## Section 3.4: Rotating Frames of Reference

### Mini Investigation: Foucault Pendulum, page 128

Answers may vary. Sample answers:

**A.** The rotation does not affect the pendulum mass. From our frame of reference, the mass swings back and forth consistently while the globe rotates beneath it.

**B.** From our frame of reference, the period of rotation does not affect the mass. To an observer on the globe, the faster the rotation of Earth, the faster the pendulum appears to move. This implies that the rotation of Earth causes the movement of the Foucault pendulum.

**C.** At the equator the pendulum would not shift at all.

### Tutorial 1 Practice, page 129

**1. (a) Given:** $d$ = 324 m or $r$ = 162 m; $F_c = F_g$

**Required:** $v$

**Analysis:** $F_c = \dfrac{mv^2}{r}$; $F_g = mg$

$$F_c = F_g$$
$$\frac{mv^2}{r} = mg$$
$$\frac{v^2}{r} = g$$
$$v = \sqrt{gr}$$

**Solution:**
$$\begin{aligned} v &= \sqrt{gr} \\ &= \sqrt{(9.8 \text{ m/s}^2)(162 \text{ m})} \\ v &= 39.8 \text{ m/s} \end{aligned}$$

**Statement:** The relative speed of the astronauts is 39.8 m/s.

**(b) Given:** $a_c = g$; $r$ = 162 m

**Required:** $T$

**Analysis:** $a_c = \dfrac{4\pi^2 r}{T^2}$

$$T = \sqrt{\frac{4\pi^2 r}{a_c}}$$

**Solution:**
$$\begin{aligned} T &= \sqrt{\frac{4\pi^2 r}{a_c}} \\ &= \sqrt{\frac{4\pi^2 (162 \cancel{\text{m}})}{\left(9.8 \dfrac{\cancel{\text{m}}}{\text{s}^2}\right)}} \\ T &= 26 \text{ s} \end{aligned}$$

**Statement:** The period of the rotation of the spacecraft is 26 s.

**2.** Yes, both would experience artificial gravity equal to about 30.0 % of Earth's gravity, or 0.300$g$. The mass cancels out in the equation to determine speed, so the effect is independent of mass.

**3. Given:** $g$ = 10.00 m/s$^2$; $a_{net}$ = 9.70 m/s$^2$; $r$ = 6.2 × 10$^6$ m

**Required:** $T$

**Analysis:** $a_c = \dfrac{4\pi^2 r}{T^2}$; the centripetal acceleration is the difference between the acceleration due to gravity and the net acceleration experienced by a falling object.

$$a_c = \frac{4\pi^2 r}{T^2}$$
$$g - a_{net} = \frac{4\pi^2 r}{T^2}$$
$$T = \sqrt{\frac{4\pi^2 r}{g - a_{net}}}$$

**Solution:** $T = \sqrt{\dfrac{4\pi^2 r}{g - a_{net}}}$

$$= \sqrt{\frac{4\pi^2\left(6.2\times10^6\ \cancel{\text{m}}\right)}{\left(10.00\ \dfrac{\cancel{\text{m}}}{\text{s}^2} - 9.70\ \dfrac{\cancel{\text{m}}}{\text{s}^2}\right)}}$$
$$= 2.856\times10^4\ \cancel{\text{s}}\times\frac{1\ \cancel{\text{min}}}{60\ \cancel{\text{s}}}\times\frac{1\ \text{h}}{60\ \cancel{\text{min}}}$$
$$T = 7.9\ \text{h}$$

**Statement:** The length of the day, or period of the planet, is 7.9 h.

**4. Given:** $m_1$ = 56 kg; $r$ = 250 m; $m_2$ = 42 kg

**Required:** $v$

**Analysis:** $F_c = \dfrac{mv^2}{r}$; $F_g = mg$; the acceleration on the space station is $\dfrac{42}{56}$ or $\dfrac{3}{4}$ that of Earth because the scale reads the astronaut's weight as 42 kg instead of 56 kg.

$$F_c = F_N$$
$$\frac{mv^2}{r} = \frac{3}{4}mg$$
$$\frac{v^2}{r} = \frac{3}{4}g$$
$$v = \sqrt{\frac{3}{4}gr}$$

