Section 4.5: The Law of Conservation of Energy

Mini Investigation: Various Energies of a Roller Coaster, page 185

Answers may vary. Sample answers:

A. The total energy graph is a straight line because total energy is conserved and it is constant.

B. At any height, *h*, the sum of the energy values on the potential energy and kinetic energy curves is equal to the value of the total energy at that height.

C. It is necessary to know the height at point A because it represents the potential energy before the roller coaster starts. This is equal to the total mechanical energy. If the height of point A were greater, the change in slopes of the potential and kinetic energy graphs would be greater, and the total mechanical energy graph would be higher, but still horizontal.

D. If the mass of the roller coaster car were greater, the total mechanical energy would be greater, and the change in slopes for the graphs for potential and kinetic energy would be greater.

Tutorial 1 Practice, page 187

1. (a) Given: $m = 0.43\text{ kg}$; $\Delta y = 18\text{ m}$; $g = 9.8\text{ m/s}^2$; $v_\text{i} = 7.4\text{ m/s}$

Required: v_f

Analysis: The total energy at the top of the hill is equal to the total energy at the bottom of the hill. At the top of the hill, the total energy is the gravitational potential energy, $mg\Delta y$, plus the kinetic energy, $\frac{1}{2}mv_\text{i}^2$. At the bottom of the hill, the total energy is equal to the kinetic energy, $\frac{1}{2}mv_\text{f}^2$, since there is no gravitational potential energy at $\Delta y = 0$.

Solution:
$$\cancel{m}g\Delta y + \frac{1}{2}\cancel{m}v_\text{i}^2 = \frac{1}{2}\cancel{m}v_\text{f}^2$$
$$2g\Delta y + v_\text{i}^2 = v_\text{f}^2$$
$$v_\text{f} = \sqrt{2g\Delta y + v_\text{i}^2}$$
$$= \sqrt{2(9.8\text{ m/s}^2)(18\text{ m}) + (7.4\text{ m/s})^2}$$
$$v_\text{f} = 2.0 \times 10^1\text{ m/s}$$

Statement: The ball's speed at the bottom of the hill is 2.0×10^1 m/s.

(b) Given: $m = 0.43\text{ kg}$; $\Delta y = 18\text{ m}$; $v_\text{i} = 4.2\text{ m/s}$; $g = 9.8\text{ m/s}^2$

Required: v_f

Analysis: The total energy when the ball is kicked up the hill is equal to the total energy when the ball reaches the bottom of the hill. The energy at any time when the ball is on its way up or down the hill does not matter. When the ball is kicked, $E_\text{T} = mg\Delta y + \frac{1}{2}mv_\text{i}^2$.

At the bottom of the hill, $E_\text{T} = \frac{1}{2}mv_\text{f}^2$.

Solution: $\cancel{m}g\Delta y + \frac{1}{2}\cancel{m}v_{\mathrm{i}}^2 = \frac{1}{2}\cancel{m}v_{\mathrm{f}}^2$

$$2g\Delta y + v_{\mathrm{i}}^2 = v_{\mathrm{f}}^2$$

$$v_{\mathrm{f}} = \sqrt{2g\Delta y + v_{\mathrm{i}}^2}$$

$$= \sqrt{2(9.8\ \text{m/s}^2)(18\ \text{m}) + (4.2\ \text{m/s})^2}$$

$$v_{\mathrm{f}} = 19\ \text{m/s}$$

Statement: The ball's speed as it reaches the bottom of the hill is 19 m/s.

2. (a) Given: $m = 0.057$ kg; $\Delta y = 1.8$ m; $g = 9.8\ \text{m/s}^2$; $v_{\mathrm{f}} = 0\ \text{m/s}$

Required: v_{i}

Analysis: Let the player's hand be the $y = 0$ reference point. The total energy when the ball is released is all kinetic energy, $\frac{1}{2}mv_{\mathrm{i}}^2$. The total energy at the highest point of the ball is all gravitational potential energy, $mg\Delta y$. Thus, $\frac{1}{2}mv_{\mathrm{i}}^2 = mg\Delta y$.

Solution: $\frac{1}{2}\cancel{m}v_{\mathrm{i}}^2 = \cancel{m}g\Delta y$

$$v_{\mathrm{i}}^2 = 2g\Delta y$$

$$v_{\mathrm{i}} = \sqrt{2g\Delta y}$$

$$= \sqrt{2(9.8\ \text{m/s}^2)(1.8\ \text{m})}$$

$$v_{\mathrm{i}} = 5.9\ \text{m/s}$$

Statement: The speed of the ball as it leaves the player's hand is 5.9 m/s.

