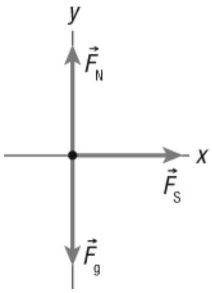
Section 2.4: Forces of Friction

Mini Investigation: Light from Friction, page 86

A. When we crushed the mints, the candy briefly gave off light.
B. The crystals of sugar in the mints rubbed together to create the friction that produces the light.

Tutorial 1 Practice, page 89

1. (a) An FBD of the top book during its acceleration is shown below.



(b) The force of static friction causes the top book to accelerate horizontally.

2. Given: $\underline{a} = 2.7 \text{ m/s}^2$

Required: μ_S

Analysis: $\vec{F}_S = \mu_S \vec{F}_N$; $\Sigma\vec{F} = m\vec{a}$

Solution: Equation for *y*-components of the force:

$$\begin{aligned}\Sigma\vec{F}_y &= 0 \text{ N}\\ \vec{F}_N - mg &= 0 \text{ N}\\ \vec{F}_N &= mg\end{aligned}$$

Equation for *x*-components of the force:

$$\begin{aligned}\Sigma\vec{F}_x &= m\vec{a}\\ \vec{F}_S &= m\vec{a}\\ \mu_S\vec{F}_N &= m\vec{a}\\ \mu_S \cancel{m}g &= \cancel{m}\vec{a}\\ \mu_S &= \frac{\vec{a}}{g}\\ &= \frac{2.7 \cancel{\text{m/s}^2}}{9.8 \cancel{\text{m/s}^2}}\\ \mu_S &= 0.28\end{aligned}$$

Statement: The smallest coefficient of static friction between dinner plates that will prevent slippage is 0.28.

3. Given: $\vec{F}_T = 28 \text{ N [forward 29° up]}$; $\mu_S = 0.45$; $\mu_K = 0.45$; $\Sigma\vec{F} = 0 \text{ N}$

Required: m

Analysis: $\Sigma\vec{F} = m\vec{a}$

Solution: Equation for *y*-components of the force:

$$\Sigma\vec{F}_y = 0 \text{ N}$$

$$\vec{F}_\text{N} + \vec{F}_\text{T}\sin\theta - mg = 0 \text{ N}$$

$$\vec{F}_\text{N} = mg - \vec{F}_\text{T}\sin\theta \quad \text{(Equation 1)}$$

Equation for *x*-components of the force:

$$\Sigma\vec{F}_x = m\vec{a}$$

$$\vec{F}_\text{T}\cos\theta - \vec{F}_\text{S} = m\vec{a}$$

$$\vec{F}_\text{T}\cos\theta - \mu_\text{S}\vec{F}_\text{N} = m\vec{a} \quad \text{(Equation 2)}$$

Substitute Equation (1) into Equation (2) and solve for *m*:

$$\vec{F}_\text{T}\cos\theta - \mu_\text{S}\vec{F}_\text{N} = 0 \text{ N}$$

$$\vec{F}_\text{T}\cos\theta - \mu_\text{S}\left(mg - \vec{F}_\text{T}\sin\theta\right) = 0 \text{ N}$$

$$\vec{F}_\text{T}(\cos\theta + \mu_\text{S}\sin\theta) = \mu_\text{S}mg$$

$$m = \frac{\vec{F}_\text{T}(\cos\theta + \mu_\text{S}\sin\theta)}{\mu_\text{S}g}$$

$$= \frac{(28 \text{ N})(\cos 29° + 0.45\sin 29°)}{0.45(9.8 \text{ m/s}^2)}$$

$$m = 6.9 \text{ kg}$$

Statement: The smallest possible mass for the box is 6.9 kg.

