Section 3.3: Centripetal Force

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1. Given: $m = 0.211$ kg; $r = 25.6$ m; $v = 21.7$ m/s

Required: F_c

Analysis: Lift is equivalent to the normal force, so it is the sum of the centripetal force and the gravitational force; $F_c = \dfrac{mv^2}{r}$; $F_g = mg$

$$F_N = F_c + F_g$$

$$F_N = \frac{mv^2}{r} + mg$$

Solution:
$$\begin{aligned} F_N &= \frac{mv^2}{r} + mg \\ &= \frac{(0.211\ \text{kg})(21.7\ \text{m/s})^2}{(25.6\ \text{m})} + (0.211\ \text{kg})(9.8\ \text{m/s}^2) \\ F_c &= 5.9\ \text{N} \end{aligned}$$

Statement: The lift on the plane at the bottom of the arc is 5.9 N.

2. Given: $r = 450$ m; $v = 97$ km/h

Required: θ

Analysis: The vertical component of the normal F_N balances the gravitational force and the horizontal component of the normal F_N represents the centripetal force, $F_c = \dfrac{mv^2}{r}$. Use the tangent ratio to determine the angle the normal makes with the vertical.

$$\begin{aligned} \tan\theta &= \frac{F_{Nx}}{F_{Ny}} \\ &= \frac{F_c}{F_g} \\ &= \frac{\left(\dfrac{mv^2}{r}\right)}{mg} \\ &= \frac{v^2}{rg} \\ \theta &= \tan^{-1}\left(\frac{v^2}{rg}\right) \end{aligned}$$

Solution: Convert the speed to metres per second:

$$\begin{aligned} v &= 97\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\ \text{s}} \\ &= 26.94\ \text{m/s (two extra digits carried)} \\ v &= 27\ \text{m/s} \end{aligned}$$

Determine the angle:

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$
$$= \tan^{-1}\left(\frac{(26.94\text{ m/s})^2}{(450\text{ m})(9.8\text{ m/s}^2)}\right)$$
$$\theta = 9.3°$$

Statement: The banking angle is 9.3°.

3. Given: m = 2.0 kg; $f = \dfrac{5.00}{2.00\text{ s}}$ = 2.50 Hz; r = 4.00 m

Required: F_T

Analysis: $F_T = ma_c$; $a_c = 4\pi^2 rf^2$; $F_c = 4\pi^2 mrf^2$

Solution: $F_c = 4\pi^2 mrf^2$
$$= 4\pi^2(2.00\text{ kg})(4.00\text{ m})(2.50\text{ Hz})^2$$
$$F_T = 2.0 \times 10^3\text{ N}$$

Statement: The magnitude of the tension in the string is 2.0×10^3 N.

4. Given: r = 150 m; $F_c = F_g$

Required: v

Analysis: $F_g = mg$; $F_c = \dfrac{mv^2}{r}$

$$F_c = F_g$$
$$\frac{mv^2}{r} = mg$$
$$\frac{v^2}{r} = g$$
$$v = \sqrt{gr}$$

Solution: $v = \sqrt{gr}$
$$= \sqrt{(9.8\text{ m/s}^2)(150\text{ m})}$$
$$v = 38\text{ m/s}$$

Statement: The speed of the barn swallow is 38 m/s.

5. I predict that the maximum speed will decrease because the road conditions are slippery.

Given: μ_s = 0.25; $r = 2.0 \times 10^2$ m; θ = 20.0°

Required: maximum v

Analysis: From Sample Problem 3: $v = \sqrt{gr\left(\dfrac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)}$

Solution: $v = \sqrt{gr\left(\dfrac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right)}$

$= \sqrt{\left(9.8 \text{ m/s}^2\right)\left(2.0\times10^2 \text{ m}\right)\left(\dfrac{\sin 20.0^\circ + (0.25)\cos 20.0^\circ}{\cos 20.0^\circ - (0.25)\sin 20.0^\circ}\right)}$

$v = 36 \text{ m/s}$

Statement: The maximum speed in slippery conditions is 36 m/s.

