Section 4.1: Work Done by a Constant Force

Tutorial 1 Practice, page 166

1. Given: F = 275 N; $\Delta d = 0.65$ m

Required: *W*

Analysis: Use the equation for work, $W = F \Delta d \cos \theta$. F and Δd are in the same direction, so the angle between them is zero, stated as $\theta = 0$.

Solution: $W = F\Delta d\cos\theta$

 $=(275 \text{ N})(0.65 \text{ m})\cos 0$

 $=(180 \text{ N} \cdot \text{m})(1)$

 $W = 180 \text{ N} \cdot \text{m}$

Statement: The weightlifter does 1.8×10^2 J of work on the weights.

2. Given: F = 9.4 N; $\Delta d = 0$ m

Required: *W*

Analysis: There is no displacement in the direction of the applied force, so no work is done.

Statement: There is no work (0 J) done on the wall.

3. Given: F = 0.73 N; $\Delta d = 0.080$ m(note that the cue stick only does work on the ball when it is in contact with the ball)

Required: *W*

Analysis: Use the equation for work, $W = F\Delta d \cos \theta$. F and Δd are in the same direction, so the angle between them is zero, $\theta = 0$.

Solution: $W = F\Delta d\cos\theta$

 $= (0.73 \text{ N})(0.080 \text{ m})\cos 0$ $= (0.0584 \text{ N} \cdot \text{m})(1)$ $W = 0.058 \text{ N} \cdot \text{m}$

Statement: The cue stick does 0.058 J of work on the ball.

4. Given: $F = 9.9 \times 10^3$ N; $\Delta d = 4.3$ m; $\theta = 12^\circ$

Required: *W*

Analysis: The work done on the car by the tow truck depends only on the component of force in the direction of the car's displacement. Use the equation for work,

 $W = F \Delta d \cos \theta$. F and Δd are at an angle of 12° to each other.

Solution: $W = F \Delta d \cos \theta$

$$=(9.9\times10^3 \text{ N})(4.3 \text{ m})\cos 12^\circ$$

$$W = 4.2 \times 10^4 \text{ N} \cdot \text{m}$$

Statement: The tow truck does 4.2×10^4 J of work on the car.

Tutorial 2 Practice, page 167

1. (a) Given: m = 56 kg; $\Delta d = 78$ m; g = -9.8 m/s² Required: W_r , the work done by the ride on the rider Analysis: The ride must counteract the force of gravity for it to move at a constant speed. $F_g = mg$, so $F_r = -mg$; $W_r = F_r \Delta d \cos \theta$. Solution: $W_r = F_r \Delta d \cos \theta$ $= -mg \Delta d \cos \theta$ $= -(56 \text{ kg})(-9.8 \text{ m/s}^2)(78 \text{ m})\cos \theta$ $= 4.3 \times 10^4 \text{ (kg} \cdot \text{m/s}^2)(\text{m})$ $W_r = 4.3 \times 10^4 \text{ N} \cdot \text{m}$ $= 4.3 \times 10^4 \text{ J}$ Statement: The work done by the ride on the rider is $4.3 \times 10^4 \text{ J}$.

(b) Given: m = 56 kg; $\Delta d = 78$ m; g = -9.8 m/s² Required: W_g , the work done by gravity on the rider Analysis: The force of gravity on the rider is $F_g = mg$; $W_g = F_g \Delta d\cos\theta$.

Solution:
$$W_g = F_g \Delta d \cos \theta$$

 $= mg \Delta d \cos \theta$
 $= (56 \text{ kg})(-9.8 \text{ m/s}^2)(78 \text{ m})\cos \theta$
 $= -4.3 \times 10^4 \text{ (kg} \cdot \text{m/s}^2)(\text{m})$
 $= -4.3 \times 10^4 \text{ N} \cdot \text{m}$
 $W_g = -4.3 \times 10^4 \text{ J}$

Statement: The work done by gravity on the rider is -4.3×10^4 J.

2. (a) Given: $F = -5.21 \times 10^{3}$ N; $\Delta d = 355$ m Required: WAnalysis: $W = F\Delta d \cos\theta$ Solution: $W = F\Delta d \cos\theta$ $= (-5.21 \times 10^{3} \text{ N})(355 \text{ m})\cos\theta$ $= -1.85 \times 10^{6} \text{ N} \cdot \text{m}$ $W = -1.85 \times 10^{6} \text{ J}$ Statement: The work done by friction on the plane's wheels is -1.85×10^{6} J.