4. Given: $\theta = 6.0°$; $\vec{v}_\text{i} = 12$ m/s [down slope]; $v_\text{f} = 0$ m/s; $\mu_\text{K} = 0.14$

Required: Δd

Analysis: $\Sigma\vec{F} = m\vec{a}$; $v_\text{f}^2 = v_\text{i}^2 + 2a\Delta d$

Solution: Equation for *y*-components of the force:

$$\Sigma\vec{F}_y = 0 \text{ N}$$

$$\vec{F}_\text{N} - mg\cos\theta = 0 \text{ N}$$

$$\vec{F}_\text{N} = mg\cos\theta$$

Equation for *x*-components of the force:

$$\Sigma\vec{F}_x = m\vec{a}_x$$

$$mg\sin\theta - \vec{F}_\text{K} = m\vec{a}_x$$

$$\cancel{m}g\sin\theta - \mu_\text{K}\cancel{m}g\cos\theta = \cancel{m}\vec{a}_x$$

$$\vec{a}_x = g(\sin\theta - \mu_\text{K}\cos\theta)$$

$$= (9.8 \text{ m/s}^2)(\sin 6.0° - 0.14\cos 6.0°)$$

$$\vec{a}_x = -0.3401 \text{ m/s}^2 \text{ (two extra digits carried)}$$

Solve for the distance travelled:

$$v_f^2 = v_i^2 + 2\vec{a}_x \Delta d$$

$$\Delta d = \frac{v_f^2 - v_i^2}{2\vec{a}_x}$$

$$= \frac{(0 \text{ m/s})^2 - (12 \text{ m/s})^2}{2(-0.3401 \text{ m/s}^2)}$$

$$\Delta d = 2.1 \times 10^2 \text{ m}$$

Statement: The sled will slide for 2.1×10^2 m before coming to rest.

5. Given: $m = 39$ kg; direction of rope [forward 21° up]; $\mu_K = 0.23$; $\Sigma\vec{F} = 0$ N

Required: $\vec{F}_T$

Analysis: $\Sigma\vec{F} = 0$ N

Solution: Equation for y-components of the force:

$$\Sigma\vec{F}_y = 0 \text{ N}$$

$$\vec{F}_N + \vec{F}_{Ty} - mg = 0 \text{ N}$$

$$\vec{F}_N = mg - \vec{F}_T \sin\theta$$

Equation for x-components of the force:

$$\Sigma\vec{F}_x = 0 \text{ N}$$

$$\vec{F}_{Tx} - \vec{F}_K = 0 \text{ N}$$

$$\vec{F}_T \cos\theta - \mu_K \vec{F}_N = 0 \text{ N}$$

$$\vec{F}_T \cos\theta - \mu_K (mg - \vec{F}_T \sin\theta) = 0 \text{ N}$$

$$\vec{F}_T (\cos\theta + \mu_K \sin\theta) = \mu_K mg$$

$$\vec{F}_T = \frac{\mu_K mg}{\cos\theta + \mu_K \sin\theta}$$

$$= \frac{(0.23)(39 \text{ kg})(9.8 \text{ m/s}^2)}{\cos 21° + 0.23 \sin 21°}$$

$$F_T = 87 \text{ N}$$

Statement: A tension of 87 N in the rope is needed to keep the box moving at a constant velocity.

6. (a) Given: $m_1 = 24$ kg; $m_2 = 14$ kg; $\mu_K = 0.32$; $\vec{F}_a = 1.8 \times 10^2$ N [forward 25° up]

Required: $\vec{a}$

Analysis: $\Sigma\vec{F}_y = 0$ N; $\Sigma\vec{F}_x = m_1\vec{a}$

Solution: Equation for *y*-components of the force:

$$\Sigma\vec{F}_y = 0\text{ N}$$

$$\vec{F}_{\text{N1}} + \vec{F}_{\text{a}y} - m_1 g = 0\text{ N}$$

$$\vec{F}_{\text{N1}} = m_1 g - \vec{F}_{\text{a}}\sin\theta$$

$$= (24\text{ kg})(9.8\text{ m/s}^2) - (180\text{ N})\sin 25°$$

$$\vec{F}_{\text{N1}} = 159.1\text{ N (two extra digits carried)}$$

$$\vec{F}_{\text{K1}} = \mu_{\text{K}}\vec{F}_{\text{N1}}$$

$$= 0.25(159.1\text{ N})$$

$$\vec{F}_{\text{K1}} = 39.78\text{ N (two extra digits carried)}$$

Equation for *x*-components of the force:

$$\Sigma\vec{F}_x = m_1\vec{a}$$

$$\vec{F}_{\text{a}x} - \vec{F}_{\text{K1}} - \vec{F}_{\text{T}} = m_1\vec{a}\quad\text{(Equation 1)}$$

