

## Section 4.7: Springs and Conservation of Energy

### Tutorial 1 Practice, page 205

1. If the ramp is not frictionless, some of the kinetic energy of the block will be transformed to thermal energy by friction as the block slides down the ramp. Thus, the block will have less kinetic energy to compress the spring, and the amount of compression will be less.

2. **Given:**  $m = 3.5 \text{ kg}$ ;  $\Delta y = 2.7 \text{ m}$ ;  $\Delta x = 26 \text{ cm} = 0.26 \text{ m}$

**Required:**  $k$

**Analysis:**  $E_g = mg\Delta y$ ;  $E_e = \frac{1}{2}k(\Delta x)^2$

Since energy is conserved, the change in potential energy of the mass must equal the change in elastic potential energy when the spring is compressed.

**Solution:** If we choose the bottom of the ramp to be the  $y = 0$  reference point, the mass will have no gravitational potential energy at the bottom of the ramp. The initial gravitational potential energy has transformed into kinetic energy. When the spring is fully compressed, the kinetic energy has transformed into elastic potential energy. Therefore, the spring's initial gravitational potential energy must equal its final elastic potential energy.

$$E_e = E_g$$

$$\frac{1}{2}k(\Delta x)^2 = mg\Delta y$$

$$\begin{aligned}k &= \frac{2mg\Delta y}{(\Delta x)^2} \\ &= \frac{2(3.5 \text{ kg})(9.8 \text{ m/s}^2)(2.7 \text{ m})}{(0.26 \text{ m})^2}\end{aligned}$$

$$k = 2700 \text{ N/m}$$

**Statement:** The spring constant is  $2.7 \times 10^3 \text{ N/m}$ .

3. **Given:**  $m = 43 \text{ kg}$ ;  $k = 3.7 \text{ kN/m} = 3700 \text{ N/m}$ ;  $\Delta x = 37 \text{ cm} = 0.37 \text{ m}$

**Required:**  $\Delta y$

**Analysis:**  $\Delta E_g = mg\Delta y$ ;  $E_e = \frac{1}{2}k(\Delta x)^2$

**Solution:** Choose the lowest point of the bounce as the  $y = 0$  reference point. At the maximum height,  $\Delta y$ , all of the elastic potential energy has converted to gravitational potential energy.

$$\begin{aligned} \Delta E_g &= E_e \\ mg\Delta y &= \frac{1}{2}k(\Delta x)^2 \\ \Delta y &= \frac{k(\Delta x)^2}{2mg} \\ &= \frac{(3700 \text{ N/m})(0.37 \text{ m})^2}{2(43 \text{ kg})(9.8 \text{ m/s}^2)} \\ &= \frac{\left(3700 \cancel{\text{ kg}} \cdot \frac{\text{m}}{\cancel{\text{s}^2}}\right)(0.37 \text{ m})}{2(43 \cancel{\text{ kg}})(9.8 \cancel{\text{ m/s}^2})} \\ \Delta y &= 0.60 \text{ m} \end{aligned}$$

**Statement:** The maximum height he reaches on the following jump is 0.60 m above the compressed point.

**4. Given:**  $m = 0.35 \text{ kg}$ ;  $h = 2.6 \text{ m}$ ;  $\Delta x = 0.14$

**Required:**  $k$

**Analysis:**  $E_g = mg\Delta y$ ;  $E_e = \frac{1}{2}k(\Delta x)^2$

The initial gravitational potential energy of the branch is equal to the final elastic potential energy of the trampoline at its lowest point.

**Solution:** Choose the lowest point of the trampoline as the  $y = 0$  reference point. At this point, all of the gravitational potential energy has transformed to elastic potential energy. Since the lowest point represents  $y = 0$ , the change in  $y$  is the height above the trampoline surface, 2.6 m, plus the maximum compression of the trampoline, 0.14 m. Therefore,  $\Delta y = 2.6 \text{ m} + 0.14 \text{ m} = 2.74 \text{ m}$ .

$$\begin{aligned} E_e &= E_g \\ \frac{1}{2}k(\Delta x)^2 &= mg\Delta y \\ k &= \frac{2mg\Delta y}{(\Delta x)^2} \\ &= \frac{2(0.35 \text{ kg})(9.8 \text{ m/s}^2)(2.74 \text{ m})}{(0.14 \text{ m})^2} \end{aligned}$$

$$k = 960 \text{ N/m}$$

**Statement:** The spring constant is 960 N/m.

**5. (a) Given:**  $m = 4.0 \text{ kg}$ ;  $\Delta y = 0.308 \text{ m}$

**Required:**  $v$

**Analysis:**  $E_g = mg\Delta y$ ;  $E_k = \frac{1}{2}mv^2$

When the mass is doubled at the top of the ramp, it is at rest, so it has only gravitational potential energy. At the bottom of the ramp, on the horizontal segment, all of the gravitational potential energy has transformed to kinetic energy.

