Section 1.3: Displacement in Two Dimensions

Tutorial 1 Practice, page 25

1. Given: $\Delta\vec{d}_1 = 1.2$ km [S]; $\Delta\vec{d}_2 = 3.1$ km [E 53° N]

Required: $\Delta\vec{d}_\text{T}$; the angle for $\Delta\vec{d}_\text{T}$, θ

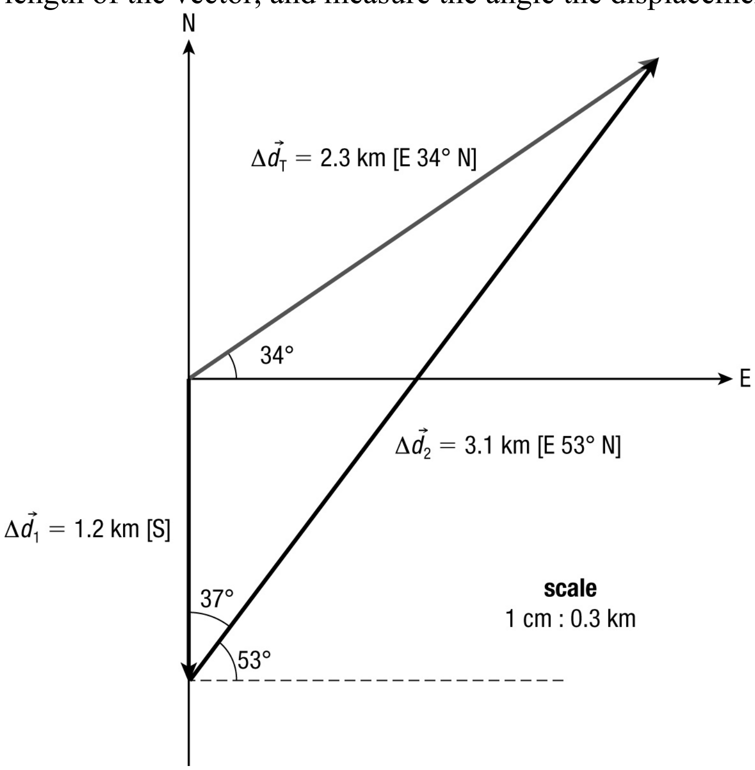
Analysis: $\Delta\vec{d}_\text{T} = \Delta\vec{d}_1 + \Delta\vec{d}_2$. Decide on a scale and then draw each vector to scale on a coordinate axis. Draw the total displacement vector.

Solution: An appropriate scale is 1 cm : 0.3 km. Calculate the lengths of the arrows for the displacement vectors:

$$\left|\Delta\vec{d}_1\right| = 1.2\ \cancel{\text{km}} \times \frac{1\ \text{cm}}{0.3\ \cancel{\text{km}}} \qquad \left|\Delta\vec{d}_2\right| = 3.1\ \cancel{\text{km}} \times \frac{1\ \text{cm}}{0.3\ \cancel{\text{km}}}$$

$$= 4.0\ \text{cm} \qquad\qquad = 10.3\ \text{cm}$$

Using a ruler and protractor, draw the two vectors, placing the tail of $\Delta\vec{d}_2$ at the tip of $\Delta\vec{d}_1$. Draw the total displacement vector $\Delta\vec{d}_\text{T}$ from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$. Measure the length of the vector, and measure the angle the displacement vector makes to the horizontal.



The measured length of the total displacement vector is 7.7 cm. Convert the length to kilometres.

$$\left|\Delta\vec{d}_\text{T}\right| = 7.7\ \cancel{\text{cm}} \times \frac{0.3\ \text{km}}{1\ \cancel{\text{cm}}} = 2.3\ \text{km}$$

The measured angle θ is $34°$.

Statement: The total displacement is 2.3 km [E 34° N].

2. Given: $\Delta\vec{d}_1 = 77.0$ m [E]; $\Delta\vec{d}_2 = 95.0$ m [S]

Required: $\Delta\vec{d}_\text{T}$; the angle for $\Delta\vec{d}_\text{T}$, θ

Analysis: $\Delta\vec{d}_\text{T} = \Delta\vec{d}_1 + \Delta\vec{d}_2$. To determine the magnitude of the displacement, use the Pythagorean theorem. To calculate angle θ, use the tangent ratio.

