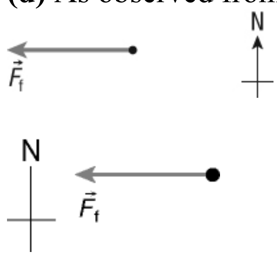
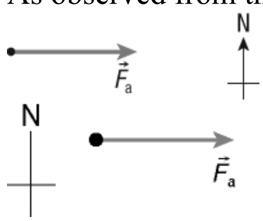
Section 3.1: Inertial and Non-inertial Frames of Reference
Tutorial 1 Practice, page 110
1. (a) When the car is moving with constant velocity, I see the ball lie still on the floor. I would see the same situation when the car is at rest.
(b) To an observer on the sidewalk, the ball appears to be moving with a constant velocity of 14 m/s [E].
(c) If the car accelerates forward, I see the ball roll backward on the floor.
(d) As observed from the frame of reference of the car:



As observed from the frame of reference of the sidewalk:



The car's frame of reference is non-inertial. I observe a fictitious force (the force pushing the ball backward, west) in the car's frame of reference.
2. (a) Given: m = 22.0 g = 0.0220 kg; θ = 32.5°
Required: a
Analysis: Look at the situation from an Earth (inertial) frame of reference. The horizontal component of the tension F_T balances the acceleration and the vertical component of the tension F_T balances the gravitational force. Express the components of the tension in terms of the horizontal and vertical applied forces. Then calculate the magnitude of the acceleration.
Solution: Vertical component of force:

$$\Sigma F_y = 0$$
$$F_T \cos\theta - mg = 0$$
$$F_T = \frac{mg}{\cos\theta}$$

Horizontal component of force:

$$\Sigma F_x = ma$$
$$F_T \sin\theta = ma$$
$$\left(\frac{\not{m}g}{\cos\theta}\right)\sin\theta = \not{m}a$$
$$g\left(\frac{\sin\theta}{\cos\theta}\right) = a$$
$$g\tan\theta = a$$
$$a = (9.8\text{ m/s}^2)\tan 32.5°$$
$$a = 6.2\text{ m/s}^2$$

Statement: The magnitude of the boat's acceleration is 6.2 m/s^2. I did not need to know the mass of the ball to make the calculation because that value was cancelled out to obtain the forces.

(b) Given: m = 22.0 g = 0.0220 kg; θ = 32.5°

Required: F_T

Analysis: From part (a), $F_T = \frac{mg}{\cos\theta}$.

Solution:
$$F_T = \frac{mg}{\cos\theta}$$
$$= \frac{(0.0220\text{ kg})(9.8\text{ m/s}^2)}{\cos 32.5°}$$
$$F_T = 0.26\text{ N}$$

Statement: The magnitude of the tension in the string is 0.26 N. I need to know the mass to make this calculation because the force is the mass multiplied by the acceleration.

3. (a) When the subway is moving at a constant velocity, there is no tension in the strap.

(b) Given: m = 14 kg; θ = 35°; a = 1.4 m/s^2

Required: F_T

Analysis: Look at the situation from an Earth (inertial) frame of reference. The horizontal component of the tension F_T balances the acceleration, so express the x-components of the tension in terms of the horizontal applied forces. The vertical forces of gravity and the normal force balance each other.

$$\Sigma F_x = ma$$
$$F_T \sin\theta = ma$$
$$F_T = \frac{ma}{\sin\theta}$$

Solution:
$$F_T = \frac{ma}{\sin\theta}$$
$$= \frac{(14\text{ kg})(1.4\text{ m/s}^2)}{\tan 35°}$$
$$F_T = 34\text{ N}$$

Statement: The tension on the strap during acceleration is 34 N.

