Section 4.2: Kinetic Energy and the Work–Energy Theorem

Tutorial 1 Practice, page 172

1. (a) Given: v_i; $v_\text{f} = 2v_\text{i}$; $E_\text{k i}$

Required: $E_\text{k f}$

Analysis:

$$E_\text{k} = \frac{1}{2}mv^2$$

Solution:

$$E_\text{k i} = \frac{1}{2}mv_\text{i}^2$$

$$\begin{aligned} E_\text{k f} &= \frac{1}{2}mv_\text{f}^2 \\ &= \frac{1}{2}m(2v_\text{i})^2 \\ &= \frac{1}{2}m(4v_\text{i}^2) \\ &= 2mv_\text{i}^2 \\ E_\text{k f} &= 4E_\text{k i} \end{aligned}$$

Statement: A car's kinetic energy increases by a factor of 4 when the car's speed doubles.

(b) Given: v_i; $v_\text{f} = 3v_\text{i}$

Required: $E_\text{k f}$

Analysis:

$$E_\text{k} = \frac{1}{2}mv^2$$

Solution:

$$E_\text{k i} = \frac{1}{2}mv_\text{i}^2$$

$$\begin{aligned} E_\text{k f} &= \frac{1}{2}mv_\text{f}^2 \\ &= \frac{1}{2}m(3v_\text{i})^2 \\ &= \frac{1}{2}m(9v_\text{i}^2) \\ &= 9\left(\frac{1}{2}mv_\text{i}^2\right) \\ E_\text{k f} &= 9E_\text{k i} \end{aligned}$$

Statement: A car's kinetic energy increases by a factor of 9 when the car's speed triples.

(c) Given: v_i; $v_f = 1.26v_i$; $E_{k\,i}$

Required: $E_{k\,f}$

Analysis:

$$E_k = \frac{1}{2}mv^2$$

Solution:

$$\begin{aligned} E_{k\,i} &= \frac{1}{2}mv_i^2 \\ E_{k\,f} &= \frac{1}{2}mv_f^2 \\ &= \frac{1}{2}m(1.26v_i)^2 \\ &= \frac{1}{2}m(1.6v_i^2) \\ &= 1.6\left(\frac{1}{2}mv_i^2\right) \\ E_{k\,f} &= 1.6E_{k\,i} \end{aligned}$$

Statement: A car's kinetic energy increases by a factor of 1.6 when the car's speed increases by 26 %.

2. Given: m = 8.0 kg; v = 2.0 m/s

Required: E_k

Analysis:

$$E_k = \frac{1}{2}mv^2$$

Solution:

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(8.0\text{ kg})(2.0\text{ m/s})^2 \\ &= 16\text{ kg}\cdot\text{m}^2/\text{s}^2 \\ E_k &= 16\text{ J} \end{aligned}$$

Statement: The bowling ball's kinetic energy is 16 J.

3. Given: v = 15 km/h; E_k = 0.83 J

Required: m

Analysis:

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ 2E_k &= mv^2 \\ m &= \frac{2E_k}{v^2} \end{aligned}$$

The speed must be converted to metres per second.

Solution: Convert 15 km/h to metres per second.

$$\left(15\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\right)\left(\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}\right) = 4.17\ \text{m/s (one extra digit carried)}$$

$$\begin{aligned} m &= \frac{2E_\text{k}}{v^2} \\ &= \frac{2(0.83\ \text{J})}{(4.17\ \text{m/s})^2} \\ &= 0.095\ \frac{\text{kg}\cdot \cancel{\text{m}^2/\text{s}^2}}{\cancel{\text{m}^2/\text{s}^2}} \\ m &= 0.095\ \text{kg} \end{aligned}$$

Statement: The bird's mass is 0.095 kg.

Tutorial 2 Practice, page 175

1. Given: $m = 22\ \text{g} = 0.022\ \text{kg};\ v_\text{i} = 0;\ v_\text{f} = 220\ \text{km / h}$

Required: W

Analysis: $W = \Delta E_\text{k}$

Solution: Convert the speed to metres per second.

$$v_\text{f} = \left(220\ \frac{\text{km}}{\text{h}}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)$$

$$v_\text{f} = 61.1\ \text{m/s (one extra digit carried)}$$

$$\begin{aligned} W &= \Delta E_\text{k} \\ &= E_\text{k f} - E_\text{k i} \\ &= \frac{1}{2}mv_\text{f}^2 - \frac{1}{2}mv_\text{i}^2 \\ &= \frac{1}{2}mv_\text{f}^2 \\ &= \frac{1}{2}(0.022\ \text{g})(61.1\ \text{m/s})^2 \\ W &= 41\ \text{J} \end{aligned}$$

Statement: The work done on the arrow by the bowstring is 41 J.