Solution: Solve for $\left|\Delta\vec{d}_\text{T}\right|$.

$$\begin{aligned}\left|\Delta\vec{d}_\text{T}\right| &= \sqrt{\left|\Delta\vec{d}_1\right|^2 + \left|\Delta\vec{d}_2\right|^2} \\ &= \sqrt{(77.0\text{ m})^2 + (95.0\text{ m})^2} \\ \left|\Delta\vec{d}_\text{T}\right| &= 122\text{ m}\end{aligned}$$

Solve for angle θ.

$$\begin{aligned}\tan\theta &= \frac{\left|\Delta\vec{d}_2\right|}{\left|\Delta\vec{d}_1\right|} \\ &= \frac{95.0\text{ m}}{77.0\text{ m}} \\ &= 1.2338 \\ \theta &= 51.0^\circ\end{aligned}$$

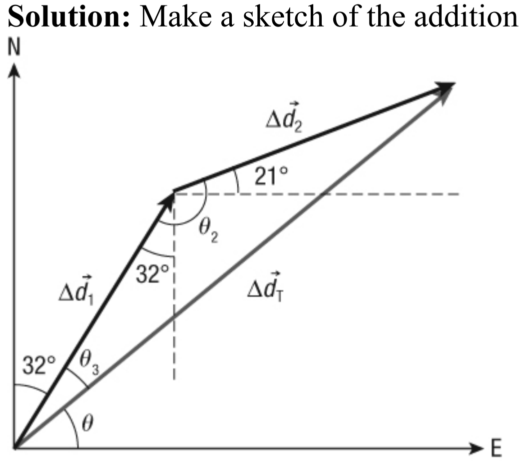
Statement: The boater's total displacement is 122 m [E 51.0° S].

3. Given: $\Delta\vec{d}_1 = 65$ km [N 32° E]; $\Delta\vec{d}_2 = 42$ km [E 21° N]

Required: $\Delta\vec{d}_\text{T}$; the angle for $\Delta\vec{d}_\text{T}$, θ

Analysis: $\Delta\vec{d}_\text{T} = \Delta\vec{d}_1 + \Delta\vec{d}_2$. To determine the magnitude of the displacement, use the cosine law. To calculate the angle θ, use the sine law.

Solution: Make a sketch of the addition of displacement vectors.



$$\begin{aligned}\theta_2 &= 21^\circ + 90^\circ + 32^\circ \\ \theta_2 &= 143^\circ\end{aligned}$$

From the cosine law:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\begin{aligned}
\left|\Delta\vec{d}_{\mathrm{T}}\right|^2 &= \left|\Delta\vec{d}_1\right|^2 + \left|\Delta\vec{d}_2\right|^2 - 2\left|\Delta\vec{d}_1\right|\left|\Delta\vec{d}_2\right|\cos\theta_2 \\
&= (65\text{ km})^2 + (42\text{ km})^2 - 2(65\text{ km})(42\text{ km})(\cos 143°) \\
&= 101.73\text{ km (three extra digits carried)} \\
\left|\Delta\vec{d}_{\mathrm{T}}\right| &= 1.0\times10^2\text{ km}
\end{aligned}$$

From the sine law:

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin\theta_3}{\left|\Delta\vec{d}_2\right|} = \frac{\sin\theta_2}{\left|\Delta\vec{d}_{\mathrm{T}}\right|}$$

$$\begin{aligned}
\sin\theta_2 &= \frac{\left|\Delta\vec{d}_2\right|\sin\theta_2}{\left|\Delta\vec{d}_{\mathrm{T}}\right|} \\
&= \frac{(42\ \cancel{\text{km}})(\sin 143°)}{101.73\ \cancel{\text{km}}} \\
&= 0.24846 \\
\theta_3 &= \sin^{-1}0.24846 \\
\theta_3 &= 14.39°\text{ (two extra digits carried)}
\end{aligned}$$

$$\theta = 90° - 32° - 14.39°$$
$$\theta = 44°$$

Statement: The helicopter travels 1.0×10^2 km [E 44° N].

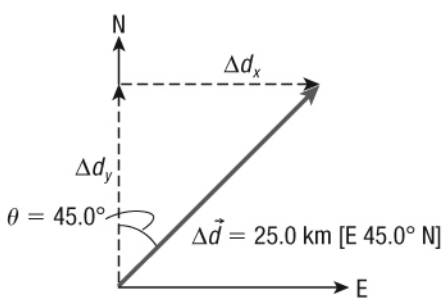
Tutorial 2 Practice, page 26

1. (a) Given: $\Delta\vec{d} = 25.0$ km [E 45.0° N]

Required: Δd_x; Δd_y

Analysis: Draw the displacement vector, and then use trigonometry to determine the components. Use east and north as positive.