Equation for *y*-components of the force:

$$\Sigma\vec{F}_y = 0\text{ N}$$

$$\vec{F}_{\text{N2}} - m_2 g = 0\text{ N}$$

$$\vec{F}_{\text{N2}} = m_2 g$$

$$= (14\text{ kg})(9.8\text{ m/s}^2)$$

$$\vec{F}_{\text{N2}} = 137.2\text{ N (two extra digits carried)}$$

$$\vec{F}_{\text{K2}} = \mu_{\text{K}}\vec{F}_{\text{N2}}$$

$$= 0.25(137.2\text{ N})$$

$$\vec{F}_{\text{K2}} = 34.3\text{ N}$$

Equation for *x*-components of the force:

$$\Sigma\vec{F}_x = m_2\vec{a}$$

$$\vec{F}_{\text{T}} - \vec{F}_{\text{K2}} = m_2\vec{a}\quad\text{(Equation 2)}$$

Add equations (1) and (2) to eliminate the tension of the rope. Solve for a.

$$\left(F_{\mathrm{T}}-F_{\mathrm{K2}}\right)+\left(F_{\mathrm{a}x}-F_{\mathrm{K1}}-F_{\mathrm{T}}\right)=m_1a+m_2a$$

$$F_{\mathrm{a}x}-F_{\mathrm{K1}}-F_{\mathrm{K2}}=(m_1+m_2)a$$

$$\begin{aligned}a&=\frac{F_{\mathrm{a}x}-F_{\mathrm{K1}}-F_{\mathrm{K2}}}{m_1+m_2}\\&=\frac{(180\ \mathrm{N})\cos 25°-39.78\ \mathrm{N}-34.3\ \mathrm{N}}{38\ \mathrm{kg}}\\&=1.797\ \mathrm{m/s^2}\text{(two extra digits carried)}\\a&=1.8\ \mathrm{m/s^2}\end{aligned}$$

Statement: The acceleration of the boxes is 1.8 m/s^2.

(b) Given: $m_2=14\ \mathrm{kg}$**;**; $\mu_{\mathrm{K}}=0.32$; $\vec{F}_{\mathrm{a}}=1.8\times10^2\ \mathrm{N}$ [forward 25° up]

Required: $\vec{F}_{\mathrm{T}}$

Analysis: $\vec{F}_{\mathrm{T}}-\vec{F}_{\mathrm{K2}}=m_2\vec{a}$

Solution: $\vec{F}_{\mathrm{T}}-\vec{F}_{\mathrm{K2}}=m_2\vec{a}$

$$\begin{aligned}\vec{F}_{\mathrm{T}}&=\mu_{\mathrm{K}}\vec{F}_{\mathrm{N2}}+m_2\vec{a}\\&=0.25(137.2)+(14\ \mathrm{kg})(1.797\ \mathrm{m/s^2})\\\vec{F}_{\mathrm{T}}&=59\ \mathrm{N}\end{aligned}$$

Statement: The tension in the rope is 59 N.

7. Given: $\mu_{\mathrm{K}}=0.20$; $\mu_{\mathrm{S}}=0.25$; $m=100.0\ \mathrm{kg}$

Required: a

Analysis: $\Sigma\vec{F}=0\ \mathrm{N}$; $\Sigma\vec{F}_x=m\vec{a}_x$

Solution: For the y-components of the force:

$$\begin{aligned}\Sigma\vec{F}_y&=0\ \mathrm{N}\\\vec{F}_{\mathrm{N}}-mg&=0\ \mathrm{N}\\\vec{F}_{\mathrm{N}}&=mg\\&=(100.0\ \mathrm{kg})(9.8\ \mathrm{m/s^2})\\\vec{F}_{\mathrm{N}}&=980\ \mathrm{N}\end{aligned}$$

For the x-components of the force:

$$\begin{aligned}\Sigma\vec{F}_x&=0\ \mathrm{N}\\\vec{F}_{\mathrm{a}}-\vec{F}_{\mathrm{S}}&=0\ \mathrm{N}\\\vec{F}_{\mathrm{a}}&=\vec{F}_{\mathrm{S}}\\&=\mu_{\mathrm{s}}\vec{F}_{\mathrm{N}}\\&=0.25(980\ \mathrm{N})\\\vec{F}_{\mathrm{a}}&=245\ \mathrm{N}\end{aligned}$$