4. Given: $\mu_s = 0.42$
Required: maximum a
Analysis: The force due to the acceleration of the train, $F = ma$, must be less than the force of static friction, $F_s = \mu_s F_N$, where $F_N = mg$ $F_N = mg$. So the maximum acceleration occurs when $F = F_s$.

$$\begin{aligned} F &= F_s \\ ma &= \mu_s F_N \\ \cancel{m}a &= \mu_s(\cancel{m}g) \\ a &= \mu_s g \end{aligned}$$

Solution:
$$\begin{aligned} a &= \mu_s g \\ &= (0.42)(9.8 \text{ m/s}^2) \\ a &= 4.1 \text{ m/s}^2 \end{aligned}$$

Statement: The maximum acceleration of the train before the passenger begins to slip along the floor is 4.1 m/s^2.

Tutorial 2 Practice, page 112

1. (a) Given: m = 55 kg; $\vec{a}$ = 2.9 m/s^2 [up]
Required: F_N
Analysis: Use up as positive and solve for the normal force when $+F_N + (-mg) = ma$.

$$\begin{aligned} +F_N + (-mg) &= ma \\ F_N &= ma + mg \end{aligned}$$

Solution:
$$\begin{aligned} F_N &= ma + mg \\ &= (55 \text{ kg})(2.9 \text{ m/s}^2) + (55 \text{ kg})(9.8 \text{ m/s}^2) \\ F_N &= 7.0 \times 10^2 \text{ N} \end{aligned}$$

Statement: The student's apparent weight is 7.0×10^2 N.
(b) Given: m = 55 kg; $\vec{a}$ = 2.9 m/s^2 [down]
Required: F_N
Analysis: Use down as positive and solve for the normal force when $-F_N + (mg) = ma$.

$$\begin{aligned} -F_N + (mg) &= ma \\ F_N &= ma - mg \end{aligned}$$

Solution:
$$\begin{aligned} F_N &= mg - ma \\ &= (55 \text{ kg})(9.8 \text{ m/s}^2) - (55 \text{ kg})(2.9 \text{ m/s}^2) \\ F_N &= 3.8 \times 10^2 \text{ N} \end{aligned}$$

Statement: The student's apparent weight is 3.8×10^2 N.

2. (a) Given: m_1 = 9.5 kg; m_2 = 2.5 kg; F_N = 70.0 N
Required: $\vec{a}$
Analysis: Use up as positive and solve for the acceleration when $+F_N + (-mg) = ma$.

$$+F_N + (-mg) = ma$$
$$a = \frac{F_N - mg}{m}$$
$$= \frac{F_N - (m_1 + m_2)g}{m_1 + m_2}$$

Solution: $a = \dfrac{F_N - (m_1 + m_2)g}{m_1 + m_2}$

$$= \frac{70.0\ \cancel{\text{kg}} \cdot \frac{\text{m}}{\text{s}^2} - (9.5\ \cancel{\text{kg}} + 2.5\ \cancel{\text{kg}})(9.8\ \text{m/s}^2)}{9.5\ \cancel{\text{kg}} + 2.5\ \cancel{\text{kg}}}$$
$$= -3.967\ \text{m/s}^2 \text{ (two extra digits carried)}$$
$$a = -4.0\ \text{m/s}^2$$

Statement: The acceleration of the elevator is 4.0 m/s^2 [down].
(b) Given: m = 2.5 kg; $\vec{a}$ = 3.967 m/s^2 [down]
Required: $\vec{F}_N$
Analysis: Use down as positive and solve for the normal force when $-F_N + (mg) = ma$.
Solution: $-F_N + (mg) = ma$

$$F_N = mg - ma$$
$$= (2.5\ \text{kg})(9.8\ \text{m/s}^2) - (2.5\ \text{kg})(3.967\ \text{m/s}^2)$$
$$F_N = 15\ \text{N}$$

Statement: The force on the smaller box is 15 N [up].
3. (a) Given: m_1 = 4.2 kg; m_2 = 2.6 kg; a = 0
Required: F_{TA}; F_{TB}
Analysis: Since this is an inertial frame of reference, the tension on rope B balances the force of gravity on block 2. The tension on rope A balances the force of gravity on block 1 and the tension on rope B. $F_g = mg$
Solution: Determine the force on rope B:

$$F_{TB} = m_2 g$$
$$= (2.6\ \text{kg})(9.8\ \text{m/s}^2)$$
$$= 25.48\ \text{N (two extra digits carried)}$$
$$F_{TB} = 25\ \text{N}$$

Determine the force on rope A:

$$F_{TA} = m_1 g + F_{TB}$$
$$= (4.2\ \text{kg})(9.8\ \text{m/s}^2) + 25.48\ \text{N}$$
$$F_{TA} = 67\ \text{N}$$

Statement: The tension on the rope A is 67 N and the tension on the rope B is 25 N.

