Section 3.2: Centripetal Acceleration

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1. Given: r = 25 km = 2.5 × 10^4 m; v = 50.0 m/s

Required: a_c

Analysis: $a_c = \dfrac{v^2}{r}$

Solution:
$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{(50.0\text{ m/s})^2}{(2.5\times 10^4\text{ m})} \\ a_c &= 0.10\text{ m/s}^2 \end{aligned}$$

Statement: The magnitude of the centripetal acceleration is 0.10 m/s^2.

2. Given: r = 1.2 m; v = 4.24 m/s

Required: $\vec{a}_c$

Analysis: $a_c = \dfrac{v^2}{r}$; Centripetal acceleration is always directed toward the centre of rotation.

Since the hammer's velocity is directed south and it is spinning clockwise, the centre of rotation is west of the hammer.

Solution:
$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{(4.24\text{ m/s})^2}{(1.2\text{ m})} \\ a_c &= 15\text{ m/s}^2 \end{aligned}$$

Statement: The centripetal acceleration is 15 m/s^2 [W].

3. Given: r = 1.4 m; a_c = 12 m/s^2

Required: v

Analysis:
$$\begin{aligned} a_c &= \frac{v^2}{r} \\ v &= \sqrt{a_c r} \end{aligned}$$

Solution:
$$\begin{aligned} v &= \sqrt{a_c r} \\ &= \sqrt{(12\text{ m/s}^2)(1.4\text{ m})} \\ v &= 4.1\text{ m/s} \end{aligned}$$

Statement: The speed of the ball is 4.1 m/s.

4. (a) Given: r = 1.08 × 10^{11} m; a_c = 1.12 × 10^{-2} m/s^2

Required: v

Analysis: $a_c = \dfrac{4\pi^2 r}{T^2}$

$$T = \sqrt{\frac{4\pi^2 r}{a_c}}$$

Solution: $T = \sqrt{\dfrac{4\pi^2 r}{a_c}}$

$$= \sqrt{\frac{4\pi^2 \left(1.08\times 10^{11}\ \cancel{\text{m}}\right)}{\left(1.12\times 10^{-2}\ \dfrac{\cancel{\text{m}}}{\text{s}^2}\right)}}$$

$$T = 1.95\times 10^{7}\ \text{s}$$

Statement: The period of Venus is 1.95×10^7 s.

(b) Convert the period in seconds to days:

$$T = 1.95\times 10^{7}\ \text{s}\times\frac{1\ \cancel{\text{min}}}{60\ \cancel{\text{s}}}\times\frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\times\frac{1\ \text{d}}{24\ \cancel{\text{h}}}$$

$$T = 226\ \text{days}$$

The period of Venus is 226 days.

5. Given: $v = 7.27 \times 10^3$ m.s; $r = 7.54 \times 10^6$ m

Required: a_c

Analysis: $a_c = \dfrac{v^2}{r}$

Solution: $a_c = \dfrac{v^2}{r}$

$$= \frac{\left(7.27\times 10^{3}\ \text{m/s}\right)^2}{\left(7.54\times 10^{6}\ \text{m}\right)}$$

$$a_c = 7.01\ \text{m/s}^2$$

Statement: The magnitude of the centripetal acceleration is 7.01 m/s^2.

6. (a) Given: $a_c = 3.3 \times 10^6\ \text{m/s}^2$; $r = 8.4\ \text{cm} = 8.4 \times 10^{-2}$ m

Required: f

Analysis: $a_c = 4\pi^2 r f^2$

$$f = \sqrt{\frac{a_c}{4\pi^2 r}}$$

Solution: $f = \sqrt{\dfrac{a_c}{4\pi^2 r}}$

$= \sqrt{\dfrac{\left(3.3 \times 10^6 \ \dfrac{\cancel{\text{m}}}{\text{s}^2}\right)}{4\pi^2\left(8.4 \times 10^{-2} \ \cancel{\text{m}}\right)}}$

$f = 1.0 \times 10^4 \text{ Hz}$

Statement: The frequency of the centrifuge is 1.0×10^4 Hz.

(b) Convert the frequency in hertz to revolutions per minute:

$f = 1.0 \times 10^4 \ \dfrac{1}{1 \ \cancel{\text{s}}} \times \dfrac{60 \ \cancel{\text{s}}}{1 \text{ min}}$

$f = 6.0 \times 10^5 \text{ rpm}$

The frequency of the centrifuge is 6.0×10^5 rpm.

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1. (a) The tension in the string provides the force to keep the puck in its circular path at constant speed, and so provides the acceleration of the puck.