Once the refrigerator is moving, an applied for 245 N produces acceleration, *a*:

$$\Sigma\vec{F}_x = m\vec{a}$$
$$\vec{F}_a - \vec{F}_K = m\vec{a}$$
$$\vec{a} = \frac{\vec{F}_a - \mu_K\vec{F}_N}{m}$$
$$= \frac{245\text{ N} - 0.20(980\text{ N})}{100.0\text{ kg}}$$
$$\vec{a} = 0.49\text{ m/s}^2$$

Statement: The acceleration when you apply minimum force needed to move the refrigerator is 0.49 m/s^2.

8. Given: $\mu_S = 0.25$; $m = 110$ kg

Required: $\vec{F}_a$

Analysis: $\vec{F}_S = \mu_S\vec{F}_N$; $\Sigma\vec{F} = 0$ N

Solution: For the *y*-components of the force:

$$\Sigma\vec{F}_y = 0\text{ N}$$
$$\vec{F}_N - mg = 0\text{ N}$$
$$\vec{F}_N = mg$$
$$= (110\text{ kg})(9.8\text{ m/s}^2)$$
$$\vec{F}_N = 1078\text{ N (two extra digits carried)}$$

For the *x*-components of the force:

$$\Sigma\vec{F}_x = 0\text{ N}$$
$$\vec{F}_a - \vec{F}_S = 0\text{ N}$$
$$\vec{F}_a = \vec{F}_S$$
$$= \mu_s\vec{F}_N$$
$$= 0.25(1078\text{ N})$$
$$\vec{F}_a = 2.7\times10^2\text{ N}$$

Statement: The minimum force required to just set the stage prop into motion is 2.7×10^2 N.

Section 2.4 Questions, page 90

1. Given: $v_i = 20$ m/s; $v_f = 0$ m/s; $\Delta d = 40$ m

Required: coefficient of friction between the tires and the road, μ_K

Analysis: $v_f{}^2 = v_i{}^2 + 2a\Delta d$; $\Sigma\vec{F}_x = m\vec{a}_x$. Choose forward as positive.

Solution: Find the acceleration $\vec{a}$:

$$v_f^2 = v_i^2 + 2\vec{a}\Delta d$$

$$\vec{a} = -\frac{v_i^2}{2\Delta d}$$

$$= -\frac{(20 \text{ m/s})^2}{2(40 \text{ m})}$$

$$\vec{a} = -5.0 \text{ m/s}^2$$

Calculate μ_K:

$$\Sigma\vec{F}_x = m\vec{a}$$

$$-\vec{F}_K = m\vec{a}$$

$$-\mu_K \cancel{m} g = \cancel{m}\vec{a}$$

$$\mu_K = -\frac{\vec{a}}{g}$$

$$= \frac{5.0 \ \cancel{\text{m/s}^2}}{9.8 \ \cancel{\text{m/s}^2}}$$

$$\mu_K = 0.5$$

Statement: The coefficient of friction between the tires and the road is 0.5.

2. Given: $v_i = 50.0$ m/s; $\Delta t = 10.0$ s; $\mu_K = 0.030$

Required: v_f

Analysis: $\Sigma\vec{F}_x = m\vec{a}$. Choose forward as positive.

Solution: Find the acceleration *a*:

$$\Sigma\vec{F}_x = m\vec{a}$$

$$-\vec{F}_K = m\vec{a}$$

$$-\mu_K \cancel{m} g = \cancel{m}\vec{a}$$

$$\vec{a} = -\mu_K g$$

$$= -0.030(9.8 \text{ m/s}^2)$$

$$\vec{a} = -0.294 \text{ m/s}^2 \text{ (one extra digit carried)}$$

Calculate v_f:

$$v_\text{f} = v_\text{i} + a\Delta t$$
$$= 50.0 \text{ m/s} + (-0.294 \text{ m/s}^2)(10.0 \text{ s})$$
$$v_\text{f} = 47 \text{ m/s}$$

Statement: The speed of the puck after 10.0 s is 47 m/s.