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**Solution:** $v = \sqrt{\frac{3}{4}gr}$

$$= \sqrt{\frac{3}{4}(9.8\ \text{m/s}^2)(162\ \text{m})}$$

$$v = 43\ \text{m/s}$$

**Statement:** The space station floor rotates at a speed of 43 m/s.

**5. Given:** $r = 6.38 \times 10^6$ m

**Required:** $v$

**Analysis:** $F_c = \frac{mv^2}{r}$ ; $F_g = mg$; the speed of the car would make the centripetal force greater than the gravitational force.

$$F_c = F_g$$

$$\frac{mv^2}{r} = mg$$

$$\frac{v^2}{r} = g$$

$$v = \sqrt{gr}$$

**Solution:** $v = \sqrt{gr}$

$$= \sqrt{(9.8\ \text{m/s}^2)(6.38 \times 10^6\ \text{m})}$$

$$v = 7.9 \times 10^3\ \text{m/s}$$

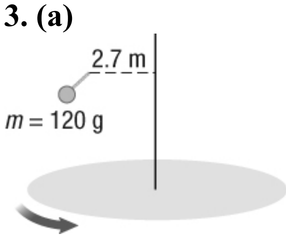
**Statement:** The car would need a speed of $7.9 \times 10^3$ m/s.

## Section 3.4 Questions, page 130

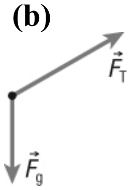
**1.** At just the right speed, the centrifugal acceleration is enough to provide enough force to keep the water in the bucket.

**2.** The spinning washing machine creates a centrifugal acceleration that forces water in the clothes to the outer wall and through pores in the wall, thus removing excess water from the clothes.

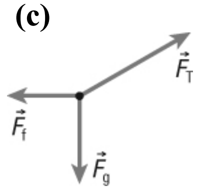
**3. (a)**




**(b)**



**(c)**




**(d) Given:** $r$ = 2.7 m; $m$ = 120 g or 0.12 kg; $T$ = 2.9 s
**Required:** $\theta$
**Analysis:** The horizontal component of the tension $F_T$ balances the centripetal force and the vertical component of the tension $F_T$ balances the gravitational force. Express the tangent ratio of the angle in terms of the applied force and the gravitational force, then solve for the angle;

$$v = \frac{2\pi r}{T}$$

$$\tan\theta = \frac{F_c}{F_g}$$

$$\theta = \tan^{-1}\left(\frac{\frac{mv^2}{r}}{mg}\right)$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

**Solution:** Determine the speed 2.7 m from the centre:

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \frac{2\pi(2.7\ \text{m})}{(3.9\ \text{s})} \\ &= 4.350\ \text{m/s (two extra digits carried)} \end{aligned}$$

$v$ = 4.3 m/s

Determine the angle the string makes with the vertical:

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{v^2}{rg}\right) \\ &= \tan^{-1}\left(\frac{\left(4.350\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)^2}{(2.7\ \cancel{\text{m}})\left(9.8\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}\right)}\right) \\ &= 35.57^\circ\ \text{(two extra digits carried)} \end{aligned}$$

$\theta = 36°$

**Statement:** The string makes a 36° angle with the vertical.
**(e) Given:** $r$ = 2.7 m; $m$ = 120 g = 0.12 kg; $\theta$ = 35.57°
**Required:** $F_T$
**Analysis:** The vertical component of the tension $F_T$ balances the gravitational force. Express the cosine ratio of the angle in terms of the tension and the gravitational force.

$$\cos\theta = \frac{F_g}{F_T}$$
$$F_T = F_g \cos\theta$$
$$F_T = mg\cos\theta$$

**Solution:** $F_T = mg\cos\theta$

$$= (0.12 \text{ kg})\left(9.8 \ \frac{\text{m}}{\text{s}^2}\right)\cos 35.57°$$

$$F_T = 0.96 \text{ N}$$

**Statement:** The tension in the string is 0.96 N.

**4. Given:** $r = 6.38 \times 10^6$ m; $T = 24$ h

**Required:** $\frac{a_c}{g}$ at the equator

**Analysis:** Earth is a non-inertial frame of reference. The acceleration of an object at the equator is the difference between the acceleration due to gravity and the centrifugal acceleration, $g - a_c$.