(b) Given: m_1 = 4.2 kg; m_2 = 2.6 kg $\vec{a}$ = 1.2 m/s^2 [up]
Required: F_{TA}; F_{TB}
Analysis: This is now a non-inertial frame of reference, so instead of just gravity, the acceleration is $g - (-a)$, or $g + a$, where down is positive.
Solution: Determine the force on rope B:

$$\begin{aligned} F_{TB} &= m_2(g+a) \\ &= (2.6 \text{ kg})(9.8 \text{ m/s}^2 + 1.2 \text{ m/s}^2) \\ &= 28.6 \text{ N (one extra digit carried)} \\ F_{TB} &= 29 \text{ N} \end{aligned}$$

Determine the force on rope A:

$$\begin{aligned} F_{TA} &= m_1(g+a) + F_{TB} \\ &= (4.2 \text{ kg})(9.8 \text{ m/s}^2 + 1.2 \text{ m/s}^2) + 28.6 \text{ N} \\ F_{TA} &= 75 \text{ N} \end{aligned}$$

Statement: The tension on the rope A is 75 N and the tension on the rope B is 29 N.
4. Given: $\vec{a}$ = 0.98 m/s^2 [down]; m = 61 kg
Required: F_N
Analysis: Use down as positive and solve for the normal force when $-F_N + (mg) = ma$.
Solution: $-F_N + (mg) = ma$

$$\begin{aligned} F_N &= mg - ma \\ &= (61 \text{ kg})(9.8 \text{ m/s}^2) - (61 \text{ kg})(0.98 \text{ m/s}^2) \\ F_N &= 5.4 \times 10^2 \text{ N} \end{aligned}$$

Statement: The passenger's apparent weight is 5.4×10^2 m/s^2.

Section 3.1 Questions, page 113

1. (a) The ball would appear to move straight up and down because I am moving with the same velocity as the other train. From my viewpoint, the other train and passenger are standing still, and the ball is not affected by the train's motion.
(b) If the trains moved in opposite directions, the ball would appear to have horizontal motion, so I would see the path as a parabola.
2. Given: a = 1.5 m/s^2
Required: θ
Analysis: The horizontal component of the tension F_T balances the acceleration, and the vertical component of the tension F_T balances the gravitational force. Express the tangent ratio of the angle in terms of the applied force and the gravitational force, then solve for the angle.

$$\begin{aligned} \tan\theta &= \frac{F_a}{F_g} \\ \theta &= \tan^{-1}\left(\frac{\cancel{m}a}{\cancel{m}g}\right) \\ \theta &= \tan^{-1}\left(\frac{a}{g}\right) \end{aligned}$$

Solution: $\theta = \tan^{-1}\left(\frac{a}{g}\right)$

$$= \tan^{-1}\left(\frac{1.5\ \frac{\cancel{m}}{\cancel{s^2}}}{9.8\ \frac{\cancel{m}}{\cancel{s^2}}}\right)$$

$$\theta = 8.7°$$

Statement: The string makes an 8.7° angle with the vertical.

3. Given: $v_f = 255$ km/h; $\Delta t = 10.0$ s

Required: θ

Analysis: Determine the acceleration using $v_f = a\Delta t$ or $a = \frac{v_f}{\Delta t}$. The horizontal component of the tension F_T balances the acceleration and the vertical component of the tension F_T balances the gravitational force. Express the tangent ratio of the angle in terms of the applied force and the gravitational force, then solve for the angle.