3. (a) Given: $m = 2.0 \times 10^2 \text{ kg}; \vec{F}_\text{a} = 3.5 \times 10^2 \text{ N}$

Required: μ_S

Analysis: $\Sigma \vec{F} = 0 \text{ N}$; $\vec{F}_\text{S} = \mu_\text{S} \vec{F}_\text{N}$

Solution: For the *y*-components of the force:

$$\Sigma \vec{F}_y = 0 \text{ N}$$
$$\vec{F}_\text{N} - mg = 0 \text{ N}$$
$$\vec{F}_\text{N} = mg$$

For the *x*-components of the force:

$$\Sigma \vec{F}_x = 0 \text{ N}$$
$$\vec{F}_\text{a} - \vec{F}_\text{S} = 0 \text{ N}$$
$$\vec{F}_\text{S} = \vec{F}_\text{a}$$
$$\mu_\text{S} \vec{F}_\text{N} = \vec{F}_\text{a}$$
$$\mu_\text{S} = \frac{\vec{F}_\text{a}}{mg}$$
$$= \frac{350 \cancel{\text{N}}}{(200 \cancel{\text{kg}})(9.8 \cancel{\text{m/s}^2})}$$
$$\mu_\text{S} = 0.18$$

Statement: The coefficient of static friction between the floor and the sofa is 0.18.

(b) Given: $m = 2.0 \times 10^2 \text{ kg}; \vec{F}_\text{a} = 3.5 \times 10^2 \text{ N}; v_\text{i} = 0 \text{ m/s}; v_\text{f} = 2.0 \text{ m/s}; \Delta t = 5.0 \text{ s}$

Required: μ_K

Analysis: $\Sigma \vec{F} = m\vec{a}$

Solution: Find the acceleration, $\vec{a}$:

$$\vec{a} = \frac{\Delta v}{\Delta t}$$
$$= \frac{2.0 \text{ m/s}}{5.0 \text{ s}}$$
$$\vec{a} = 0.40 \text{ m/s}^2$$

Using the x-components of the force:

$$\Sigma\vec{F}_x = m\vec{a}$$
$$\vec{F}_a - \vec{F}_K = m\vec{a}$$
$$\vec{F}_K = \vec{F}_a - m\vec{a}$$
$$\mu_K \vec{F}_N = \vec{F}_a - m\vec{a}$$
$$\mu_K = \frac{\vec{F}_a - m\vec{a}}{mg}$$
$$= \frac{350\ \text{N} - (200\ \cancel{\text{kg}})(0.40\ \cancel{\text{m/s}^2})}{(200\ \cancel{\text{kg}})(9.8\ \cancel{\text{m/s}^2})}$$
$$\mu_K = 0.14$$

Statement: The coefficient of kinetic friction between the sofa and the floor is 0.14.

4. (a) Given: $\mu_S = 0.29$

Required: θ

Analysis: $\Sigma\vec{F} = 0\ \text{N}$

Solution: For the y-components of the force:

$$\Sigma\vec{F}_y = 0\ \text{N}$$
$$\vec{F}_N - mg\cos\theta = 0\ \text{N}$$
$$\vec{F}_N = mg\cos\theta$$

For the x-components of the force:

$$\Sigma\vec{F}_x = 0\ \text{N}$$
$$mg\sin\theta - \vec{F}_S = 0\ \text{N}$$
$$mg\sin\theta = \mu_S \vec{F}_N$$
$$\cancel{mg}\sin\theta = \mu_S \cancel{mg}\cos\theta$$
$$\tan\theta = \mu_S$$
$$\theta = \tan^{-1}(0.29)$$
$$\theta = 16^\circ$$

Statement: The crate just begins to slip when the angle of inclination, θ, is 16°.