Use $a_c = \frac{4\pi^2 r}{T^2}$ to determine the centrifugal acceleration, then calculate its ratio with $g$.

**Solution:** Determine the centripetal acceleration at the equator:

$$a_c = \frac{4\pi^2 r}{T^2}$$
$$= \frac{4\pi^2(6.38\times10^6 \ \cancel{\text{m}})}{\left(24 \ \cancel{\text{h}} \times \frac{60 \ \cancel{\text{min}}}{1 \ \cancel{\text{h}}} \times \frac{60 \text{ s}}{1 \ \cancel{\text{min}}}\right)^2}$$
$$= \frac{4\pi^2(6.38\times10^6 \ \cancel{\text{m}})}{(86\ 400 \text{ s})^2}$$
$$a_c = 0.0337 \text{ m/s}^2$$

Therefore, the ratio of the centrifugal acceleration to $g$ is:

$$\frac{a_c}{g} = \frac{(0.0337 \text{ m/s}^2)}{(9.8 \text{ m/s}^2)}$$
$$\frac{a_c}{g} = 0.0034$$

**Statement:** The acceleration at the equator is 0.34 % less that $g$.

**5. Given:** $d = 10$ m or $r = 5$ m; $T = 30$ s; $\Delta d = 1.7$ m
**Required:** compare $a_c$ at $r$ and $r - \Delta d$

**Analysis:** $a_c = \dfrac{4\pi^2 r}{T^2}$

**Solution:** Determine the centripetal acceleration at the astronaut's feet:

$$\begin{aligned} a_c &= \frac{4\pi^2 r}{T^2} \\ &= \frac{4\pi^2(5\text{ m})}{(30\text{ s})^2} \\ a_c &= 0.07\text{ m/s}^2 \end{aligned}$$

Determine the centripetal acceleration at the astronaut's head:

$$\begin{aligned} a_c &= \frac{4\pi^2(r - \Delta d)}{T^2} \\ &= \frac{4\pi^2(5\text{ m} - 1.7\text{ m})}{(30\text{ s})^2} \\ a_c &= 0.05\text{ m/s}^2 \end{aligned}$$

**Statement:** The movie did not get the physics right. The acceleration experienced by the astronaut is in the range of 0.05 m/s$^2$ to 0.07 m/s$^2$ instead of 9.8 m/s$^2$.

**6. (a) Given:** $r = 100$ m; $a_c = g$
**Required:** $T$

**Analysis:**
$$\begin{aligned} a_c &= \frac{4\pi^2 r}{T^2} \\ g &= \frac{4\pi^2 r}{T^2} \\ T &= \sqrt{\frac{4\pi^2 r}{g}} \end{aligned}$$

**Solution:**
$$\begin{aligned} T &= \sqrt{\frac{4\pi^2 r}{g}} \\ &= \sqrt{\frac{4\pi^2(100\ \cancel{\text{m}})}{\left(9.8\ \dfrac{\cancel{\text{m}}}{\text{s}^2}\right)}} \\ &= 20.07\text{ s (two extra digits carried)} \\ T &= 2.0 \times 10^1\text{ s} \end{aligned}$$

**Statement:** The period of rotation is $2.0 \times 10^1$ s.

**(b) Given:** $r$ = 100 m; $a_c = g$
**Required:** $v$

**Analysis:** $a_c = \dfrac{v^2}{r}$

$$v = \sqrt{a_c r}$$

**Solution:** $v = \sqrt{a_c r}$

$$= \sqrt{\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(100\ \text{m})}$$

$$= 31.30\ \text{m/s (two extra digits carried)}$$

$$v = 31\ \text{m/s}$$

**Statement:** The speed of rotation is 31 m/s.
**(c) Given:** $r$ = 100 m; $v_i$ = 31.30 m/s; $v$ = –4.2 m/s
**Required:** $F_N$

**Analysis:** $F_c = \dfrac{mv}{r^2}$; $F_N = F_c$

**Solution:** $F_N = \dfrac{mv^2}{r}$

$$= \frac{m(31.30\ \text{m/s} - 4.2\ \text{m/s})^2}{(100\ \text{m})}$$

$$= m(7.34\ \text{m/s}^2)$$

$$F_N = 7.3m$$

**Statement:** The apparent weight is 7.3 times the mass.
**(d) Given:** $r$ = 100 m; $v_i$ = 31.30 m/s; $v$ = +4.2 m/s
**Required:** $F_N$

