Section 4.3: Gravitational Potential Energy

Tutorial 1 Practice, page 180

1. Given: m = 0.02 kg; Δd = 8.0 m; g = 9.8 m/s^2

Required: ΔE_g

Analysis: Use the gravitational potential energy equation, $\Delta E_g = mg\Delta y$. Let the y = 0 reference point be the ground.

Solution:
$$\begin{aligned}\Delta E_g &= mg\Delta y\\ &= (0.02\ \text{kg})(9.8\ \text{m/s}^2)(8.0\ \text{m})\\ \Delta E_g &= 1.6\ \text{J}\end{aligned}$$

Statement: The change in potential energy between the branch and the ground is 1.6 J.

2. Given: $\Delta E_g = 660$ J; $\Delta y = 2.2$ m; $g = 9.8\ \text{m/s}^2$

Required: m

Analysis: Rearrange the gravitational potential energy equation, $\Delta E_g = mg\Delta y$, to solve for m.

Solution:
$$\begin{aligned}\Delta E_g &= mg\Delta y\\ m &= \frac{\Delta E_g}{g\Delta y}\\ &= \frac{660\ \text{J}}{(9.8\ \text{m/s}^2)(2.2\ \text{m})}\\ m &= 31\ \text{kg}\end{aligned}$$

Statement: The mass of the loaded barbell is 31 kg.

3. Given: height of each book, $h = 3.6\ \text{cm} = 0.036\ \text{m}$; number of extra books = 2

Required: W

Analysis: $\Delta E_g = mg\Delta y$

The 11th book is moved 10 × 3.6 cm and the 12th book is moved 11 × 3.6 cm.

Solution:
$$\begin{aligned}\Delta E_g &= mg\Delta y\\ &= (1.6\ \text{kg})(9.8\ \text{m/s}^2)[10(0.036\ \text{m}) + 11(0.036\ \text{m})]\\ \Delta E_g &= 12\ \text{J}\end{aligned}$$

Statement: The work done by the student to stack the two extra books is 12 J.

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1. (a) Given: $m = 2.5$ kg; $g = 9.8\ \text{m/s}^2$; $\Delta y = 2.0$ m

Required: E_k

Analysis: The kinetic energy of the wood when it hits the table is equal to the potential energy of the wood before it falls. $E_k = \Delta E_g = mg\Delta y$

Solution: $E_k = E_g$

$$= mg\Delta y$$

$$= (2.5 \text{ kg})(9.8 \text{ m/s}^2)(2.0 \text{ m})$$

$$E_k = 49 \text{ J}$$

Statement: The kinetic energy of the piece of wood as it hits the table is 49 J.

(b) Given: $m = 2.5 \text{ kg}; E_k = 49 \text{ J}$

Required: v

Analysis: $E_k = \frac{1}{2}mv^2$; solve for v

Solution: $E_k = \frac{1}{2}mv^2$

$$\frac{2E_k}{m} = v^2$$

$$v = \sqrt{\frac{2E_k}{m}}$$

$$= \sqrt{\frac{2(49 \text{ J})}{2.5 \text{ kg}}}$$

$$v = 6.3 \text{ m/s}$$

Statement: The speed of the wood as it hits the table is 6.3 m/s.

2. Given: $g = 9.8 \text{ m/s}^2; m = 5.0 \text{ kg}; \Delta y = 553 \text{ m}$

Required: E_g

Analysis: $E_g = mg\Delta y$

Solution: $E_g = mg\Delta y$

$$= (5.0 \text{ kg})(9.8 \text{ m/s}^2)(553 \text{ m})$$

$$E_g = 2.7 \times 10^4 \text{ J}$$

Statement: The gravitational potential energy of the Canada goose is 2.7×10^4 J.

3. (a) Given: $m = 175 \text{ g} = 0.175 \text{ kg}$; $\Delta y = 1.05 \text{ m}$; $g = -9.8 \text{ m/s}^2$

Required: gravitational potential energy of the puck, E_g

Analysis: $E_g = mg\Delta y$

Solution: $E_g = mg\Delta y$

$$= (0.175 \text{ kg})(9.8 \text{ m/s}^2)(1.05 \text{ m})$$

$$E_g = 1.8 \text{ J}$$

Statement: The gravitational potential energy of the puck is 1.8 J.

(b) Given: E_g = 1.8 J

Required: change in gravitational potential energy of puck, ΔE_g

Analysis: Since the gravitational potential energy of the puck when it hits the ice is equal to 0, it is expressed as $\Delta E_g = -E_g$.

Solution: $\Delta E_g = -E_g$

$\Delta E_g = -1.8 \text{ J}$

Statement: The change in gravitational potential energy of the puck is −1.8 J.

(c) Given: $\Delta E_g = -1.8 \text{ J}$

Required: work done by the puck, W

Analysis: Since work and energy use the same units, W is equal to the change in gravitational potential energy of the puck.

Solution: $W = \Delta E_g$

$W = -1.8 \text{ J}$

Statement: The work done on the puck by gravity is 1.8 J.

4. The total work done is 0 J. The work done by gravity while you lift the cat is exactly balanced by the work done by gravity while you lower the cat.