(b) Given: $\mu_K = 0.26$; $\theta = 16.17^\circ$

Required: $\vec{a}$

Analysis: $\Sigma\vec{F}_x = m\vec{a}$

Solution:
$$\Sigma\vec{F}_x = m\vec{a}$$
$$mg\sin\theta - \vec{F}_\text{K} = m\vec{a}$$
$$\cancel{m}g\sin\theta - \mu_\text{K}\cancel{m}g\cos\theta = \cancel{m}\vec{a}$$
$$\vec{a} = g(\sin\theta - \mu_\text{K}\cos\theta)$$
$$= (9.8\text{ m/s}^2)(\sin 16.17° - 0.26\cos 16.17°)$$
$$\vec{a} = 0.28\text{ m/s}^2$$
Statement: The crate accelerates at 0.28 m/s^2 [down the incline] when the coefficient of kinetic friction is 0.26.

5. (a) Two situations in which friction is helpful for an object moving on a horizontal surface are when running, so you can push yourself forward, and when walking on a snowy field so you can use traction from the snow to move yourself forward.

(b) Two situations in which it would be ideal if there were no friction when an object moves across a horizontal surface are: shooting the puck across the ice when playing hockey; and when trying to move a sled over a snowy sidewalk.

6. (a) Given: $m_1 = 45\text{ kg}$; $m_2 = 12\text{ kg}$; $\mu_\text{S} = 0.45$; $\mu_\text{K} = 0.35$

Required: $\vec{F}_\text{g2} \le \mu\vec{F}_\text{N}$

Analysis: To determine if this system is in static equilibrium, you need to determine the magnitude of the static friction, F_S, for mass m_1 and compare it to the tension in the string, $\vec{F}_\text{g2}$, for mass m_2.

Find F_S for mass m_1.
$$\vec{F}_\text{S} = \mu_\text{S}\vec{F}_\text{N}$$
$$= \mu_\text{S}m_1g$$
$$= 0.45(45\text{ kg})(9.8\text{ m/s}^2)$$
$$\vec{F}_\text{S} = 200\text{ N}$$
Find $\vec{F}_\text{g2}$ for mass m_2.
$$\vec{F}_\text{g2} = m_2g$$
$$= (12\text{ kg})(9.8\text{ m/s}^2)$$
$$\vec{F}_\text{g2} = 120\text{ N}$$
Statement: Yes, the system is in static equilibrium. The tension in the string for mass m_2 is not sufficient to break the force of static friction for mass m_1, so there is no acceleration.

(b) Given: $\vec{F}_\text{g2} = 120\text{ N}$

Required: $\vec{F}_\text{T}$

Analysis: As long as the two masses remain at rest, the tension in the string is equal to the force of gravity on mass m_2, $\vec{F}_\text{T} = \vec{F}_\text{g2} = m_2g$.

Solution: $\vec{F}_{\mathrm{T}} = \vec{F}_{\mathrm{g2}}$

$$\vec{F}_{\mathrm{T}} = 120 \text{ N}$$

Statement: The tension in the string is 120 N.

(c) Given: $m_1 = 45$ kg; $m_2 = 32$ kg; $\mu_{\mathrm{K}} = 0.35$

Required: $\vec{a}$

Analysis: $\Sigma\vec{F} = m\vec{a}$

Solution: Find $\vec{F}_{\mathrm{T}}$ for mass m_1.

$$\Sigma\vec{F}_x = m_1\vec{a}$$

$$\vec{F}_{\mathrm{T}} - \vec{F}_{\mathrm{K}} = m_1\vec{a}$$

$$\vec{F}_{\mathrm{T}} - \mu_{\mathrm{K}} m_1 g = m_1\vec{a}$$

Find $\vec{F}_{\mathrm{T}}$ for mass m_2.

$$\Sigma\vec{F}_y = m_2\vec{a}$$

$$m_2 g - \vec{F}_{\mathrm{T}} = m_2\vec{a}$$

Solve for $\vec{a}$:

$$\left(\vec{F}_{\mathrm{T}} - \mu_{\mathrm{K}} m_1 g\right) + \left(m_2 g - \vec{F}_{\mathrm{T}}\right) = m_1\vec{a} + m_2\vec{a}$$

$$(m_2 - \mu_{\mathrm{K}} m_1)g = (m_1 + m_2)\vec{a}$$

$$\vec{a} = \frac{(m_2 - \mu_{\mathrm{K}} m_1)g}{m_1 + m_2}$$

$$= \frac{(32 \text{ kg} - 0.35(45 \text{ kg}))(9.8 \text{ m/s}^2)}{77 \text{ kg}}$$

$$\vec{a} = 2.1 \text{ m/s}^2$$

Statement: The acceleration of the system is 2.1 m/s^2, eliminating the string tension.