2. Given: $m = 3.8\times 10^4\text{kg};\ v_\text{i} = 1.5\times 10^4\text{m / s};\ F = 2.2\times 10^5\text{N};\ \Delta d = 2.8\times 10^6\text{m}$

Required: v_f

Analysis: $E_\text{kf} = E_\text{ki} + \Delta E_\text{k}; E_\text{ki} = \frac{1}{2}mv_\text{i}^2; E_\text{kf} = \frac{1}{2}mv_\text{f}^2;\ \Delta E_\text{k} = F\Delta d$

Solution: $E_{kf} = E_{ki} + \Delta E_k$

$$= \frac{1}{2}mv_i^2 + F\Delta d$$

$$= \frac{1}{2}(3.8\times10^4 \text{ kg})(1.5\times10^4 \text{ m/s})^2 + (2.2\times10^5 \text{ N})$$

$$= 4.275\times10^{12} \text{ J} + 6.16\times10^{11} \text{ J}$$

$$E_{kf} = 4.891\times10^{12} \text{ J (two extra digits carried)}$$

$$E_{kf} = \frac{1}{2}mv_f^2$$

$$\frac{2}{m}E_{kf} = v_f^2$$

$$v_f = \sqrt{\frac{2}{m}E_{kf}}$$

$$= \sqrt{\frac{2}{3.8\times10^4 \text{ kg}}(4.891\times10^{12} \text{ J})}$$

$$v_f = 1.6\times10^4 \text{ m/s}$$

Statement: The final speed of the probe is 1.6×10^4 m/s.

3. Given: $v_i = 2.2 \text{ m/s};\ v_f = 0;\ F_f = 15 \text{ N}$

Required: m

Analysis: Friction opposes the motion of the disc, so θ is 180°, and $\cos\theta$ is –1. The work done by friction is

$$W = F\Delta d\cos\theta$$

$$= (15 \text{ N})(12 \text{ m})\cos 180°$$

$$W = -180 \text{ J}$$

Solution: The work–energy theorem tells us that the change in kinetic energy will equal the work done, or –180 J. The final velocity is zero, so the final kinetic energy is zero.

$$W = E_{kf} - E_{ki}$$

$$W = 0 - \frac{1}{2}mv_{ki}^2$$

$$\frac{1}{2}mv_{ki}^2 = -W$$

$$m = -\frac{2W}{v_{ki}^2}$$

$$= -\left(\frac{2(-180 \text{ J})}{(2.2 \text{ m/s})^2}\right)$$

$$m = 74 \text{ kg}$$

Statement: The skater's mass is 74 kg.

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1. Answers may vary. Sample answer:
Yes, it is possible. For example, an elephant can have a mass of up to 12 000 kg. Its slow walking speed might be 2 m/s. Thus, its kinetic energy is

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(12\,000 \text{ kg})(2 \text{ m/s})^2 \\ &= 24\,000 \text{ J} \end{aligned}$$

A small cheetah might have a mass of 35 kg, and its top running speed is about 120 km/h, which is about 33 m/s. Its kinetic energy is

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(35 \text{ kg})(33 \text{ m/s})^2 \\ &= 19\,000 \text{ J} \end{aligned}$$

Yes, it is possible that an elephant walking slowly could have greater kinetic energy than the cheetah.

2. (a) Given: m_c = 5.0 kg; m_m = 0.035 kg; $E_{k\,c} = 100E_{k\,m}$; mouse running at a constant speed, v_m

Required: Will the cat catch up with the mouse?

Analysis: Determine the cat's speed relative to the mouse's speed using the fact that the cat's kinetic energy is 100 times the mouse's kinetic energy.

Solution:

$$\begin{aligned} E_{k\,c} &= 100E_{k\,m} \\ \frac{1}{2}m_c v_c^2 &= 100\left(\frac{1}{2}m_m v_m^2\right) \\ m_c v_c^2 &= 100 m_m v_m^2 \\ (5.0 \cancel{\text{ kg}})v_c^2 &= 100(0.035 \cancel{\text{ kg}})v_m^2 \\ 5.0v_c^2 &= 3.5v_m^2 \\ v_c^2 &= 0.70v_m^2 \\ v_c &= \sqrt{0.70}v_m \\ v_c &= 0.84v_m \end{aligned}$$

Statement: Since the cat's speed is less than the mouse's speed, the cat will never catch up to the mouse.