5. Given: $\Delta y = -5.4 \text{ m}$; $\Delta E_g = -3.1 \times 10^3 \text{ J}$; $g = 9.8 \text{ m/s}^2$

Required: m

Analysis: $E_g = mg\Delta y$

Solution: At the mat, the pole vaulter's gravitational potential energy is 0 J. Thus, $\Delta E_g = -E_g$.

$$\Delta E_g = -E_g$$

$$\Delta E_g = -mg\Delta y$$

$$m = -\frac{\Delta E_g}{g\Delta y}$$

$$= \frac{-3.1 \times 10^3 \text{ J}}{(9.8 \text{ m/s}^2)(-5.4 \text{ m})}$$

$$m = 59 \text{ kg}$$

Statement: The pole vaulter's mass is 59 kg.

6. Given: $m = 0.46 \text{ kg}; \Delta E_g = 155 \text{ J}; g = 9.8 \text{ m/s}^2$

Required: Δy

Analysis: $\Delta E_g = mg\Delta y$

Solution: $\Delta E_g = mg\Delta y$

$$\Delta y = \frac{\Delta E_g}{mg}$$

$$= \frac{155 \text{ J}}{(0.46 \text{ kg})(9.8 \text{ m/s}^2)}$$

$$\Delta y = 34 \text{ m}$$

Statement: The maximum height of the ball above the tee is 34 m.

7. Given: $m = 59$ kg; $\Delta y = 1.3$ km $= 1300$ m; $\theta = 14°$; $g = 9.8$ m/s^2

Required: E_g

Analysis: $\sin\theta = \dfrac{\Delta y}{d}$; $E_g = mg\Delta y$

$$\Delta y = d \sin\theta$$

Solution: $\Delta y = d \sin\theta$

$$= (1300 \text{ m}) \sin 14°$$

$$\Delta y = 314.498 \text{ m (four extra digits carried)}$$

$$E_g = mg\Delta y$$

$$= (59 \text{ kg})(9.8 \text{ m/s}^2)(314.498 \text{ m})$$

$$E_g = 1.8 \times 10^5 \text{ J}$$

Statement: The snowboarder's gravitational potential energy is 1.8×10^5 J.

8. (a) The work done on the first box is zero, because it doesn't move. The second box is lifted a height of Δy, the third is lifted a height of $2\Delta y$, the fourth is lifted a height of $3\Delta y$, and so on until the *N*th box, which is lifted a height of $(N - 1)\Delta y$. Therefore, the work done to raise the last box to the top of the pile is expressed as $mg(N - 1)\Delta y$.

(b) As in Sample Problem 3 of Tutorial 1 on page 180, the gravitational potential energy of the stack of boxes is the sum of the gravitational potential energies of the individual boxes.

$$\Delta E_g = mg[0 \times \Delta y + 1 \times \Delta y + 2 \times \Delta y + 3 \times \Delta y + \cdots + (N-1)\Delta y]$$

$$\Delta E_g = mg\Delta y[0 + 1 + 2 + 3 + \cdots + (N-1)]$$

The sum of an arithmetic sequence is given by the formula $S_n = \dfrac{n}{2}[2a_1 + (n-1)d]$. To find the sum of the sequence $0 + 1 + 2 + 3 + \ldots + (N-1)$, substitute $n = N$, $a_1 = 0$, and $d = 1$.

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$S_N = \frac{N}{2}[2(0) + (N-1)(1)]$$

$$S_N = \frac{N(N-1)}{2}$$

Therefore, the gravitational potential energy that is stored in the entire pile is expressed as:

$$\Delta E_g = \frac{mg\Delta y N(N-1)}{2}$$

$$\Delta E_g = mgN(N-1)\frac{\Delta y}{2}$$

9. Answers may vary. Sample answer:

Given: $E_c = 1.3\times10^8$ J; 3.79 L = 1 gal; $g = 9.8$ m / s^2 ; 30 students in class; average mass of each student = 70 kg

Required: Δy

Analysis: Find the chemical potential energy, $E_{c\,1}$, in 1 L of gas by dividing E_c by 3.79. Find the total mass of the class of students. Solve the equation $E_g = mg\Delta y$ for Δy.

Solution:
$$E_{c\,1} = \frac{E_c}{3.79}$$
$$= \frac{1.3\times10^8\ \text{J}}{3.79}$$
$$E_{c\,1} = 3.43\times10^7\ \text{J (one extra digit carried)}$$

There are 3.43×10^7 J of chemical potential energy in 1 L of gas. Assuming that there are 30 students in the class, each with an average mass of 70 kg, this equals a total mass of $30\times70\ \text{kg} = 2100\ \text{kg}$.

Therefore,
$$E_g = mg\Delta y$$
$$\Delta y = \frac{E_g}{mg}$$
$$= \frac{3.43\times10^7\ \text{J}}{(2100\ \text{kg})(9.8\ \text{m/s}^2)}$$
$$\Delta y = 1700\ \text{m}$$

Statement: The chemical potential energy of the gas could lift the students 1700 m if it could all be converted to gravitational potential energy.

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