(b) The centripetal acceleration is half as large because centripetal acceleration depends on the inverse of the radius: $\dfrac{1}{2}a_c = \dfrac{v^2}{2r}$.

(c) The centripetal acceleration is four times as great because centripetal acceleration depends on the square of the speed: $4a_c = \dfrac{(2v)^2}{r}$.

2. The centripetal acceleration for the first athlete's hammer is four times greater than that of the second athlete. Centripetal acceleration depends on the square of the speed: $a_c = \dfrac{v^2}{r}$. So if the hammer spins two times as fast, the centripetal acceleration is 2^2, or 4, times larger: $4a$.

3. Given: $r = 0.42$ m; $T = 1.5$ s

Required: a_c

Analysis: $a_c = \dfrac{4\pi^2 r}{T^2}$

Solution: $a_c = \dfrac{4\pi^2 r}{T^2}$

$= \dfrac{4\pi^2(0.42 \text{ m})}{(1.5 \text{ s})^2}$

$a_c = 7.4 \text{ m/s}^2$

Statement: The magnitude of the centripetal acceleration of the lasso is 7.4 m/s^2.

4. Given: $v = 28$ m/s; $r = 135$ m

Required: a_c

Analysis: $a_c = \dfrac{v^2}{r}$

Solution: $a_c = \frac{v^2}{r}$

$= \frac{(28 \text{ m/s})^2}{(135 \text{ m})}$

$a_c = 5.8 \text{ m/s}^2$

Statement: The magnitude of the centripetal acceleration is 5.8 m/s^2.

5. Given: $r = 6.38 \times 10^6$ m; $T = 1$ day or 86 400 s

Required: a_c

Analysis: $a_c = \frac{4\pi^2 r}{T^2}$

Solution: $a_c = \frac{4\pi^2 r}{T^2}$

$= \frac{4\pi^2 (6.38 \times 10^6 \text{ m})}{(86\,400 \text{ s})^2}$

$a_c = 3.37 \times 10^{-2} \text{ m/s}^2$

Statement: The centripetal acceleration at Earth's equator is 3.37×10^{-2} m/s^2.

6. Given: $a_c = 25$ m/s^2; $r = 2.0$ m

Required: f

Analysis: $a_c = 4\pi^2 r f^2$

$f = \sqrt{\frac{a_c}{4\pi^2 r}}$

Solution: $f = \sqrt{\frac{a_c}{4\pi^2 r}}$

$= \sqrt{\frac{\left(25 \frac{\cancel{\text{m}}}{\text{s}^2}\right)}{4\pi^2 (2.0 \cancel{\text{m}})}}$

$f = 0.56 \text{ Hz}$

Statement: The minimum frequency of the cylinder is 0.56 Hz.

7. Given: $v = 22$ m/s; $a_c = 7.8$ m/s^2

Required: r

Analysis: $a_c = \frac{v^2}{r}$

$r = \frac{v^2}{a_c}$

Solution: $r = \dfrac{v^2}{a_c}$

$$= \frac{(22\ \text{m/s})^2}{(7.8\ \text{m/s}^2)}$$

$$r = 62\ \text{m}$$

Statement: The radius of the curve is 62 m.

8. Given: C = 478 m; a_c = 0.146 m/s^2

Required: v

Analysis: $C = 2\pi r$ or $r = \dfrac{C}{2\pi}$;

$$a_c = \frac{v^2}{r}$$

$$v = \sqrt{a_c r}$$

$$v = \sqrt{a_c\left(\frac{C}{2\pi}\right)}$$

Solution: $v = \sqrt{a_c\left(\dfrac{C}{2\pi}\right)}$

$$= \sqrt{\frac{(0.146\ \text{m/s}^2)(478\ \text{m})}{2\pi}}$$

$$= 3.333\ \frac{\cancel{\text{m}}}{\text{s}} \times \frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}} \times \frac{60\ \cancel{\text{min}}}{1\ \text{h}} \times \frac{1\ \text{km}}{1000\ \cancel{\text{m}}}$$

$$v = 12.0\ \text{km/h}$$

Statement: The speed of the jogger is 12.0 km/h.