(b) Analysis: The cat's speed must be greater than the mouse's speed for the cat to catch up. Let the factor by which the cat's kinetic energy is greater than the mouse's kinetic energy be x. Then $E_{kc} = xE_{km}$.

Solution:

$$E_{\text{k c}} = xE_{\text{k m}}$$

$$\frac{1}{2}m_{\text{c}}v_{\text{c}}^2 = x\left(\frac{1}{2}m_{\text{m}}v_{\text{m}}^2\right)$$

$$m_{\text{c}}v_{\text{c}}^2 = xm_{\text{m}}v_{\text{m}}^2$$

$$(5.0\ \cancel{\text{kg}})v_{\text{c}}^2 = x(0.035\ \cancel{\text{kg}})v_{\text{m}}^2$$

$$5.0v_{\text{c}}^2 = 0.035xv_{\text{m}}^2$$

$$v_{\text{c}}^2 = 0.0070xv_{\text{m}}^2$$

$$v_{\text{c}} = \sqrt{0.0070x}(v_{\text{m}})$$

For the cat's speed to be greater than the mouse's speed, $\sqrt{0.0070x} > 1$.

$$\sqrt{0.0070x} > 1$$

$$0.0070x > 1$$

$$x > 140$$

Statement: For the cat to catch up with the mouse, its kinetic energy must be greater than 140 times the kinetic energy of the mouse.

3. Given: m = 1.5×10^3 kg; v_{i} = 11 m/s; v_{f} = 25 m/s; Δd = 0.20 km

Required: W

Analysis: $W = \Delta E_k$

Solution: $W = \Delta E_{\text{k}}$

$$= E_{\text{k f}} - E_{\text{k i}}$$

$$= \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_{\text{i}}^2$$

$$= \frac{1}{2}m(v_{\text{f}}^2 - v_{\text{i}}^2)$$

$$= \frac{1}{2}(1.5\times10^3\ \text{kg})((25\ \text{m/s})^2 - (11\ \text{m/s})^2)$$

$$W = 380\,000\ \text{J}$$

Statement: The work done on the car is 3.8×10^5 J.

4. Given: $m = 9.1\times10^3\text{kg}$; $v_{\text{i}} = 98\ \text{km / h}$; $v_{\text{f}} = 27\ \text{km / h}$

Required: W

Analysis: $W = \Delta E_{\text{k}}$

Solution: Convert the speeds to metres per second.

$$v_{\text{i}} = \left(98\frac{\cancel{\text{km}}}{\cancel{\text{h}}}\right)\left(\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\right)$$

$v_{\text{i}} = 27.2$ m/s (one extra digit carried)

$$v_f = \left(27\frac{\cancel{\text{km}}}{\cancel{\text{h}}}\right)\left(\frac{1000\text{ m}}{1\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\text{ s}}\right)$$

$$v_f = 7.5\text{ m/s}$$

$$\begin{aligned}W &= \Delta E_k \\ &= E_{k\,f} - E_{k\,i} \\ &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\ &= \frac{1}{2}m(v_f^2 - v_i^2) \\ &= \frac{1}{2}(9.1\times10^3\text{ kg})((7.5\text{ m/s})^2 - (27.2\text{ m/s})^2) \\ W &= -3\,100\,000\text{ J}\end{aligned}$$

Statement: The work done on the truck is -3.1×10^6 J.

5. Given: $E_{k1} = E_{k2}$; $v_2 = 2.5v_1$

Required: $m_1 : m_2$

Analysis: $E_k = \frac{1}{2}mv^2$;

Solution: Substitute the given values into the equation $E_{k1} = E_{k2}$.

$$\begin{aligned}E_{k\,1} &= E_{k\,2} \\ \cancel{\frac{1}{2}}m_1v_1^2 &= \cancel{\frac{1}{2}}m_2v_2^2 \\ m_1v_1^2 &= m_2(2.5v_1)^2 \\ m_1\cancel{v_1^2} &= 6.25m_2\cancel{v_1^2} \\ m_1 &= 6.2m_2 \\ \frac{m_1}{m_2} &= 6.2 \\ m_1 : m_2 &= 6.2:1\end{aligned}$$

Statement: The ratio of the slower mass to the faster mass is 6.2 : 1.