Solution:

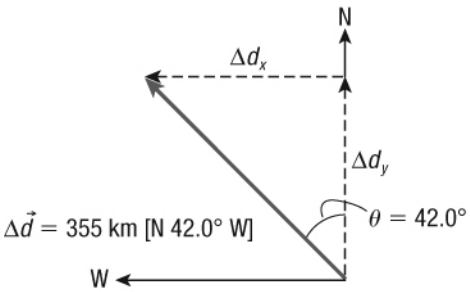


$$\Delta d_x = +\left|\Delta \vec{d}\right|\cos\theta \qquad \Delta d_y = +\left|\Delta \vec{d}\right|\sin\theta$$
$$= +(25.0 \text{ km})(\cos\ 45.0°) \qquad = +(25.0 \text{ km})(\sin\ 45.0°)$$
$$\Delta d_x = +17.7 \text{ km} \qquad \Delta d_y = +17.7 \text{ km}$$

Statement: The components of the displacement are $\Delta d_x = 17.7$ km [E] and $\Delta d_y = 17.7$ km [N].

(b) Given: $\Delta \vec{d} = 355$ km [N 42.0° W]

Required: Δd_x; Δd_y
Analysis: Draw the displacement vector, and then use trigonometry to determine the components. Use east and north as positive.
Solution:

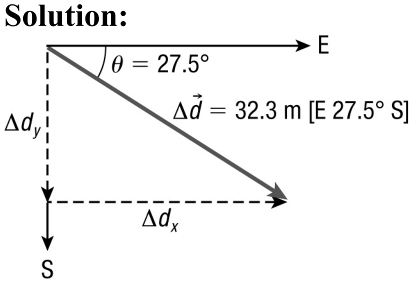


$$\Delta d_x = -\left|\Delta \vec{d}\right|\sin\theta \qquad \Delta d_y = +\left|\Delta \vec{d}\right|\cos\theta$$
$$= -(355 \text{ km})(\sin\ 42.0°) \qquad = +(355 \text{ km})(\cos\ 42.0°)$$
$$\Delta d_x = -238 \text{ km} \qquad \Delta d_y = +264 \text{ km}$$

Statement: The components of the displacement are $\Delta d_x = 238$ km [W] and $\Delta d_y = 264$ km [N].

(c) Given: $\Delta \vec{d} = 32.3$ m [E 27.5° S]

Required: Δd_x; Δd_y
Analysis: Draw the displacement vector, and then use trigonometry to determine the components. Use east and north as positive.
Solution:



$$\Delta d_x = +\left|\Delta \vec{d}\right|\cos\theta \qquad \Delta d_y = -\left|\Delta \vec{d}\right|\sin\theta$$
$$= +(32.3 \text{ m})(\cos\ 27.5°) \qquad = -(32.3 \text{ m})(\sin\ 27.5°)$$
$$\Delta d_x = +28.7 \text{ m} \qquad \Delta d_y = -14.9 \text{ m}$$

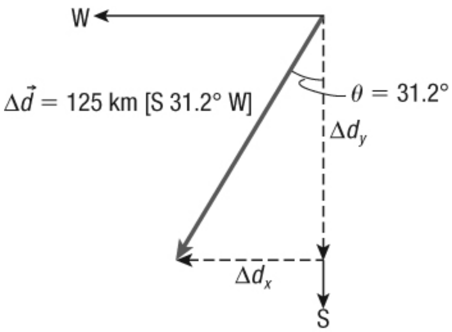
Statement: The components of the displacement are $\Delta d_x = 28.7$ m [E] and $\Delta d_y = 14.9$ m [S].

(d) Given: $\Delta\vec{d} = 125$ km [S 31.2° W]

Required: Δd_x; Δd_y

Analysis: Draw the displacement vector, and then use trigonometry to determine the components. Use east and north as positive.

Solution:



$$\begin{aligned} \Delta d_x &= -\left|\Delta\vec{d}\right|\sin\theta \\ &= -(125\text{ km})(\sin 31.2°) \\ \Delta d_x &= -64.8\text{ km} \end{aligned}$$

$$\begin{aligned} \Delta d_y &= -\left|\Delta\vec{d}\right|\cos\theta \\ &= -(125\text{ km})(\cos 31.2°) \\ \Delta d_y &= -107\text{ km} \end{aligned}$$

Statement: The components of the displacement are $\Delta d_x = 64.8\text{ km [W]}$ and $\Delta d_y = 107\text{ km [S]}$.