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1. Given: d = 24 m or r = 12 m; $F_{\text{net}} = \dfrac{1}{3}F_g$

Required: v

Analysis: $F_c = \dfrac{mv^2}{r}$; $F_g = mg$; $F_{\text{net}} = F_c + F_g$

$$
\begin{aligned}
F_{\text{net}} &= \frac{1}{3}F_g \\
F_c + F_g &= \frac{1}{3}F_g \\
F_c &= -\frac{2}{3}F_g \\
\frac{mv^2}{r} &= -\frac{2}{3}mg \\
v^2 &= -\frac{2}{3}\frac{\cancel{m}gr}{\cancel{m}} \\
v &= \sqrt{-\frac{2}{3}gr}
\end{aligned}
$$

Solution: $v = \sqrt{-\dfrac{2}{3}gr}$

$= \sqrt{-\dfrac{2}{3}\left(-9.8 \text{ m/s}^2\right)\left(12 \text{ m}\right)}$

$v = 8.9 \text{ m/s}$

Statement: The speed of the roller coaster is 8.9 m/s.

2. (a)

(b) Given: m = 1000.0 kg; r = 40.0 m; v = 15 m/s
Required: F_N

Analysis: $F_c = \dfrac{mv^2}{r}$; $F_g = mg$;

$$F_N = F_g + F_c$$
$$F_N = mg + \frac{mv^2}{r}$$

Solution: $F_N = mg + \dfrac{mv^2}{r}$

$$= (1000.0\text{ kg})(9.8\text{ m/s}^2) + \frac{(1000.0\text{ kg})(15\text{ m/s})^2}{(40.0\text{ m})}$$
$$F_N = 1.5 \times 10^4\text{ N}$$

Statement: The magnitude of the normal force is 1.5×10^4 N.
(c) Given: m = 1000.0 kg; r = 40.0 m; v = 15 m/s; $F_{net} = 0$
Required: v

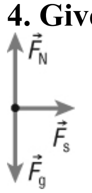
Analysis: $F_c = \dfrac{mv^2}{r}$; $F_g = mg$

$$F_{net} = F_g + F_c$$
$$0 = mg + \frac{mv^2}{r}$$
$$0 = g + \frac{v^2}{r}$$
$$v = \sqrt{-gr}$$

Solution: $v = \sqrt{-gr}$

$$= \sqrt{-(-9.8\text{ m/s}^2)(40.0\text{ m})}$$
$$v = 20\text{ m/s}$$

Statement: The speed required to make the driver feel weightless is 20 m/s.
3. (a) When the banking angle increases, the maximum speed of a car also increases because the horizontal component of the normal force has increased.
(b) When the coefficient of friction increases, the maximum speed of a car also increases because the force due to friction, which points into the turn, has increased.
(c) When the mass of the car increases, the maximum speed of a car also increases because the normal force and the force due to friction both increase.
4. Given: $r = 1.2 \times 10^2$ m; $\mu_s = 0.72$; $F_{net} = 0$



Required: maximum v

Analysis: $F_c = \frac{mv^2}{r}$; $F_g = mg$; $F_s = \mu_s F_N$

$$F_c = F_s$$
$$\frac{mv^2}{r} = \mu_s F_N$$
$$\frac{\cancel{m}v^2}{r} = \mu_s \cancel{m}g$$
$$\frac{v^2}{r} = \mu_s g$$
$$v = \sqrt{\mu_s g r}$$

Solution: $v = \sqrt{\mu_s g r}$

$$= \sqrt{(0.72)(9.8\ \text{m/s}^2)(1.2 \times 10^2\ \text{m})}$$
$$v = 29\ \text{m/s}$$

Statement: The maximum speed of the car is 29 m/s.

5. (a) The banking angle creates a horizontal component of the normal force, which is only vertical on a horizontal round. This horizontal component increases the net force pushing into the curve. Thanks to this force into the curve, cars can navigate the turn at higher speeds without losing friction.

(b) Drivers must go much more slowly because the coefficient of static friction is dramatically reduced and the net force pushing into the curve is much less.

(c) Answers may vary. Sample answer: The banking angle must work for conditions where the coefficient of static friction is high and when it is low. Making the angles significantly larger would allow for greater speeds but would be much more dangerous in slippery conditions.

6. Given: $m_1 = 0.26$ kg; $m_2 = 0.68$ kg; $r = 1.2$ m

Required: v

Analysis: $F_c = \frac{mv^2}{r}$; $F_g = mg$; The tension in the string equals the gravitational force on m_2 and the centripetal force on m_1.

$$F_c = F_g$$
$$\frac{m_1 v^2}{r} = m_2 g$$
$$v = \sqrt{\frac{m_2 g r}{m_1}}$$

Solution: $v = \sqrt{\frac{m_2 g r}{m_1}}$

$$= \sqrt{\frac{(0.68\ \text{kg})(9.8\ \text{m/s}^2)(1.2\ \text{m})}{(0.26\ \text{kg})}}$$
$$v = 5.5\ \text{m/s}$$

Statement: The speed of the air puck is 5.5 m/s.