7. (a) Given: $\theta = 42°$

Required: μ_{S}

Analysis: $\Sigma\vec{F} = 0$ N

Solution: For the y-components of the force:

$$\Sigma\vec{F}_y = 0 \text{ N}$$

$$\vec{F}_{\mathrm{N}} - mg\cos\theta = 0 \text{ N}$$

$$\vec{F}_{\mathrm{N}} = mg\cos\theta$$

For the x-components of the force:

$$\Sigma\vec{F}_x = 0\text{ N}$$
$$mg\sin\theta - \vec{F}_S = 0\text{ N}$$
$$mg\sin\theta = \mu_S\vec{F}_N$$
$$\cancel{mg}\sin\theta = \mu_S\cancel{mg}\cos\theta$$
$$\tan\theta = \mu_S$$
$$\mu_S = \tan 42°$$
$$\mu_S = 0.90$$

Statement: The coefficient of static friction is 0.90.

(b) Given: $\theta = 35°$

Required: μ_K

Analysis: $\Sigma\vec{F}_x = 0\text{ N}$

Solution:

$$\Sigma\vec{F}_x = 0\text{ N}$$
$$mg\sin\theta - \vec{F}_K = 0\text{ N}$$
$$\cancel{mg}\sin\theta = \mu_K\cancel{mg}\cos\theta$$
$$\mu_K = \tan\theta$$
$$= \tan 35°$$
$$\mu_K = 0.70$$

Statement: The coefficient of kinetic friction is 0.70.

8. Given: $m_1 = 8.0\text{ kg}; m_2 = 12\text{ kg}; \theta_1 = 26°; \theta_2 = 39°; \mu_K = 0.21$

Required: $\vec{a}$

Analysis: $\Sigma\vec{F} = m\vec{a}$

Solution:

For the y-components of the force (mass m_1):

$$\Sigma\vec{F}_y = 0\text{ N}$$
$$\vec{F}_{N1} - m_1 g\cos\theta_1 = 0\text{ N}$$
$$\vec{F}_{N1} = m_1 g\cos\theta_1$$

For the y-components of the force (mass m_2):

$$\Sigma\vec{F}_y = 0\text{ N}$$
$$\vec{F}_{N2} - m_2 g\cos\theta_2 = 0\text{ N}$$
$$\vec{F}_{N2} = m_2 g\cos\theta_2$$

For the x-components of the force (mass m_1):

$$\Sigma\vec{F}_x = m_1\vec{a}$$

$$\vec{F}_\text{T} - m_1 g\sin\theta_1 - \vec{F}_\text{K1} = m_1\vec{a}$$

$$\vec{F}_\text{T} - m_1 g\sin\theta_1 - \mu_\text{K} m_1 g\cos\theta_1 = m_1\vec{a}$$

$$\vec{F}_\text{T} - 49.17\text{ N} = m_1\vec{a} \text{ (two extra digits carried)}$$

For the x-components of the force (mass m_2):

$$\Sigma\vec{F}_x = m_2\vec{a}$$

$$m_2 g\sin\theta_2 - \vec{F}_\text{K2} - \vec{F}_\text{T} = m_2\vec{a}$$

$$m_2 g\sin\theta_2 - \mu_\text{K} m_2 g\cos\theta_2 - \vec{F}_\text{T} = m_2\vec{a}$$

$$54.82\text{ N} - \vec{F}_\text{T} = m_2\vec{a} \text{ (two extra digits carried)}$$

Add the final equations for mass m_1 and mass m_2 to eliminate the string tension.

$$(\vec{F}_\text{T} - 49.17\text{ N}) + (54.82\text{ N} - \vec{F}_\text{T}) = (m_1 + m_2)\vec{a}$$

$$\vec{a} = \frac{54.82\text{ N} - 49.17\text{ N}}{20\text{ kg}}$$

$$\vec{a} = 0.28\text{ m/s}^2$$

Statement: The acceleration of the two masses (as a system) is 0.28 m/s^2 [clockwise].