6. Given: $m_c = 1.2 \times 10^3$ kg; $m_s = 4.1 \times 10^3$ kg; $v_c = 99$ km/h; $E_{k\,c} = E_{k\,s}$

Required: v_s

Analysis: Convert the speed to kilometres per hour. Substitute $E_k = \frac{1}{2}mv^2$ into the equation $E_{kc} = E_{ks}$ and solve for v_s.

Solution: $v_c = \left(99\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$

$v_c = 27.5$ m/s (one extra digit carried)

$$E_{k\,c} = E_{k\,s}$$

$$\frac{1}{2}m_c v_c^2 = \frac{1}{2}m_s v_s^2$$

$$v_s^2 = \frac{m_c v_c^2}{m_s}$$

$$v_s^2 = \frac{(1.2\times10^3\text{ kg})(27.5\text{ m/s})^2}{(4.1\times10^3\text{ kg})}$$

$$v_s = \sqrt{\frac{(1.2\times10^3\text{ kg})(27.5\text{ m/s})^2}{(4.1\times10^3\text{ kg})}}$$

$= 14.9$ m/s (one extra digit carried)

Convert the speed back to kilometres per hour.

$$v_s = \left(14.9\frac{\text{m}}{\text{s}}\right)\left(\frac{1\text{ km}}{1000\text{ m}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)$$

$v_s = 54$ km/h

Statement: The speed of the SUV is 54 km/h.

7. Given: $m_a = 0.020$ kg; $v_a = 250$ km / h; $m_b = 0.14$ kg; $E_{ka} = E_{kb}$

Required: v_b

Analysis: $E_k = \frac{1}{2}mv^2$; convert speed to metres per second; substitute into $E_{ka} = E_{kb}$

Solution: $v_a = 250\ \frac{\text{km}}{\text{h}}\cdot\frac{1000\text{ m}}{1\text{ km}}\cdot\frac{1\text{ h}}{3600\text{ s}}$

$v_a = 69.4$ m/s (one extra digit carried)

$$E_{k\,a} = E_{k\,b}$$

$$\frac{1}{2}m_a v_a^2 = \frac{1}{2}m_b v_b^2$$

$$\frac{m_a v_a^2}{m_b} = v_b^2$$

$$v_b = \sqrt{\frac{m_a v_a^2}{m_b}}$$

$$= \sqrt{\frac{(0.020\ \text{kg})(69.4\ \text{m/s})^2}{0.14\ \text{kg}}}$$

$$v_b = 26.2\ \text{m/s}$$

Convert the speed back to kilometres per hour.

$$v_b = \left(26.2\frac{\text{m}}{\text{s}}\right)\left(\frac{1\ \text{km}}{1000\ \text{m}}\right)\left(\frac{3600\ \text{s}}{1\ \text{h}}\right)$$

$$v_b = 94\ \text{km/h}$$

Statement: The speed of the baseball is 94 km/h.

8. Given: $v = 150\ \text{km / h}$; $m = 0.16\ \text{kg}$; $\Delta d = 0.25\ \text{m}$

Required: F

Analysis: Convert the speed to metres per second; $E_k = \frac{1}{2}mv^2$; $W = F\Delta d$; $W = \Delta E_k$; $\Delta E_k = E_{kf} - E_{ki}$. The puck has no initial velocity, so it has no initial kinetic energy, that is, $E_{ki} = 0$.

Solution: $v = 150\ \frac{\text{km}}{\text{h}} \cdot \frac{1000\ \text{m}}{1\ \text{km}} \cdot \frac{1\ \text{h}}{3600\ \text{s}}$

$v = 41.7$ m/s (one extra digit carried)

$$W = F\Delta d$$

$$F = \frac{W}{\Delta d}$$

$$
\begin{aligned}
W &= \Delta E_{\text{k}} \\
&= E_{\text{k f}} - E_{\text{k i}} \\
&= E_{\text{k f}} \\
&= \frac{1}{2}mv^2 \\
&= \frac{1}{2}(0.16 \text{ kg})(41.7 \text{ m/s})^2 \\
W &= 139 \text{ J (one extra digit carried)}
\end{aligned}
$$

$$
\begin{aligned}
F &= \frac{W}{\Delta d} \\
&= \frac{139 \text{ J}}{0.25 \text{ m}} \\
F &= 560 \text{ N}
\end{aligned}
$$

Statement: The average force exerted on the puck by the player is 560 N.