(b) Given: $F = -5.21 \times 10^3 \text{ N}; W = -1.52 \times 10^6 \text{ J}$

Required: Δd

Analysis: $W = F \Delta d \cos \theta$ and $W = F \Delta d \cos \theta$; $\Delta d = \frac{W}{F \cos \theta}$

Solution:
$$\Delta d = \frac{W}{F \cos \theta}$$
$$= \frac{-1.52 \times 10^6 \text{ J}}{(-5.21 \times 10^3 \text{ N})(\cos 0)}$$
$$= \frac{-1.52 \times 10^6 \text{ N} \cdot \text{m}}{(-5.21 \times 10^3 \text{ N})(\cos 0)}$$
$$\Delta d = 292 \text{ m}$$
Statement: The distance travelled by the plane is 292 m.
3. Given: $F = 5.9 \text{ N}$; $\theta = 150^\circ$; $\Delta d = 3.5 \text{ m}$
Required: W

Solution: $W = F \Delta d \cos \theta$ = (5.9 N)(3.5 m)(cos150°) = -18 N · m W = -18 J

Statement: The work done on the skier by the snow is -18 J.

Tutorial 3 Practice, page 168

1. Given: $\theta = 90^{\circ}$

Required: *W*

Analysis: The gravitational pull of Earth causes the Moon to experience centripetal acceleration during its orbit. At each moment, the Moon's instantaneous velocity is at an angle of 90° to the centripetal force. During a very short time interval, the very small displacement of the Moon is also at an angle of 90° to the centripetal force. We can break one orbit of the Moon into a series of many small displacements, each occurring during a very short time interval. During each time interval, the centripetal force and the displacement are perpendicular. The total work done during one orbit will equal the sum of the work done during each small displacement. For each small segment, use the work equation, $W = F\Delta d \cos \theta$, with $\theta = 90^\circ$.

Solution:

direction of Moon's motion



$$W = F\Delta d\cos\theta$$

= $F\Delta d\cos90^{\circ}$
= $F\Delta d(0)$
 $W = 0$ J

Statement: Summing the work done during all segments of the orbit gives a total of W = 0 J during each revolution. The centripetal force exerted by Earth does zero work on the Moon during the revolution.

Tutorial 4 Practice, page 169

1. Given: $\Delta d = 223 \text{ m}$; $F_{\rm h} = 122 \text{ N}$; $\theta_{\rm h} = 37^{\circ}$; $F_{\rm f} = 72.3 \text{ N}$; $\theta_{\rm f} = 180^{\circ}$ **Required:** $W_{\rm h}$, $W_{\rm f}$, $W_{\rm T}$ **Analysis:** $W = F\Delta d\cos\theta$. The total work done is the sum of the work done by the individual forces Solution: $W_{\rm h} = F_{\rm h} \Delta d \cos \theta$ = (122 N)(223 m)cos37° $W_{\rm h} = 2.17 \times 10^4$ J (one extra digit carried) $W_{\rm f} = F_{\rm f} \Delta d \cos \theta$ = (72.3 N)(223 m)cos180° $W_{\rm f} = -1.61 \times 10^4$ J (one extra digit carried)

$$W_{\rm T} = W_{\rm h} + W_{\rm f}$$

= 2.17×10⁴ J + (-1.61×10⁴ J)
 $W_{\rm T} = 5.6 \times 10^3$ J

Statement: The work done by the hiker is 2.2×10^4 J. The work done by friction is -1.6×10^4 J. The total work done is 5.6×10^3 J.

2. Given: $W_{\rm T} = 2.42 \times 10^4 \,\text{J}$; $F_{\rm h} = 122 \,\text{N}$; $\theta_{\rm h} = 37^\circ$; $F_{\rm f} = 72.3 \,\text{N}$; $\theta_{\rm f} = 180^\circ$

Required: Δd

Analysis:
$$W_{\rm T} = W_{\rm h} + W_{\rm f}$$

 $W_{\rm T} = F_{\rm h} \Delta d \cos\theta + F_{\rm f} \Delta d \cos\theta$
 $W_{\rm T} = \Delta d (F_{\rm h} \cos\theta + F_{\rm f} \cos\theta)$
 $\Delta d = \frac{W_{\rm T}}{F_{\rm h} \cos\theta + F_{\rm f} \cos\theta}$
Solution: $\Delta d = \frac{W_{\rm T}}{F_{\rm h} \cos\theta + F_{\rm f} \cos\theta}$
 $= \frac{2.42 \times 10^4 \text{ J}}{(122 \text{ N})(\cos 37^\circ) + (72.3 \text{ N})(\cos 180^\circ)}$
 $\Delta d = 963 \text{ m}$

Statement: The hiker pulled the sled 963 m.

Section 4.1 Questions, page 170

1. The bottom rope does more work on the box, because it is in the same direction as the displacement. Only the horizontal force component of the top rope does any work on the box.

2. No. There is no work done on an object by a centripetal force, because for each small displacement, the force is perpendicular to the direction of the displacement.

3. Given: F = 12.6 N; $\Delta d = 14.2$ m; $\theta = 21.8^{\circ}$ Required: W Analysis: $W = F\Delta d \cos \theta$ Solution: $W = F\Delta d \cos \theta$

 $= (12.6 \text{ N})(14.2 \text{ m})\cos 21.8^{\circ}$

W = 166 J

Statement: The shopper does 166 J of work on the cart.