(b) Given: $m = 0.057$ kg; $v_{\mathrm{i}\,2} = v_{\mathrm{i}}$; $g = 9.8\ \text{m/s}^2$

Required: $\Delta y_2 : \Delta y_1$

Analysis: $\frac{1}{2}mv_{\mathrm{i}}^2 = mg\Delta y$

Solution:

$$\frac{1}{2}\cancel{m}v_{\mathrm{i}\,1}^2 = \cancel{m}g\Delta y_1 \qquad \frac{1}{2}\cancel{m}v_{\mathrm{i}\,2}^2 = \cancel{m}g\Delta y_2$$

$$\frac{1}{2}v_{\mathrm{i}\,1}^2 = g\Delta y_1 \qquad \frac{1}{2}v_{\mathrm{i}\,2}^2 = g\Delta y_2$$

$$\Delta y_1 = \frac{v_{\mathrm{i}\,1}^2}{2g} \qquad \Delta y_2 = \frac{v_{\mathrm{i}\,2}^2}{2g}$$

$$\frac{\Delta y_2}{\Delta y_1} = \frac{\frac{v_{i\,2}^2}{2g}}{\frac{v_{i\,1}^2}{2g}}$$

$$= \frac{v_{i\,2}^2}{\cancel{2g}} \times \frac{\cancel{2g}}{v_{i\,1}^2}$$

$$= \frac{v_{i\,2}^2}{v_{i\,1}^2}$$

$$= \frac{\left(\frac{1}{4} v_{i\,1}\right)^2}{v_{i\,1}^2}$$

$$= \frac{\frac{1}{16} \cancel{v_{i\,1}^2}}{\cancel{v_{i\,1}^2}}$$

$$\frac{\Delta y_2}{\Delta y_1} = \frac{1}{16}$$

Statement: The ratio of the maximum rise of the ball after leaving the player's hand to the maximum rise in (a) is 1:16.

Tutorial 2 Practice, page 190

1. (a) Given: $v = 1.4\ \text{m/s}; \Delta y = 5.0\ \text{m}; m = 65\ \text{kg}; g = 9.8\ \text{m/s}^2$

Required: P

Analysis: The work done to get to the top of the ladder is equal to the gravitational potential energy at the top of the ladder, $W = mg\Delta y$. The time, t, taken to get to the top of the ladder is the distance, Δy, divided by the speed, v. $P = \frac{W}{t}$

Solution: $t = \frac{\Delta y}{v}$

$$= \frac{5.0\ \text{m}}{1.4\ \text{m/s}}$$

$t = 3.57$ s (one extra digit carried)

$$P = \frac{W}{t}$$
$$= \frac{mg\Delta y}{t}$$
$$= \frac{(65\text{ kg})(9.8\text{ m/s}^2)(5.0\text{ m})}{3.57\text{ s}}$$
$$P = 890\text{ W}$$

Statement: The firefighter's power output while climbing the ladder is 890 W.

(b) From (a), it takes the firefighter 3.6 s to climb the ladder.

2. Given: $v_{f2} = 2v_{f1}$; $t_2 = t_1$; $v_i = 0\text{ m/s}$; $m_2 = m_1$

Required: ratio of power needed, $P_2 : P_1$

Analysis: We are told that the Grand Prix car accelerates to twice the speed of the car in Sample 1, which can be expressed as $v_{f2} = 2v_{f1}$. We are also told that the Grand Prix car accelerates to this speed in the same amount of time as the car in Sample 1, which is stated as 7.7 s. We will assume that the two cars are equal in mass, at 1.1×10^3 kg.

Solution: $P_1 = \frac{mv_{f1}^2}{2t}$ $\quad P_2 = \frac{mv_{f2}^2}{2t}$

$$\frac{P_2}{P_1} = \frac{\frac{\cancel{m}v_{f2}^2}{\cancel{2t}}}{\frac{\cancel{m}v_{f1}^2}{\cancel{2t}}}$$
$$= \frac{v_{f2}^2}{v_{f1}^2}$$
$$= \frac{(2v_{f1})^2}{v_{f1}^2}$$
$$= \frac{4\cancel{v_{f1}^2}}{\cancel{v_{f1}^2}}$$
$$\frac{P_2}{P_1} = \frac{4}{1}$$

Statement: The ratio of the power needed by the Grand Prix car to the power needed by the car in Sample Problem 1 is 4:1.