9. (a) Given: r = 0.300 m; f = 60.0 rpm

Required: T

Analysis: $T = \dfrac{1}{f}$

Solution: Convert the frequency to hertz:

$$f = 60.0\ \text{rpm}$$

$$= 60.0\ \frac{1}{1\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\ \text{s}}$$

$$f = 1.00\ \text{Hz}$$

Determine the period:

$$T = \frac{1}{f}$$
$$= \frac{1}{1.00 \text{ Hz}}$$
$$T = 1.00 \text{ s}$$

Statement: The period of the bicycle wheel is 1.00 s.
(b) Given: r = 0.300 m; f = 1.00 Hz
Required: $\vec{a}_\text{c}$
Analysis: $a_\text{c} = 4\pi^2 rf^2$; Centripetal acceleration is always directed toward the centre of rotation. Since the wheel's velocity is directed west and it is spinning clockwise, the centre of rotation is north of the point.
Solution: $a_\text{c} = 4\pi^2 rf^2$

$$= 4\pi^2(0.300 \text{ m})(1.00 \text{ Hz})^2$$
$$a_\text{c} = 11.8 \text{ m/s}^2$$

Statement: The centripetal acceleration of a point on the edge of that wheel is 11.8 m/s^2 [N] if it is moving westward at that instant.
10. (a) Given: T = 27.3 days; a_c = 2.7 × 10^{-3} m/s^2
Required: r
Analysis: $a_\text{c} = \dfrac{4\pi^2 r}{T^2}$
Solution: Convert the period to seconds:

$$T = 27.3 \text{ days} \times \frac{24 \cancel{\text{h}}}{1 \cancel{\text{day}}} \times \frac{60 \cancel{\text{min}}}{1 \cancel{\text{h}}} \times \frac{60 \text{ s}}{1 \cancel{\text{min}}}$$
$$= 2.3587 \times 10^6 \text{ s (two extra digits carried)}$$
$$T = 2.36 \times 10^6 \text{ s}$$

Determine the radius:

$$a_\text{c} = \frac{4\pi^2 r}{T^2}$$
$$r = \frac{a_\text{c} T^2}{4\pi^2}$$
$$= \frac{\left(2.7 \times 10^{-3} \dfrac{\text{m}}{\cancel{\text{s}^2}}\right)(2.3587 \times 10^6 \cancel{\text{s}})^2}{4\pi^2}$$
$$r = 3.8 \times 10^8 \text{ m}$$

Statement: The radius of the curve is 3.8 × 10^8 m.
(b) The values are the same to two significant digits. Any difference beyond that may be because the orbit is not perfectly circular or the speed is not constant.

11. (a) Given: $a_c = 711\ \text{m/s}^2$; $r = 1.21$ m
Required: v

Analysis: $a_c = \dfrac{v^2}{r}$; $v = \sqrt{a_c r}$

Solution:
$$
\begin{aligned}
v &= \sqrt{a_c r} \\
&= \sqrt{(711\ \text{m/s}^2)(1.21\ \text{m})} \\
v &= 29.3\ \text{m/s}
\end{aligned}
$$

Statement: The speed of the hammer is 29.3 m/s.

(b) Given: $\Delta d = -2.0$ m; $v_i = 29.3$ m/s; $\theta = 42°$
Required: Δd_x

Analysis: Use $v_f^2 = v_i^2 + 2a\Delta d$ to calculate the y-component of the final speed, then calculate the time of flight $v_f = v_i + a\Delta t$. Finally, calculate the range using $\Delta d = v\Delta t$.

Solution: Determine the y-component of the final speed:

$$
\begin{aligned}
v_f^2 &= v_i^2 + 2a\Delta d \\
v_{fy}^2 &= v_{iy}^2 + 2g\Delta d \\
v_{fy} &= \sqrt{v_{iy}^2 + 2g\Delta d} \\
&= \sqrt{\left((29.3\ \text{m/s})(\sin 42°)\right)^2 + 2(9.8\ \text{m/s}^2)(-2.0\ \text{m})} \\
&= 18.58\ \text{m/s (two extra digits carried)} \\
v_{fy} &= 19\ \text{m/s}
\end{aligned}
$$

Determine the time of flight:

$$
\begin{aligned}
v_{fy} &= v_{iy} + a\Delta t \\
\Delta t &= \frac{v_{fy} - v_{iy}}{a} \\
&= \frac{18.58\ \dfrac{\cancel{\text{m}}}{\text{s}} - \left(-29.3\ \dfrac{\cancel{\text{m}}}{\text{s}}\right)(\sin 42°)}{9.8\ \dfrac{\cancel{\text{m}}}{\text{s}^2}} \\
\Delta t &= 3.896\ \text{s (two extra digits carried)}
\end{aligned}
$$

Determine the range of the ball:

$$
\begin{aligned}
\Delta d_x &= v_x \Delta t \\
&= v_i \Delta t \cos\theta \\
&= \left(29.3\ \frac{\text{m}}{\cancel{\text{s}}}\right)(3.896\ \cancel{\text{s}})(\cos 42°) \\
\Delta d_x &= 85\ \text{m}
\end{aligned}
$$

Statement: The range of the ball is 85 m.