**Solution:** Let the bottom of the ramp be the  $y = 0$  reference point. Therefore, the gravitational potential energy at the top of the ramp is equal to the kinetic energy along the horizontal part of the ramp.

$$E_k = E_g$$

$$\frac{1}{2}mv^2 = mg\Delta y$$

$$v = \sqrt{2g\Delta y}$$
$$= \sqrt{2(9.8 \text{ m/s}^2)(0.308 \text{ m})}$$

$$v = 2.5 \text{ m/s}$$

**Statement:** The speed of the block as it returns along the horizontal surface is 2.5 m/s.

**(b)** No, the block does not have the same kinetic energy as before along the horizontal surface. The kinetic energy is equal to the gravitational potential energy, which is greater than before.

**(c) Given:**  $m_d = 2m$

**Required:**  $\Delta x_d$

**Analysis:** The elastic potential energy of the spring at its greatest compression is equal to the gravitational potential energy at the top of the ramp.

**Solution:** Block in Sample Problem 3: Block with Mass Doubled

$$mg\Delta y = \frac{1}{2}k(\Delta x)^2$$

$$\Delta x = \sqrt{\frac{2mg\Delta y}{k}}$$

$$m_d g \Delta y = \frac{1}{2} k (\Delta x_d)^2$$

$$2mg\Delta y = \frac{1}{2} k (\Delta x_d)^2$$

$$\Delta x_d = \sqrt{\frac{4mg\Delta y}{k}}$$

$$= \sqrt{\frac{2(2mg\Delta y)}{k}}$$

$$= \sqrt{2} \sqrt{\frac{2mg\Delta y}{k}}$$

$$\Delta x_d = \sqrt{2}(\Delta x)$$

Since the gravitational potential energy is doubled, the compression increases, but it is not doubled. It increases by a factor of  $\sqrt{2}$ . So the new value of  $\Delta x$  is  $\sqrt{2}(0.22 \text{ m})$  or 0.31 m.

**Statement:** The new value of  $\Delta x$  is 31 cm.

**(d) Given:**  $\mu_k = 0.15$ ;  $\Delta y = 0.308 \text{ m}$ ;  $\Delta d = 0.62 \text{ m}$ ;  $k = 250 \text{ N / m}$

**Required:**  $\Delta x$

**Analysis:**  $\vec{F}_f = \mu \vec{F}_N = \mu mg$ ;  $W_f = F_f \Delta d$ ;  $E_k = E_g - W_f$ ;  $E_e = E_k$

**Solution:** Let the bottom of the ramp be the  $y = 0$  reference point.

$$\vec{F}_f = \mu mg$$

$$= 0.15(4.0 \text{ kg})(9.8 \text{ m/s}^2)$$

$$\vec{F}_f = 5.88 \text{ N (one extra digit carried)}$$

$$W_f = F_f \Delta d$$

$$= (5.88 \text{ N})(0.62 \text{ m})$$

$$W_f = 3.65 \text{ J (one extra digit carried)}$$

$$E_k = E_g - W_f$$

$$= mg\Delta y - W_f$$

$$= (4.0 \text{ kg})(9.8 \text{ m/s}^2)(0.308 \text{ m}) - 3.65 \text{ J}$$

$$E_k = 8.42 \text{ J (one extra digit carried)}$$

$$E_e = E_k$$

$$\frac{1}{2}k(\Delta x)^2 = 8.42 \text{ J}$$

$$\begin{aligned}\Delta x &= \sqrt{\frac{2(8.42 \text{ J})}{k}} \\ &= \sqrt{\frac{2(8.42 \text{ N} \cdot \text{m})}{250 \text{ N/m}}}\end{aligned}$$

$$\Delta x = 0.26 \text{ m}$$

**Statement:** The new value of the compression is 0.26 m.

### Research This: Perpetual Motion Machines, page 206

Answers may vary. Sample answers:

**A.** I chose a metronome. A spring is wound tight, and as it unwinds, the elastic potential energy is converted to kinetic energy, forcing the metronome wand to swing. As the wand swings to its highest point, kinetic energy transforms to potential energy. As the wand swings down again, the gravitational potential energy converts back to kinetic energy. The spring contributes kinetic energy at the bottom of each swing, until the spring is fully unwound. Eventually the metronome stops due to air resistance.