4. Given: F = 22.8 N; $\Delta d = 52.6$ m; $W = 9.53 \times 10^2$ J Required: θ Analysis: $W = F\Delta d \cos\theta$ $\cos\theta = \frac{W}{F\Delta d}$ Solution: $\cos\theta = \frac{W}{F\Delta d}$ $= \frac{9.53 \times 10^2 \text{ J}}{(22.8 \text{ N})(52.6 \text{ m})}$ = 0.7946 (one extra digit carried) $\theta = 37.4^\circ$

Statement: The angle between the rope and the horizontal is 37.4°.

5. (a) Given: $\theta = 30^{\circ}$; m = 24 kg; g = -9.8 m/s²

Required: component of gravitational force along the ramp's surface **Analysis:** Draw a FBD of the situation.



Let F_{g_1} represent the component of gravitational force along the ramp's surface, and F_{g_2} represent the component perpendicular to the ramp's surface. Since the angle between F_{g_1} and F_g is 60° (from the FBD), $F_{g_1} = F_g \cos 60^\circ$.

Solution: $F_{g1} = F_g \cos 60^\circ$

$$= mg \cos 60^{\circ}$$

= (24 kg)(-9.8 m/s²) cos 60°
 $F_{g1} = -117.6$ N (two extra digits carried)

Statement: The component of gravitational force along the ramp's surface is 120 N down the ramp.

(b) Analysis: The force up the ramp must exactly balance the component of gravitational force along the ramp's surface for the crate to move up the ramp at a constant speed. **Statement:** The force required is 120 N up the ramp.

(c) Given: $F = 117.6 \text{ N}; \Delta d = 23 \text{ m}$ Required: WAnalysis: $W = F\Delta d \cos \theta$ **Solution:** $W = F\Delta d\cos\theta$

 $=(117.6 \text{ N})(23 \text{ m})\cos 0^{\circ}$

= 2700 J

Statement: The work done to push the crate up the ramp is 2.7×10^3 J.

(d) Given: $\mu_{\rm k} = 0.25$; m = 24 kg; $\theta = 30^{\circ}$; $\Delta d = 16$ m

Required: W_k ; W_T

Analysis: The worker has to overcome the force of friction and the force of gravity along the ramp, so the total work done by the worker is the sum of the work done by gravity and the work done by friction.

Solution: The work done by gravity is the force of gravity along the ramp times the distance, which is $mg \cos 60^{\circ}(\Delta d)$. The work done by friction is the force of friction along the ramp times the distance, which is $\mu_k mg \cos 30^{\circ}(\Delta d)$. The total work done by the worker is the sum of these two.

$$W_{w} = W_{g} + W_{k}$$

= $(mg\cos 60^{\circ})(\Delta d) + (\mu_{k}mg\cos 30^{\circ})(\Delta d)$
= $mg\Delta d(\cos 60^{\circ} + \mu_{k}\cos 30^{\circ})$
= $(24 \text{ kg})(9.8 \text{ m/s}^{2})(16 \text{ m})(\cos 60^{\circ} + 0.25\cos 30^{\circ})$
 $W_{w} = 2700 \text{ J [up the ramp]}$

$$W_{k} = F_{k}\Delta d$$

= $\mu_{k}F_{N}\Delta d$
= $(0.25)(24 \text{ kg})(9.8 \text{ m/s}^{2})(\cos 30^{\circ})(16 \text{ m})$
 $W_{k} = 810 \text{ J} \text{ [down the ramp]}$

The total work done by the system is equal to the work done by the worker plus the work done by friction.

$$W_{\rm T} = W + W_{\rm k}$$

= 2700 J + (-810 J)
 $W_{\rm T} = 1900$ J

Statement: The work done by the worker is 2.7×10^3 J up the ramp. The work done by kinetic friction is 8.1×10^2 J down the ramp. The total work done is 1.9×10^3 J up the ramp.

6. Given: $F_{\rm b} = 75$ N; $\theta_{\rm b} = 32^{\circ}$; $F_{\rm g} = 75$ N; $\theta_{\rm g} = 22^{\circ}$; $\Delta d = 13$ m Required: $W_{\rm T}$ Analysis: $W_{\rm T} = W_{\rm b} + W_{\rm g}$; $W_{\rm b} = F_{\rm b}\Delta d\cos\theta_{\rm b}$; $W_{\rm g} = F_{\rm g}\Delta d\cos\theta_{\rm g}$ Solution: $W_{\rm T} = W_{\rm b} + W_{\rm g}$ $= F_{\rm b}\Delta d\cos\theta_{\rm b} + F_{\rm g}\Delta d\cos\theta_{\rm g}$ $= (75 \text{ N})(13 \text{ m})\cos 32^{\circ} + (75 \text{ N})(13 \text{ m})\cos 22^{\circ}$ $W_{\rm T} = 1700 \text{ J}$

Statement: The total work done by the boy and the girl together is 1.7×10^3 J.