Tutorial 3 Practice, page 28

1. Given: $\Delta\vec{d}_1 = 276.9$ km [W 76.70° S]; $\Delta\vec{d}_2 = 675.1$ km [W 11.45° S]

Required: $\Delta\vec{d}$

Analysis: $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2$. Determine the *x*- and *y*-components of the given displacement vectors. Add these *x*- and *y*-components to calculate the *x*- and *y*-components of the total displacement. Finally, use the Pythagorean theorem and tangent ratio to determine the total displacement vector. Use east and north as positive.

Solution: For the first vector,

$$\begin{aligned} \Delta d_{1x} &= -\left|\Delta\vec{d}_1\right|\cos\theta \\ &= -(276.9\text{ km})(\cos 76.70°) \\ \Delta d_{1x} &= -63.701\text{ km (one extra digit carried)} \end{aligned}$$

$$\begin{aligned} \Delta d_{1y} &= -\left|\Delta\vec{d}_1\right|\sin\theta \\ &= -(276.9\text{ km})(\sin 76.70°) \\ \Delta d_{1y} &= -269.473\text{ km (two extra digits carried)} \end{aligned}$$

For the second vector,

$$\begin{aligned} \Delta d_{2x} &= -\left|\Delta\vec{d}_2\right|\cos\theta \\ &= -(675.1\text{ km})(\cos 11.45°) \\ \Delta d_{2x} &= -661.664\text{ km (two extra digits carried)} \end{aligned}$$

$$\begin{aligned}\Delta d_{2y} &= -\left|\Delta \vec{d}_2\right| \sin\theta \\ &= -(675.1\text{ km})(\sin 11.45°)\end{aligned}$$

$\Delta d_{2y} = -134.016$ km (two extra digits carried)

Add the horizontal components.

$$\begin{aligned}\Delta d_x &= \Delta d_{1x} + \Delta d_{2x} \\ &= -63.701\text{ km} + (-661.664\text{ km})\end{aligned}$$

$$\Delta d_x = -725.365\text{ km}$$

Add the vertical components.

$$\begin{aligned}\Delta d_y &= \Delta d_{1y} + \Delta d_{2y} \\ &= -269.473\text{ km} + (-134.016\text{ km})\end{aligned}$$

$$\Delta d_y = -403.489\text{ km}$$

Combine the total displacement components to determine the total displacement.

$$\left|\Delta \vec{d}\right| = \sqrt{\Delta d_x^2 + \Delta d_y^2}$$

$$\left|\Delta \vec{d}\right| = \sqrt{(-725.365\text{ km})^2 + (-403.489\text{ km})^2}$$

$$\left|\Delta \vec{d}\right| = 830.0\text{ km}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{\left|\Delta d_y\right|}{\left|\Delta d_x\right|}\right) \\ &= \tan^{-1}\left(\frac{403.489\ \cancel{\text{km}}}{725.365\ \cancel{\text{km}}}\right)\end{aligned}$$

$$\theta = 29.09°$$

Statement: The total displacement of the airplane is 830.0 km [W 29.09° S].

2. Given: $\Delta \vec{d}_1 = 120$ km [N 32° W]; $\Delta \vec{d}_2 = 150$ km [W 24° N]

Required: $\Delta \vec{d}$

Analysis: $\Delta \vec{d} = \Delta \vec{d}_1 + \Delta \vec{d}_2$. Determine the *x*- and *y*-components of the given displacement vectors. Add these *x*- and *y*-components to calculate the *x*- and *y*-components of the total displacement. Finally, use the Pythagorean theorem and tangent ratio to determine the total displacement vector. Use east and north as positive.

Solution: For the first vector,

$$\begin{aligned}\Delta d_{1x} &= -\left|\Delta \vec{d}_1\right| \sin\theta \\ &= -(120\text{ km})(\sin 32°)\end{aligned}$$

$\Delta d_{1x} = -63.59$ km (two extra digits carried)

$$\begin{aligned}\Delta d_{1y} &= +\left|\Delta \vec{d}_1\right| \cos\theta \\ &= +(120\text{ km})(\cos 32°)\end{aligned}$$

$\Delta d_{1y} = +101.77$ km (three extra digits carried)

For the second vector,

$$\Delta d_{2x} = -\left|\Delta \vec{d}_2\right| \cos\theta$$
$$= -(150\text{ km})(\cos 24°)$$
$$\Delta d_{2x} = -137.03\text{ km (three extra digits carried)}$$

$$\Delta d_{2y} = +\left|\Delta \vec{d}_2\right| \sin\theta$$
$$= +(150\text{ km})(\sin 24°)$$
$$\Delta d_{2y} = +61.01\text{ km (two extra digits carried)}$$