3. Given: $\Delta d = 190\text{ m}; t = 4\text{ min }50\text{ s } = 290\text{ s}; m = 62\text{ kg}; g = 9.8\text{ m/s}^2$

Required: P

Analysis: $P = \frac{W}{t}$; $W = mg\Delta y$

Solution: $P = \dfrac{W}{t}$

$$= \frac{mg\Delta y}{t}$$

$$= \frac{(62\ \text{kg})(9.8\ \text{m/s}^2)(190\ \text{m})}{290\ \text{s}}$$

$$P = 0.40\ \text{kW}$$

Statement: The racer's average power output during the race is 0.40 kW.

Section 4.5 Questions, page 191

1. (a) Given: $v_\text{i} = 11\ \text{m/s};\ g = 9.8\ \text{m/s}^2$

Required: maximum height that the ball will reach, Δy

Analysis: The kinetic energy when the child tosses the ball is equal to the gravitational potential energy at the ball's maximum height, expressed as $\dfrac{1}{2}mv_\text{i}^2 = mg\Delta y$.

Solution: $\dfrac{1}{2}\cancel{m}v_\text{i}^2 = \cancel{m}g\Delta y$

$$\frac{1}{2}v_\text{i}^2 = g\Delta y$$

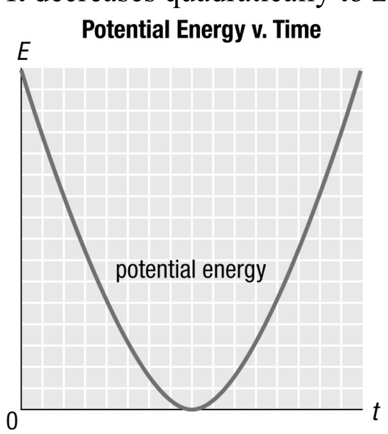
$$\Delta y = \frac{v_\text{i}^2}{2g}$$

$$= \frac{(11\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)}$$

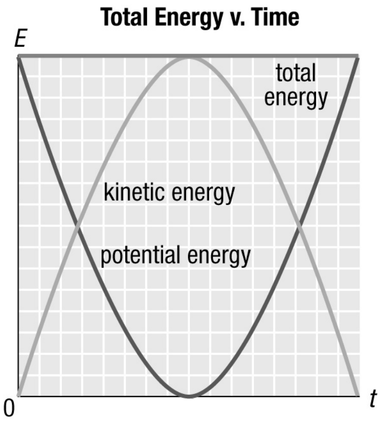
$$\Delta y = 6.2\ \text{m}$$

Statement: The maximum height that the ball will reach is 6.2 m.

(b) As the ball leaves the child's hand, the gravitational potential energy is zero. It increases quadratically to its maximum when the ball reaches its maximum height. It decreases quadratically to zero as the ball returns to the level of the child's hand.



(c) The graph has this shape because, as the ball leaves the child's hand, the kinetic energy is at its maximum. It decreases quadratically to zero when the ball reaches its maximum height. It increases quadratically to its maximum as the ball returns to the level of the child's hand. The total energy is conserved, so it is a constant horizontal line equal to the maximum kinetic energy or potential energy.



2. Answers may vary. Sample answers:
(a) The kinetic energy is the greatest just before the apple hits the ground.
(b) The gravitational potential energy is the greatest as the apple leaves the branch.
3. (a) The law of conservation of energy states that energy can neither be created nor destroyed in an isolated system; it can only change form. Assuming the puck and surface form an isolated system, the energy of the hockey puck is conserved. The kinetic energy of the puck is transformed to thermal energy by friction.
(b) The initial kinetic energy is transformed to thermal energy by friction as the puck slows down to a stop.
4. (a) Given: $m = 110$ kg; $\Delta y = 210$ m; $g = 9.8$ m / s^2
Required: W
Analysis: The work done by gravity is equal to the gravitational potential energy at the top of the hill, expressed as $E_g = mg\Delta y$.

Solution:
$$\begin{aligned} W &= E_g \\ &= mg\Delta y \\ &= (110 \text{ kg})(9.8 \text{ m/s}^2)(210 \text{ m}) \\ W &= 2.3\times 10^5 \text{ J} \end{aligned}$$

Statement: The work done by gravity on the skier is 2.3×10^5 J.
(b) Given: $m = 110$ kg; $\Delta y = 210$ m; $g = 9.8$ m / s^2; $v_i = 0$ m / s
Required: v_f
Analysis: Because the skier has no initial velocity, the total energy at the top of the hill is all potential energy. The total energy at the bottom of the hill is all kinetic energy. The total energy at the top of the hill is equal to the total energy at the bottom of the hill, or $mg\Delta y = \frac{1}{2}mv_f^2$. Solve for v_f and substitute.