**B.** The design of the machine has been improved over time by creating quartz metronomes, electronic metronomes, computer metronomes, and even metronome apps for smart phones.

**C.** The improvements have been the results of all three developments: new materials; new technology; and new scientific discoveries.

### Section 4.7 Questions, page 208

1. The total mechanical energy of the system increases. Energy has been added by the person outside the system of the mass and spring.

2. **Given:**  $k = 520 \text{ N/m}$ ;  $m = 4.5 \text{ kg}$ ;  $\Delta x = 0.35 \text{ m}$

**Required:**  $v$

**Analysis:** When the spring is compressed, but the mass is at rest, the mass has only elastic potential energy. As the spring is released, the elastic potential energy transforms to kinetic energy. When the mass is no longer touching the spring, all of the energy is kinetic.

$$E_e = \frac{1}{2}k(\Delta x)^2; E_k = \frac{1}{2}mv^2$$

**Solution:** The kinetic energy when the mass leaves the spring is equal to the elastic potential energy when the spring is at its maximum compression.

$$\begin{aligned}
 E_k &= E_e \\
 \frac{1}{2}mv^2 &= \frac{1}{2}k(\Delta x)^2 \\
 v &= \sqrt{\frac{k(\Delta x)^2}{m}} \\
 &= \sqrt{\frac{(520 \text{ N/m})(0.35 \text{ m})^2}{4.5 \text{ kg}}} \\
 v &= 3.8 \text{ m/s}
 \end{aligned}$$

**Statement:** The speed of the mass when it leaves the spring is 3.8 m/s [away from the spring].

**3. (a) Given:**  $k = 5.2 \times 10^2 \text{ N/m}$ ;  $\Delta x = 5.2 \text{ cm} = 0.052 \text{ m}$

**Required:**  $E_e$

**Analysis:**  $E_e = \frac{1}{2}k(\Delta x)^2$

**Solution:**  $E_e = \frac{1}{2}k(\Delta x)^2$   
 $= \frac{1}{2}(5.2 \times 10^2 \text{ N/m})(0.052 \text{ m})^2$

$E_e = 0.703 \text{ J}$  (one extra digit carried)

**Statement:** The elastic potential energy of the compressed spring is 0.70 J.

**(b) Given:**  $E_e = 0.703 \text{ J}$ ;  $m = 8.4 \text{ g} = 0.0084 \text{ kg}$ ;  $\Delta x = 5.2 \text{ cm} = 0.052 \text{ m}$

**Required:**  $v$

**Analysis:**  $E_g = mg\Delta y$ ;  $E_k = \frac{1}{2}mv^2$

All of the elastic potential energy of the compressed spring has transformed to gravitational potential energy and kinetic energy as the pilot ejects.

**Solution:** The kinetic energy of the pilot as it ejects and the additional gravitational potential energy is equal to the elastic potential energy of the compressed spring.

$$\begin{aligned}
 E_g + E_k &= E_e \\
 mg\Delta x + \frac{1}{2}mv^2 &= E_e \\
 v &= \sqrt{\frac{2E_e - 2mg\Delta x}{m}} \\
 &= \sqrt{\frac{2(0.703 \text{ J}) - 2(0.0084 \text{ kg})(9.8 \text{ m/s}^2)(0.052 \text{ m})}{0.0084 \text{ kg}}} \\
 &= \sqrt{\frac{2(0.660 \text{ 194 } \cancel{\text{kg}} \cdot \text{m}^2/\text{s}^2)}{0.0084 \cancel{\text{kg}}}} \\
 v &= 13 \text{ m/s}
 \end{aligned}$$

**Statement:** The speed of the pilot as it ejects upward from the airplane is 13 m/s above the launch point of the compressed spring.