Add the horizontal components.

$$\Delta d_x = \Delta d_{1x} + \Delta d_{2x}$$
$$= -63.59\text{ km} + (-137.03\text{ km})$$
$$\Delta d_x = -200.62\text{ km}$$

Add the vertical components.

$$\Delta d_y = \Delta d_{1y} + \Delta d_{2y}$$
$$= +101.77\text{ km} + 61.01\text{ km}$$
$$\Delta d_y = +162.78\text{ km}$$

Combine the total displacement components to determine the total displacement.

$$\left|\Delta \vec{d}\right| = \sqrt{\Delta d_x^2 + \Delta d_y^2}$$
$$= \sqrt{(200.59\text{ km})^2 + (162.78\text{ km})^2}$$
$$\left|\Delta \vec{d}\right| = 260\text{ km}$$

$$\theta = \tan^{-1}\left(\frac{\left|\Delta d_y\right|}{\left|\Delta d_x\right|}\right)$$
$$= \tan^{-1}\left(\frac{162.78\text{ km}}{200.59\text{ km}}\right)$$
$$\theta = 39°$$

Statement: The total displacement of the trip is 260 km [W 39° N].

3. Given: $\Delta \vec{d}_1 = 12\text{ km}$ [N]; $\Delta \vec{d}_2 = 14\text{ km}$ [N 22° E]; $\Delta \vec{d}_3 = 11\text{ km}$ [E]

Required: $\Delta \vec{d}$

Analysis: $\Delta \vec{d} = \Delta \vec{d}_1 + \Delta \vec{d}_2 + \Delta \vec{d}_3$. Determine the *x*- and *y*-components of the given displacement vectors. Add these *x*- and *y*-components to calculate the *x*- and *y*-components of the total displacement. Finally, use the Pythagorean theorem and tangent ratio to determine the total displacement vector. Use east and north as positive.

Solution: For the first vector,

$$\Delta d_{1x} = 0\text{ km}$$
$$\Delta d_{1y} = +12\text{ km}$$

For the second vector,

$$\Delta d_{2x} = +\left|\Delta\vec{d}_2\right|\sin\theta$$
$$= +(14 \text{ km})(\sin 22°)$$
$$\Delta d_{2x} = +5.244 \text{ km (two extra digits carried)}$$
$$\Delta d_{2y} = +\left|\Delta\vec{d}_2\right|\cos\theta$$
$$= +(14 \text{ km})(\cos 22°)$$
$$\Delta d_{2y} = +12.98 \text{ km (two extra digits carried)}$$

For the third vector,

$$\Delta d_{3x} = 11 \text{ km}$$
$$\Delta d_{3y} = 0 \text{ km}$$

Add the horizontal components.

$$\Delta d_x = \Delta d_{1x} + \Delta d_{2x} + \Delta d_{3x}$$
$$= 0 \text{ km} + 5.244 \text{ km} + 11 \text{ km}$$
$$\Delta d_x = +16.244 \text{ km}$$

Add the vertical components.

$$\Delta d_y = \Delta d_{1y} + \Delta d_{2y} + \Delta d_{3y}$$
$$= +12 \text{ km} + 12.98 \text{ km} + 0 \text{ km}$$
$$\Delta d_y = +24.98 \text{ km}$$

Combine the total displacement components to determine the total displacement.

$$\left|\Delta\vec{d}\right| = \sqrt{\Delta d_x^2 + \Delta d_y^2}$$
$$= \sqrt{(16.244 \text{ km})^2 + (24.98 \text{ km})^2}$$
$$\left|\Delta\vec{d}\right| = 3.0\times 10^1 \text{ km}$$
$$\theta = \tan^{-1}\left(\frac{\left|\Delta\vec{d}_y\right|}{\left|\Delta\vec{d}_x\right|}\right)$$
$$= \tan^{-1}\left(\frac{24.98 \text{ km}}{16.244 \text{ km}}\right)$$
$$\theta = 57°$$

Statement: The total displacement of the helicopter is 3.0×10^1 km [E 57° N].