Solution: $\cancel{m}g\Delta y = \frac{1}{2}\cancel{m}v_{\text{f}}^2$

$$2g\Delta y = v_{\text{f}}^2$$
$$v_{\text{f}} = \sqrt{2g\Delta y}$$
$$= \sqrt{2(9.8\text{ m/s}^2)(210\text{ m})}$$
$$v_{\text{f}} = 64\text{ m/s}$$

Statement: The skier's speed when he reaches the bottom of the hill is 64 m/s.

5. Given: $m = 62\text{ kg};\ v_{\text{i}} = 8.1\text{ m / s};\ g = 9.8\text{ m / s}^2;\ \Delta y = 3.7\text{ m}$

Required: v_{f}

Analysis: The total energy as the snowboarder leaves the ledge is the sum of her gravitational potential energy, $mg\Delta y$, and her kinetic energy, $\frac{1}{2}mv_{\text{i}}^2$. The total energy when she lands is all kinetic energy, $\frac{1}{2}mv_{\text{f}}^2$, if we take $\Delta y = 0$ at the landing point. Thus, $mg\Delta y + \frac{1}{2}mv_i^2 = \frac{1}{2}mv_{\text{f}}^2$. Solve for v_{f} and substitute the given values.

Solution: $\cancel{m}g\Delta y + \frac{1}{2}\cancel{m}v_{\text{i}}^2 = \frac{1}{2}\cancel{m}v_{\text{f}}^2$

$$2g\Delta y + v_{\text{i}}^2 = v_{\text{f}}^2$$
$$v_{\text{f}} = \sqrt{2g\Delta y + v_{\text{i}}^2}$$
$$= \sqrt{2(9.8\text{ m/s}^2)(3.7\text{ m}) + (8.1\text{ m/s})^2}$$
$$v_{\text{f}} = 12\text{ m/s}$$

Statement: The snowboarder's speed at the moment she hits the ground is 12 m/s.

6. Given: $\Delta y = 3.5\text{ m};\ \theta = 40°;\ g = 9.8\text{ m / s}^2$

Required: the speed in the *y*-direction

Analysis: $mg\Delta y = \frac{1}{2}mv^2$

The energy equations give the speed in the direction along the jump, so we need to use components to solve for the vertical velocity: $v_y = \frac{v}{\sin\theta}$.

Solution: $\cancel{m}g\Delta y = \frac{1}{2}\cancel{m}v^2$

$$2g\Delta y = v^2$$
$$v = \sqrt{2g\Delta y}$$
$$= \sqrt{2(9.8\text{ m/s}^2)(3.5\text{ m})}$$
$$v = 8.28\text{ m/s (one extra digit carried)}$$

$$v_y = \frac{v}{\sin\theta}$$
$$= \frac{8.28 \text{ m/s}}{\sin 40°}$$
$$v_y = 13 \text{ m/s}$$

Statement: The dolphin's minimum speed is 13 m/s.

7. (a) Yes, the mechanical energy of the roller coaster is conserved because there is no friction.

(b) Given: $m = 640$ kg; $v_\text{i} = 0; \Delta y_\text{A} = 30.0$ m; $g = 9.8 \text{ m/s}^2$

Required: E_T, the total mechanical energy

Analysis: Because the car starts from rest, its total mechanical energy is equal to its potential energy at point A: $E_\text{T} = mg\Delta y_\text{A}$.

Solution: $E_\text{T} = mg\Delta y_\text{A}$
$$= (640 \text{ kg})(9.8 \text{ m/s}^2)(30.0 \text{ m})$$
$$E_\text{T} = 1.9 \times 10^5 \text{ J}$$

Statement: The total mechanical energy at point A is 1.9×10^5 J.

(c) The total mechanical energy is conserved, so it is the same at point B as it is at point A: 1.9×10^5 J.

(d) Given: $m = 640$ kg; $\Delta y_\text{B} = 15.0$ m; $g = 9.8 \text{ m/s}^2$; $\Delta y_\text{A} = 30.0$ m

Required: v_B; v_C

Analysis for v_B: The total mechanical energy at point B is the sum of the kinetic energy, $\frac{1}{2}mv_\text{B}^2$, and the potential energy, $mg\Delta y_\text{B}$. The total mechanical energy is equal to the potential energy at point A: $mg\Delta y_\text{A}$. Therefore, $\frac{1}{2}mv_\text{B}^2 + mg\Delta y_\text{B} = mg\Delta y_\text{A}$.

