}$, so $\vec{F}=\frac{\Delta \vec{p}}{\Delta t}$.
Solution: $\vec{F}=\frac{\Delta \vec{p}}{\Delta t}$

$$
\begin{aligned}
& =\frac{5.93 \mathrm{~N} \cdot \ngtr[\mathrm{E}]}{0.023 \not x} \\
\vec{F} & =2.6 \times 10^{2} \mathrm{~N}[\mathrm{E}]
\end{aligned}
$$

Statement: The average force is $2.6 \times 10^{2} \mathrm{~N}[\mathrm{E}]$.