**(c) Given:**  $m = 8.4 \text{ g} = 0.0084 \text{ kg}$ ;  $k = 5.2 \times 10^2 \text{ N/m}$ ;  $\Delta x = 5.2 \text{ cm} = 0.052 \text{ m}$

**Required:**  $\Delta y$

**Analysis:** Let the point where the pilot ejects be the  $y = 0$  reference point. All of the elastic potential energy of the compressed spring has transformed to gravitational potential energy at the pilot's maximum height.

$$E_e = \frac{1}{2}k(\Delta x)^2; E_g = mg\Delta y$$

**Solution:** The elastic potential energy of the compressed spring is equal to the gravitational potential energy at the pilot's maximum height.

$$E_g = E_e$$

$$mg\Delta y = \frac{1}{2}k(\Delta x)^2$$

$$\Delta y = \frac{k(\Delta x)^2}{2mg}$$

$$= \frac{\left(5.2 \times 10^2 \cancel{\text{ kg}} \cdot \frac{\text{ m}}{\cancel{\text{ s}^2}}\right)(0.052 \text{ m})^2}{2(0.0084 \cancel{\text{ kg}})(9.8 \cancel{\text{ m/s}^2})}$$

$$\Delta y = 8.5 \text{ m}$$

**Statement:** The maximum height that the pilot will reach is 8.5 m above the launch point of the compressed spring.

**4. Given:**  $k = 1.2 \times 10^2 \text{ N/m}$ ;  $m = 82 \text{ g} = 0.082 \text{ kg}$ ;  $\Delta y = 3.4 \text{ cm} = 0.034 \text{ m}$

**Required:**  $\Delta x$

**Analysis:** The elastic potential energy of the spring transforms to kinetic energy and then to gravitational potential energy as it comes to rest at the top of the ramp.

$$E_e = \frac{1}{2}k(\Delta x)^2; E_g = mg\Delta y$$

**Solution:** Let the bottom of the ramp represent the  $y = 0$  reference point. The elastic potential energy of the spring is equal to the gravitational potential energy at the top of the ramp.

$$E_e = E_g$$

$$\frac{1}{2}k(\Delta x)^2 = mg\Delta y$$

$$\Delta x = \sqrt{\frac{2mg\Delta y}{k}}$$

$$= \sqrt{\frac{2(0.082 \text{ kg})(9.8 \text{ m/s}^2)(0.034 \text{ m})}{1.2 \times 10^2 \text{ N/m}}}$$

$$\Delta x = 0.021 \text{ m}$$

**Statement:** The distance of the spring's compression is 0.021 m.

**5. Given:**  $m = 75 \text{ kg}$ ;  $k = 6.5 \text{ N/m}$

**Required:**  $v$

**Analysis:** Let the  $y = 0$  reference point be 19 m below the platform. Since the unstretched bungee cord is 11 m long, and the cord is stretched 19 m below the platform,  $\Delta x = 19 \text{ m} - 11 \text{ m} = 8 \text{ m}$ . The gravitational potential energy is transformed to elastic potential energy and kinetic energy at this point.

$$E_g = mg\Delta y; E_k = \frac{1}{2}mv^2; E_e = \frac{1}{2}k(\Delta x)^2$$

**Solution:** The gravitational potential energy at the platform is equal to the sum of the kinetic energy and the elastic potential energy 19 m below the platform.

$$\begin{aligned}E_g &= E_k + E_e \\mg\Delta y &= \frac{1}{2}mv^2 + \frac{1}{2}k(\Delta x)^2 \\ \frac{1}{2}mv^2 &= mg\Delta y - \frac{1}{2}k(\Delta x)^2 \\ v &= \sqrt{\frac{2}{m}\left(mg\Delta y - \frac{1}{2}k(\Delta x)^2\right)} \\ &= \sqrt{\frac{2}{(75 \text{ kg})}\left[(75 \text{ kg})(9.8 \text{ m/s}^2)(19 \text{ m}) - \frac{1}{2}(65.5 \text{ N/m})(8.0 \text{ m})^2\right]} \\ v &= 18 \text{ m/s}\end{aligned}$$

**Statement:** The speed of the bungee jumper at 19 m below the bridge is 18 m/s.

**6. Given:**  $k = 5.0 \text{ N/m}$ ;  $m = 0.25 \text{ kg}$ ;  $\Delta x = 14 \text{ cm} = 0.14 \text{ m}$

**Required:**  $h_{\text{max}}$ ;  $v_{\text{max}}$ ;  $a_{\text{max}}$

**Analysis:** Let the  $y = 0$  reference point be the rest position of the spring. In simple harmonic motion, the maximum height is the opposite of the lowest point.

The maximum velocity occurs as the box passes through the rest position. At this point,

there is only kinetic energy. Thus,  $\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv^2$ .

The maximum acceleration occurs at the maximum height. At this point, the spring force is equal to the applied force, so  $k\Delta x = ma$ .