Section 1.3 Questions, page 29

1. (a) Given: $\Delta\vec{d}_1 = 7.81$ km [E 50° N]; $\Delta\vec{d}_2 = 5.10$ km [W 11° N]

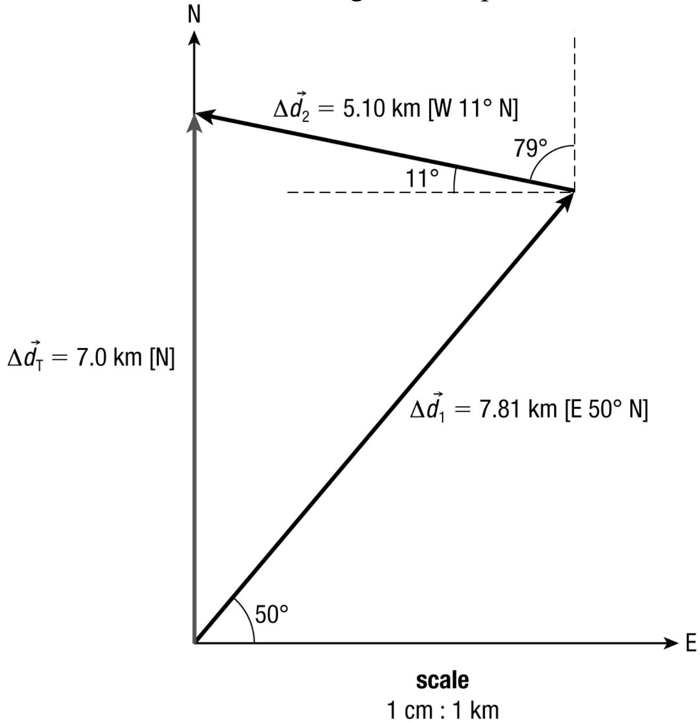
Required: $\Delta\vec{d}$; the angle for $\Delta\vec{d}$, θ

Analysis: $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2$ Decide on a scale and then draw each vector to scale on a coordinate axis. Draw the total displacement vector.

Solution: An appropriate scale is 1 cm : 1 km. Calculate the lengths of the arrows for the displacement:

$$\Delta\vec{d}_1 : \frac{7.81\ \cancel{\text{km}}}{1\ \cancel{\text{km}}\ /\text{cm}} = 7.81\ \text{cm};\ \ \Delta\vec{d}_2 = \frac{5.10\ \cancel{\text{km}}}{1\ \cancel{\text{km}}\ /\text{cm}} = 5.10\ \text{cm}$$

Using a ruler and protractor, draw the two vectors placing the tail of $\Delta\vec{d}_2$ at the tip of $\Delta\vec{d}_1$. Draw the total displacement vector $\Delta\vec{d}$ from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$. Measure the length of the vector, and measure the angle the displacement vector makes to the horizontal.



The measured length of the total displacement vector is 7.0 cm, measured to the nearest millimetre. Convert to kilometres.

$$\Delta\vec{d} = 7.0\ \cancel{\text{cm}} \times \frac{1\ \text{km}}{1\ \cancel{\text{cm}}}$$

$$= 7.0\ \text{km}$$

The measured angle θ is $0°$, measured to the nearest degree.

Statement: The total displacement is 7.0 km [N].

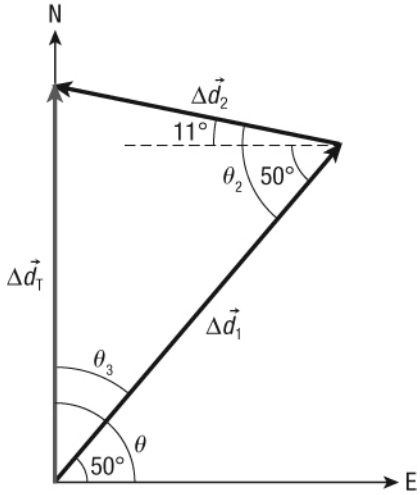
(b) Both an algebraic solution and a component solution are given.

(i) Algebraic Method:

Required: $\Delta\vec{d}$; the angle for $\Delta\vec{d}$, θ

Analysis: $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2$. To determine the magnitude of the displacement, use the cosine law. To calculate the angle θ, use the sine law.

Solution: Make a sketch of the addition of displacement vectors.