Solution: Determine the acceleration of the plane:

$$a = \frac{v_f}{\Delta t}$$

$$= \frac{255\ \frac{\cancel{km}}{\cancel{h}}}{10.0\ \text{s}} \times \frac{1000\ \text{m}}{1\ \cancel{km}} \times \frac{1\ \cancel{h}}{60\ \cancel{min}} \times \frac{1\ \cancel{min}}{60\ \text{s}}$$

$$= 7.0833\ \text{m/s}^2\ \text{(two extra digits carried)}$$

$$a = 7.08\ \text{m/s}^2$$

Determine the angle the string makes with the vertical:

$$\tan\theta = \frac{F_a}{F_g}$$

$$\theta = \tan^{-1}\left(\frac{\cancel{m}a}{\cancel{m}g}\right)$$

$$= \tan^{-1}\left(\frac{a}{g}\right)$$

$$= \tan^{-1}\left(\frac{7.0833\ \frac{\cancel{m}}{\cancel{s^2}}}{9.8\ \frac{\cancel{m}}{\cancel{s^2}}}\right)$$

$$\theta = 35.9°$$

Statement: The string makes an 35.9° angle with the vertical.

4. Given: $\theta = 16°$

Required: a

Analysis: Look at the situation from an Earth (inertial) frame of reference. The horizontal component of the tension F_T balances the acceleration and the vertical component of the tension F_T must balance the gravitational force since the cork ball does not move. Express the components of the tension in terms of the horizontal and vertical applied forces. Then calculate the magnitude of the acceleration.

Vertical component of force:

$$\Sigma F_y = 0$$

$$F_T \cos\theta - mg = 0$$

$$F_T = \frac{mg}{\cos\theta}$$

Horizontal component of force:

$$\Sigma F_x = ma$$

$$F_T \sin\theta = ma$$

$$\left(\frac{\cancel{m}g}{\cos\theta}\right)\sin\theta = \cancel{m}a$$

$$g\left(\frac{\sin\theta}{\cos\theta}\right) = a$$

$$g\tan\theta = a$$

Solution: $a = g\tan\theta$

$$= \left(9.8\ \text{m/s}^2\right)\tan 16^\circ$$

$$a = 2.8\ \text{m/s}^2$$

Statement: The magnitude of the car's acceleration is 2.8 m/s^2.

5. Given: v_f = 6.0 m/s; Δt = 10.0 s; m = 64 kg

Required: F_N

Analysis: Determine the upward acceleration using $v_f = a\Delta t$ or $a = \frac{v_f}{\Delta t}$. Use up as positive and solve for the normal force when $+F_N + (-mg) = ma$.

$$+F_N + \left(-mg\right) = ma$$

$$F_N = ma + mg$$

Solution: Determine the upward acceleration:

$$a = \frac{v_f}{\Delta t}$$

$$= \frac{6.0\ \text{m/s}}{10.0\ \text{s}}$$

$$a = 0.60\ \text{m/s}^2$$

Determine the apparent weight:

$$F_N = ma + mg$$

$$= \left(64\ \text{kg}\right)\left(0.6\ \text{m/s}^2\right) + \left(64\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right)$$

$$F_N = 6.7\times 10^2\ \text{N}$$

Statement: The passenger's apparent weight is 6.7×10^2 N.

6. Given: F_N = 255 N; m = 52 kg
Required: a
Analysis: Use up as positive and solve for the acceleration when $+F_N + (-mg) = ma$.

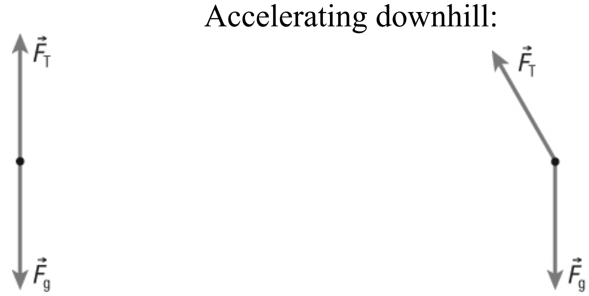
$$+F_N + (-mg) = ma$$
$$a = \frac{F_N - mg}{m}$$

Solution: $a = \frac{F_N - mg}{m}$

$$= \frac{255\ \cancel{\text{kg}} \cdot \frac{\text{m}}{\text{s}^2} - (52\ \cancel{\text{kg}})(9.8\ \text{m/s}^2)}{52\ \cancel{\text{kg}}}$$
$$a = -4.9\ \text{m/s}^2$$

Statement: The acceleration of the ride is 4.9 m/s^2 [down].