**Solution:** The maximum height is 14 cm [above rest position].

For the maximum velocity,

$$\begin{aligned}\frac{1}{2}k(\Delta x)^2 &= \frac{1}{2}mv^2 \\ v &= \sqrt{\frac{k(\Delta x)^2}{m}} \\ &= \sqrt{\frac{(5.0 \text{ N/m})(0.14 \text{ m})^2}{0.25 \text{ kg}}} \\ v &= 0.63 \text{ m/s}\end{aligned}$$



For the maximum acceleration,

$$k\Delta x = ma$$

$$a = \frac{k\Delta x}{m}$$
$$= \frac{(5.0 \text{ N/m})(0.14 \text{ m})}{0.25 \text{ kg}}$$

$$a = 2.8 \text{ m/s}^2$$

**Statement:** The maximum height is 14 cm [above rest position]. The maximum speed is 0.63 m/s. The maximum acceleration is 2.8 m/s<sup>2</sup> [toward rest position].

**7. Given:**  $m = 0.22 \text{ kg}$ ;  $k = 280 \text{ N/m}$ ;  $\Delta x = 11 \text{ cm} = 0.11 \text{ m}$

**Required:**  $h$

**Analysis:** Let the  $y = 0$  reference point be the fully compressed position of the spring. When the block is dropped, all the energy is in the form of gravitational potential energy. At the full compression of the spring, all the energy has transformed to elastic potential energy.

$$E_g = mg\Delta y; E_e = \frac{1}{2}k(\Delta x)^2$$

**Solution:** The gravitational potential energy as the block is dropped is equal to the elastic potential energy at full compression of the spring.

$$E_g = E_e$$

$$mg\Delta y = \frac{1}{2}k(\Delta x)^2$$

$$\Delta y = \frac{k(\Delta x)^2}{2mg}$$

$$= \frac{(280 \text{ N/m})(0.11 \text{ m})^2}{2(0.22 \text{ kg})(9.8 \text{ m/s}^2)}$$

$$\Delta y = 0.79 \text{ m}$$

This height represents the total distance from where the block was dropped to the lowest point of the spring, so subtract the maximum compression of the spring, 0.11 m, to get the height from which the block was dropped.

$$h = \Delta y - \Delta x$$

$$= 0.79 \text{ m} - 0.11 \text{ m}$$

$$h = 0.68 \text{ m}$$

**Statement:** The block was dropped from a height of 0.68 m.

**8. (a) Given:**  $m = 1.0 \text{ kg}$ ;  $v = 1.0 \text{ m/s}$ ;  $k = 1000.0 \text{ N/m}$

**Required:**  $\Delta x$

**Analysis:** The kinetic energy of the block transforms fully to elastic potential energy at the maximum compression of the spring, because the block is at rest at this point.

$$E_k = \frac{1}{2}mv^2; E_e = \frac{1}{2}k(\Delta x)^2$$

**Solution:** The kinetic energy of the block before it hits the spring is equal to the elastic potential energy of the spring at its maximum compression.

$$E_e = E_k$$

$$\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv^2$$

$$\Delta x = \sqrt{\frac{mv^2}{k}}$$

$$= \sqrt{\frac{(1.0 \text{ kg})(1.0 \text{ m/s})^2}{1000.0 \text{ N/m}}}$$

$$\Delta x = 0.032 \text{ m}$$

**Statement:** The maximum compression of the spring is 0.032 m.

(b) The block will travel to the maximum compression of the spring, 0.032 m before coming to rest.

**9. (a) Given:**  $m = 6.0 \text{ kg}$ ;  $v = 3.0 \text{ m/s}$ ;  $k = 1250 \text{ N/m}$

**Required:**  $\Delta x$

**Analysis:** The kinetic energy of the block transforms fully to elastic potential energy at the maximum compression of the spring, because the block is at rest at this point.

$$E_k = \frac{1}{2}mv^2; E_e = \frac{1}{2}k(\Delta x)^2$$

**Solution:**  $E_e = E_k$

$$\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv^2$$

$$\Delta x = \sqrt{\frac{mv^2}{k}}$$

$$= \sqrt{\frac{(6.0 \text{ kg})(3.0 \text{ m/s})^2}{1250 \text{ N/m}}}$$

$$\Delta x = 0.208 \text{ m (one extra digit carried)}$$

**Statement:** The maximum distance the spring is compressed is 0.21 m.

(b) **Given:**  $\Delta x = 14 \text{ cm} = 0.14 \text{ m}$ ;  $m = 6.0 \text{ kg}$ ;  $v_i = 3.0 \text{ m/s}$ ;  $k = 1250 \text{ N/m}$