The angle θ_2 is

$\theta_2 = 50° + 11°$

$\theta_2 = 61°$

From the cosine law,

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\left|\Delta\vec{d}\right|^2 = \left|\Delta\vec{d}_1\right|^2 + \left|\Delta\vec{d}_2\right|^2 - 2\left|\Delta\vec{d}_1\right|\left|\Delta\vec{d}_2\right|\cos\theta_2$$

$$= (7.81 \text{ km})^2 + (5.10 \text{ km})^2 - 2(7.81 \text{ km})(5.10 \text{ km})(\cos\ 61°)$$

$$\left|\Delta\vec{d}\right|^2 = 48.385 \text{ m}^2$$

$$\left|\Delta\vec{d}\right| = 6.956 \text{ m (two extra digits carried)}$$

$$\left|\Delta\vec{d}\right| = 7.0 \text{ m}$$

From the sine law,

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin\theta_3}{\left|\Delta\vec{d}_2\right|} = \frac{\sin\theta_2}{\left|\Delta\vec{d}\right|}$$

$$\sin\theta_3 = \frac{\left|\Delta\vec{d}_2\right|\sin\theta_2}{\left|\Delta\vec{d}\right|}$$

$$= \frac{(5.10\ \cancel{\text{m}})(\sin\ 61°)}{(6.956\ \cancel{\text{m}})}$$

$$= 0.6412$$

$$\theta_3 = \sin^{-1} 0.6412$$

$$\theta_3 = 39.88°$$

$\theta = 50° + 39.88°$

$\theta = 90°$

Statement: The total displacement is 7.0 km [N].

(ii) Component Method:

Required: $\Delta\vec{d}$

Analysis: $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2$. Determine the *x*- and *y*-components of the given displacement vectors. Add these *x*- and *y*-components to calculate the *x*- and *y*-components of the total displacement. Finally, use the Pythagorean theorem and tangent ratio to determine the total displacement vector. Use east and north as positive.

Solution: For the first vector,

$$\Delta d_{1x} = +\left|\Delta\vec{d}_1\right|\cos\theta$$
$$= +(7.81 \text{ km})(\cos 50°)$$
$$\Delta d_{1x} = -5.020 \text{ km (two extra digits carried)}$$

$$\Delta d_{1y} = +\left|\Delta\vec{d}_1\right|\sin\theta$$
$$= +(7.81 \text{ km})(\sin 50°)$$
$$\Delta d_{1y} = +5.983 \text{ km (two extra digits carried)}$$

For the second vector,

$$\Delta d_{2x} = -\left|\Delta\vec{d}_2\right|\cos\theta$$
$$= -(5.10 \text{ km})(\cos 11°)$$
$$\Delta d_{2x} = -5.006 \text{ km (two extra digits carried)}$$

$$\Delta d_{2y} = +\left|\Delta\vec{d}_2\right|\sin\theta$$
$$= +(5.10 \text{ km})(\sin 11°)$$
$$\Delta d_{2y} = +0.9732 \text{ km (two extra digits carried)}$$

Add the horizontal components.

$$\Delta d_x = \Delta d_{1x} + \Delta d_{2x}$$
$$= +5.020 \text{ km} + (-5.006 \text{ km})$$
$$\Delta d_x = +0.014 \text{ km}$$

Add the vertical components.

$$\Delta d_y = \Delta d_{1y} + \Delta d_{2y}$$
$$= +5.983 \text{ km} + 0.9732 \text{ km}$$
$$\Delta d_y = +6.9562 \text{ km}$$

Combine the total displacement components to determine the total displacement.

$$\left|\Delta\vec{d}\right| = \sqrt{\Delta d_x^2 + \Delta d_y^2}$$
$$= \sqrt{(0.014 \text{ km})^2 + (6.9562 \text{ km})^2}$$
$$\left|\Delta\vec{d}\right| = 7.0 \text{ km}$$

$$\theta = \tan^{-1}\left(\frac{|\Delta d_y|}{|\Delta d_x|}\right)$$
$$= \tan^{-1}\left(\frac{6.9562\ \cancel{\text{km}}}{0.014\ \cancel{\text{km}}}\right)$$
$$\theta = 90°$$

Statement: The total displacement of the trip is 7.0 km [N].

(c) Answers may vary. Sample answer: The answers obtained by the three methods were the same (no percent difference). It is reasonable that the methods in part (b) gave the same results because care was taken to carry extra digits while calculating. In both cases, the final answer was given to two significant digits, consistent with the given information. It is somewhat surprising that the result of part (a) agreed with the mathematical methods because it is difficult to draw or measure an angle to 1° precision. It is also difficult to draw arrows tip to tail to 1 mm precision.

2. Given: $\Delta\vec{d}_1 = 5.0$ cm [E 30.0° N]; $\Delta\vec{d}_2 = 7.5$ cm [E]; $\Delta\vec{d}_3 = 15.0$ cm [E 10.0° S]

Required: $\Delta\vec{d}$; the angle for $\Delta\vec{d}$, θ

Analysis: $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2 + \Delta\vec{d}_3$. Decide on a scale and then draw each vector to scale on a coordinate axis. Draw the total displacement vector.