Solution for v_B: $\frac{1}{2}\cancel{m}v_\text{B}^2 + \cancel{m}g\Delta y_\text{B} = \cancel{m}g\Delta y_\text{A}$
$$v_\text{B}^2 + 2g\Delta y_\text{B} = 2g\Delta y_\text{A}$$
$$v_\text{B}^2 = 2g\Delta y_\text{A} - 2g\Delta y_\text{B}$$
$$v_\text{B} = \sqrt{2g(\Delta y_\text{A} - \Delta y_\text{B})}$$
$$= \sqrt{2(9.8 \text{ m/s}^2)(30.0 \text{ m} - 15 \text{ m})}$$
$$v_\text{B} = 17 \text{ m/s}$$

Statement for v_B: The speed of the car when it reaches point B is 17 m/s.

Analysis for v_C: The total mechanical energy at point C is all kinetic energy, $\frac{1}{2}mv_\text{C}^2$.

The total mechanical energy is equal to the potential energy at point A: $mg\Delta y_\text{A}$.

Thus, $\frac{1}{2}mv_\text{C}^2 = mg\Delta y_\text{A}$.

Solution for v_C: $\frac{1}{2}\cancel{m}v_C^2 = \cancel{m}g\Delta y_A$

$$v_C^2 = 2g\Delta y_A$$

$$v_C = \sqrt{2g(\Delta y_A)}$$

$$= \sqrt{2(9.8\text{ m/s}^2)(30.0\text{ m})}$$

$$v_C = 24\text{ m/s}$$

Statement for v_C: The speed of the car when it reaches point C is 24 m/s.

(e) Given: $\Delta y_A = 30.0$ m; $\Delta y_B = 15.0$ m; $g = 9.8\text{ m/s}^2$

Required: v_B; v_C

Analysis: The total energy at A is equal to the total energy at B and at C. At A, the total energy consists of kinetic energy and gravitational potential energy. At B, the total energy also consists of kinetic energy and gravitational potential energy. At C, the total energy consists of kinetic energy only: $E_k = \frac{1}{2}mv^2; E_g = mg\Delta y$

Solution:

$$E_{TB} = E_{TA}$$

$$\cancel{m}g\Delta y_B + \frac{1}{2}\cancel{m}v_B^2 = \cancel{m}g\Delta y_A + \frac{1}{2}\cancel{m}v_A^2$$

$$\frac{1}{2}v_B^2 = g\Delta y_A + \frac{1}{2}v_A^2 - g\Delta y_B$$

$$v_B^2 = 2g\Delta y_A + v_A^2 - 2g\Delta y_B$$

$$v_B = \sqrt{2g\Delta y_A + v_A^2 - 2g\Delta y_B}$$

$$= \sqrt{2(9.8\text{ m/s}^2)(30.0\text{ m}) + (12\text{ m/s})^2 - 2(9.8\text{ m/s}^2)(15.0\text{ m})}$$

$$v_B = 21\text{ m/s}$$

$$E_{TC} = E_{TA}$$

$$\frac{1}{2}\cancel{m}v_C^2 = \cancel{m}g\Delta y_A + \frac{1}{2}\cancel{m}v_A^2$$

$$\frac{1}{2}v_C^2 = g\Delta y_A + \frac{1}{2}v_A^2$$

$$v_C^2 = 2g\Delta y_A + v_A^2$$

$$v_C = \sqrt{2g\Delta y_A + v_A^2}$$

$$= \sqrt{2(9.8\text{ m/s}^2)(30.0\text{ m}) + (12\text{ m/s})^2}$$

$$v_C = 27\text{ m/s}$$

Statement: The speed at B is 21 m/s, and the speed at C is 27 m/s.

8. Given: $m = 52\text{ kg};\ t = 24\text{ s};\ \Delta y = 18\text{ m};\ g = 9.8\text{ m/s}^2$
Required: P

Analysis: $P = \dfrac{W}{t};\ W = mg\Delta y$

Solution:
$$
\begin{aligned}
P &= \frac{W}{t} \\
&= \frac{mg\Delta y}{t} \\
&= \frac{(52\text{ kg})(9.8\text{ m/s}^2)(18\text{ m})}{24\text{ s}} \\
P &= 380\text{ W}
\end{aligned}
$$

Statement: The power the woman exerts is 3.8×10^2 W.