**Required:**  $v_f$ ;  $a$

**Analysis:** Some of the kinetic energy of the block transforms to elastic potential energy as the spring compresses. So the initial kinetic energy transforms to final kinetic energy and elastic potential energy.

$$E_k = \frac{1}{2}mv^2; E_e = \frac{1}{2}k(\Delta x)^2$$

For the acceleration, the only force acting on the block is the spring force, which is equal to  $k\Delta x$  away from the spring.

$$\vec{F} = m\vec{a}$$

**Solution:** The initial kinetic energy is equal to the sum of the final kinetic energy and the elastic potential energy.

$$\begin{aligned}
 E_{ki} &= E_{kf} + E_e \\
 \frac{1}{2}mv_i^2 &= \frac{1}{2}mv_f^2 + \frac{1}{2}k(\Delta x)^2 \\
 mv_f^2 &= mv_i^2 - k(\Delta x)^2 \\
 v_f &= \sqrt{\frac{mv_i^2 - k(\Delta x)^2}{m}} \\
 &= \sqrt{\frac{(6.0 \text{ kg})(3.0 \text{ m/s})^2 - (1250 \text{ N/m})(0.14 \text{ m})^2}{6.0 \text{ kg}}} \\
 v_f &= 2.2 \text{ m/s}
 \end{aligned}$$

For the acceleration:

$$\begin{aligned}
 \vec{F} &= m\vec{a} \\
 k\Delta x \text{ [away from the spring]} &= m\vec{a} \\
 \vec{a} &= \frac{k\Delta x \text{ [away from the spring]}}{m} \\
 &= \frac{(1250 \text{ N/m})(0.14 \text{ m}) \text{ [away from the spring]}}{6.0 \text{ kg}} \\
 \vec{a} &= 29 \text{ m/s}^2 \text{ [away from the spring]}
 \end{aligned}$$

**Statement:** When the spring is compressed, the speed of the block is 2.2 m/s, and the acceleration is 29 m/s<sup>2</sup> [away from the spring].

**10. Given:**  $k = 440 \text{ N/m}$ ;  $\Delta x = 45 \text{ cm} = 0.45 \text{ m}$ ;  $m = 57 \text{ g} = 0.057 \text{ kg}$ ;  $d_y = 1.2 \text{ m}$

**Required:**  $d_x$ , the horizontal distance

**Analysis:** Find the speed of the ball as it leaves the machine, and then use projectile motion equations to determine when it will hit the ground.

Let the  $y = 0$  reference point be the height at which the ball leaves the machine. Thus, there is no gravitational potential energy at this point. The elastic potential energy of the spring when it is at its maximum compression transforms to kinetic energy when the ball leaves the machine.

$$E_k = \frac{1}{2}mv^2; \quad E_e = \frac{1}{2}k(\Delta x)^2$$

Once  $v$  has been determined, use projectile motion equations to determine the distance,  $d_x$ , the ball travels. Since the ball is projected horizontally, the initial launch angle is  $0^\circ$ ,

and there is no vertical component of velocity:  $v_y = 0$ . Use the equation  $d_y = v_y t - \frac{1}{2}gt^2$

to determine  $t$ , and then use the equation  $d_x = v_x t$  to determine  $d_x$ .

**Solution:** The elastic potential energy of the spring when it is at its maximum compression is equal to the kinetic energy when the ball leaves the machine.

$$E_k = E_e$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k(\Delta x)^2$$

$$v = \sqrt{\frac{k(\Delta x)^2}{m}}$$

$$= \sqrt{\frac{(440 \text{ N/m})(0.45 \text{ cm})^2}{0.057 \text{ g}}}$$

$$v = 39.5 \text{ m/s (one extra digit carried)}$$

Now, use the projectile motion equations.

$$d_y = v_y t - \frac{1}{2}gt^2$$

$$d_y = -\frac{1}{2}gt^2$$

$$t = \sqrt{\frac{-2d_y}{g}}$$

$$= \sqrt{\frac{-2(-1.2 \text{ m})}{9.8 \text{ m/s}^2}}$$

$$t = 0.495 \text{ s (one extra digit carried)}$$

$$d_x = v_x t$$

$$= (39.5 \text{ m/s})(0.495 \text{ s})$$

$$d_x = 2.0 \times 10^1 \text{ m}$$

**Statement:** The horizontal distance that the tennis ball can travel before hitting the ground is  $2.0 \times 10^1 \text{ m}$ .