**Analysis:** $F_c = \dfrac{mv}{r^2}$; $F_N = F_c$

**Solution:** $F_N = \dfrac{mv^2}{r}$

$$= \frac{m(31.30\ \text{m/s} + 4.2\ \text{m/s})^2}{(100\ \text{m})}$$

$$= m(12.6\ \text{m/s}^2)$$

$$F_N = 13m$$

**Statement:** The apparent weight is 13 times the mass.
**(e)** Running with the direction of the rotation is a better workout because you experience a greater centrifugal force and it requires more effort or exertion.
**7. (a) Given:** $m$ = 65 kg; $r$ = 150 m; $F_N$ = 540 N
**Required:** $a_c$

**Analysis:** $F_N = F_c$; $F_c = ma_c$; $a_c = \dfrac{F_N}{m}$

**Solution:** $a_c = \dfrac{F_N}{m}$

$$= \frac{(540\ \text{N})}{(65\ \text{kg})}$$

$$= 8.308\ \text{m/s}^2\ \text{(two extra digits carried)}$$

$$a_c = 8.3\ \text{m/s}^2$$

**Statement:** The acceleration of objects near the floor of the space station is 8.3 m/s$^2$.

**(b) Given:** $r$ = 150 m; $a_c$ = 8.308 m/s$^2$

**Required:** $v$

**Analysis:** $a_c = \dfrac{v^2}{r}$

$$v = \sqrt{a_c r}$$

**Solution:** $v = \sqrt{a_c r}$

$$= \sqrt{\left(8.308\ \frac{\text{m}}{\text{s}^2}\right)(150\ \text{m})}$$

$$= 35.30\ \text{m/s (two extra digits carried)}$$

$$v = 35\ \text{m/s}$$

**Statement:** The speed of rotation of the outer rim is 35 m/s.

**(c) Given:** $r$ = 150 m; $a_c$ = 8.308 m/s$^2$

**Required:** $T$

**Analysis:** $a_c = \dfrac{4\pi^2 r}{T^2}$

$$g = \frac{4\pi^2 r}{T^2}$$

$$T = \sqrt{\frac{4\pi^2 r}{g}}$$

**Solution:** $T = \sqrt{\dfrac{4\pi^2 r}{g}}$

$$= \sqrt{\frac{4\pi^2(150\ \cancel{\text{m}})}{\left(8.308\ \frac{\cancel{\text{m}}}{\text{s}^2}\right)}}$$

$$T = 27\ \text{s}$$

**Statement:** The period of rotation of the space station is 27 s.

**8. (a) Given:** $r$ = 3.4 cm or 0.034 m; $f = 1.1 \times 10^3$ Hz

**Required:** $a_c$

**Analysis:** $a_c = 4\pi^2 r f^2$

**Solution:** $a_c = 4\pi^2 rf^2$

$$= 4\pi^2(0.034\ \text{m})(1.1\times 10^3\ \text{Hz})^2$$

$$a_c = 1.6\times 10^6\ \text{m/s}^2$$

**Statement:** From Earth's frame of reference, the magnitude of the centripetal acceleration is $1.6 \times 10^6$ m/s$^2$.

**(b)** Answers may vary. Sample answer: Centrifuges need high frequencies to get the greatest possible acceleration. A high centrifugal force moves the denser particles to the bottom of a test tube, perfectly separating mixed solutions such as plasma and red blood cells.

**(c)** Answers may vary. Sample answer: By separating particles, medical researchers can study the particles in their pure form.

**9.** Answers may vary. Sample answer: A large-scale centrifuge, like all centrifuges, spins to separate a mixture into its components. In a large-scale centrifuge, a wastewater mixture is spun and water is separated from the heavier mixture, often called sludge, which settles on the bottom. The thickened mixture is moved to another facility for treatment while the water is sent on for different treatment before returning to the environment. By separating water from the heavier mixture, these two components of wastewater can receive the appropriate treatment before returning to the environment.