7. (a) At rest: Accelerating downhill:



When the car is at rest, the dice are aligned with the vertical (with respect to level ground), and make an angle of 17° with respect to the normal (perpendicular to the roof). The only forces acting on the dice are gravity and tension. When the car is accelerating, a horizontal fictitious force pointing to the rear of the car deflects the dice so that they are aligned with the normal, and make an angle of 17° with respect to the vertical, as viewed from the level ground.

(b) Given: θ = 17°
Required: a
Analysis: Look at the situation from an inertial frame of reference on the same angle as the hill. The "vertical" component of the gravitational force F_g balances the tension and the "horizontal" component of the gravitational force F_g balances the force applied by the acceleration. Express the horizontal component of F_g in terms of the applied forces. Then calculate the magnitude of the acceleration.

$$\Sigma F_x = 0$$
$$F_g \sin\theta - ma = 0$$
$$mg \sin\theta - ma = 0$$
$$g \sin\theta - a = 0$$
$$a = g \sin\theta$$

Solution: $a = g \sin\theta$

$$= (9.8\ \text{m/s}^2)\sin 17°$$
$$a = 2.9\ \text{m/s}^2$$

Statement: The magnitude of the car's acceleration is 2.9 m/s^2.

8. (a) Given: m_1 = 1.8 kg; m_2 = 1.2 kg; m_2 = 1.2 kg; F_N = 70.0 N
Required: F_a

Analysis: Since mass 1 does not slide, the acceleration due to the horizontal force must balance the acceleration due to the tension, which equals m_2g. Use $F = ma$ to determine the acceleration.

$$\frac{F_a}{m_1 + m_2 + m_3} = \frac{F_T}{m_1}$$

$$\frac{F_a}{m_1 + m_2 + m_3} = \frac{m_2 g}{m_1}$$

$$F_a = \frac{m_2 g}{m_1}(m_1 + m_2 + m_3)$$

Solution: $F_a = \frac{m_2 g}{m_1}(m_1 + m_2 + m_3)$

$$\frac{(1.2\text{ kg})(9.8\text{ m/s}^2)}{(1.8\text{ kg})}(1.8\text{ kg} + 1.2\text{ kg} + 3.0\text{ kg})$$

$$F_a = 39\text{ N}$$

Statement: The applied force is 39 N.
(b) Given: m_1 = 1.2 kg; m_2 = 2.8 kg; θ = 25°
Required: F_a
Analysis: Since mass 1 does not slide, the applied force on mass 1, the gravitational force, and the normal force must balance each other: $\Sigma F = 0$. Use $F = ma$ to determine the acceleration. Since the applied force is entirely horizontal and the gravitational force is entirely vertical, use the tangent ratio to relate them.
Determine the applied acceleration:

$$F = ma$$

$$F_a = (m_1 + m_2)a$$

$$a = \frac{F_a}{m_1 + m_2}$$

Determine the horizontal acceleration of mass 1:

$$\tan\theta = \frac{\cancel{m_1} a}{\cancel{m_1} g} = \frac{a}{g}$$

$$a = g\tan\theta$$

Determine the applied force:

$$\frac{F_a}{m_1 + m_2} = g\tan\theta$$

$$F_a = g\tan\theta(m_1 + m_2)$$

Solution: $F_\text{a} = g\tan\theta\left(m_1 + m_2\right)$

$\left(9.8\text{ m/s}^2\right)\left(\tan 25°\right)\left(1.2\text{ kg} + 2.8\text{ kg}\right)$

$F_\text{a} = 17\text{ N}$

Statement: The applied force is 17 N.