9. Given: $m = 5.31\times 10^{-26}\text{kg};\ E_{\text{k}} = 6.25\times 10^{-21}\text{J}$

Required: v

Analysis: $E_{\text{k}} = \frac{1}{2}mv^2$; solve for v and substitute.

Solution:

$$
\begin{aligned}
E_{\text{k}} &= \frac{1}{2}mv^2 \\
\frac{2}{m}E_{\text{k}} &= v^2 \\
v &= \sqrt{\frac{2}{m}E_{\text{k}}} \\
&= \sqrt{\frac{2}{5.31\times 10^{-26} \text{ kg}}(6.25\times 10^{-21} \text{ J})} \\
v &= 485 \text{ m/s}
\end{aligned}
$$

Statement: The speed of the molecule is 485 m/s.

10. Given: $F_{\text{a}} = 15 \text{ N}; m = 3.9 \text{ kg};\ \mu_{\text{k}} = 0.25;\ v_{\text{i}} = 0.0 \text{ m / s};\ \Delta d = 12 \text{ m}$

Required: v_{f}

Analysis: $F_{\text{f}} = \mu_{\text{k}}F_{\text{N}};\ F_{\text{N}} = mg;\ F = F_{\text{a}} - F_{\text{f}};\ W = \Delta E_{\text{k}};$ since $v_{\text{i}} = 0.0 \text{ m / s}; \Delta E_{\text{k}} = E_{\text{kf}};$

$E_{\text{kf}} = \frac{1}{2}mv_{\text{f}}^2$

Solution: $F_f = \mu_k F_N$

$= 0.25mg$

$= 0.25(3.9 \text{ kg})(-9.8 \text{ m/s}^2)$

$F_f = -9.56 \text{ N}$ (one extra digit carried)

$F = F_a + F_f$

$= 15 \text{ N} + (-9.56 \text{ N})$

$F = 5.44 \text{ N}$ (one extra digit carried)

$W = F\Delta d$

$= (5.44 \text{ N})(12 \text{ m})$

$W = 65.3 \text{ J}$

$W = E_k$

$$W = \frac{1}{2}mv_f^2$$

$$\frac{2}{m}W = v_f^2$$

$$v_f = \sqrt{\frac{2}{m}W}$$

$$= \sqrt{\frac{2}{3.9 \text{ kg}}(65.3 \text{ J})}$$

$v_f = 5.8 \text{ m/s}$

Statement: The final speed of the block is 5.8 m/s.

11. Given: $m = 5.55 \times 10^3 \text{kg}$; $v_1 = 2.81 \text{ km / s}$ or 2810 m / s; $v_2 = 3.24 \text{ km / s}$ or 3240 m / s

Required: W_g

Analysis: $W_g = \Delta E_k$; $\Delta E_k = E_{k2} - E_{k1}$; $E_k = \frac{1}{2}mv^2$

Solution: $W_g = \Delta E_k$

$$= E_{k\,2} - E_{k\,1}$$

$$= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$= \frac{1}{2}m(v_2^2 - v_1^2)$$

$$= \frac{1}{2}(5.55 \times 10^3 \text{ kg})((3240 \text{ m/s})^2 - (2810 \text{ m/s})^2)$$

$$W_g = 7.22 \times 10^9 \text{ J}$$

Statement: The work done by gravity on the satellite is 7.22×10^9 J.

12. (a) It is a quadratic function.

(b) The graph passes through the origin because when the speed is zero, the kinetic energy is zero.

(c) Given: From the graph, when the speed, v, is 2 m/s, the kinetic energy, E_k, is 4 J.

Required: m

Analysis: $E_k = \frac{1}{2}mv^2$; solve for m.

Solution: $E_k = \frac{1}{2}mv^2$

$$\frac{2}{v^2}E_k = m$$

$$m = \frac{2}{v^2}E_k$$

$$= \frac{2}{(2 \text{ m/s})^2}(4 \text{ J})$$

$$m = 2 \text{ kg}$$

Statement: The mass of the robot is 2 kg.

(d) Substitute the mass of the robot into the equation for kinetic energy, omitting units.

$$E_k = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(2)v^2$$

$$E_k = v^2$$