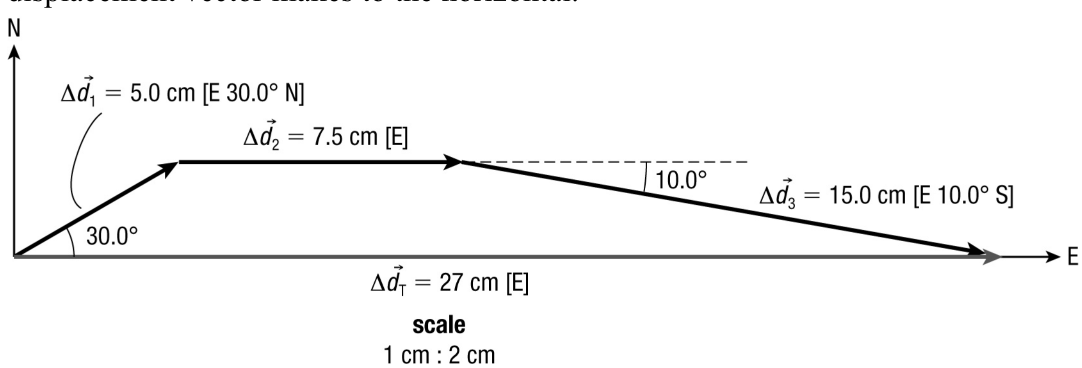
Solution: An appropriate scale is 1 cm : 2 cm. The lengths of the arrows for the displacement vectors are

$$\Delta\vec{d}_1 = 5.0\ \cancel{\text{cm}} \times \frac{1\text{ cm}}{2\ \cancel{\text{cm}}} = 2.5\text{ cm}$$

$$\left|\Delta\vec{d}_2\right| = \left(7.5\ \cancel{\text{cm}}\right)\frac{1\text{ cm}}{2\ \cancel{\text{cm}}} = 3.75\text{ cm}$$

$$\left|\Delta\vec{d}_3\right| = \left(15.0\ \cancel{\text{cm}}\right)\frac{1\text{ cm}}{2\ \cancel{\text{cm}}} = 7.5\text{ cm}$$

Using a ruler and protractor, draw the three vectors placing the tail of $\Delta\vec{d}_2$ at the tip of $\Delta\vec{d}_1$, and the tail of $\Delta\vec{d}_3$ at the tip of $\Delta\vec{d}_2$. Draw the total displacement vector $\Delta\vec{d}$ from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_3$. Measure the length of the displacement vector, and measure the angle the displacement vector makes to the horizontal.



The measured length of the total displacement vector is 13.5 cm, to the nearest millimetre.

Convert back to the original scale.

$$\left|\Delta\vec{d}\right| = 13.5\text{ cm}\times\frac{2\ \cancel{\text{cm}}}{1\ \cancel{\text{cm}}} = 27\text{ cm}$$

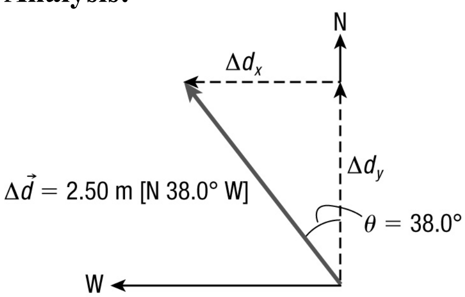
The measured angle θ is $0°$, to the nearest degree.

Statement: The total displacement is 27 cm [E].

3. Given: $\Delta\vec{d} = 2.50\text{ m [N 38.0° W]}$

Required: Δd_x; Δd_y

Analysis:



Draw the displacement vector, and then use trigonometry to determine the components. Use east and north as positive.

Solution:

$$\begin{aligned}\Delta d_x &= -\left|\Delta\vec{d}\right|\sin\theta \\ &= -(2.50\text{ m})(\sin 38.0°) \\ \Delta d_x &= -1.54\text{ m}\end{aligned}$$

$$\begin{aligned}\Delta d_y &= +\left|\Delta\vec{d}\right|\cos\theta \\ &= +(2.50\text{ km})(\cos 38.0°) \\ \Delta d_y &= +1.97\text{ m}\end{aligned}$$

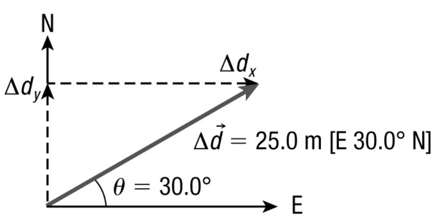
Statement: The components of the displacement are $\Delta d_x = 1.54\text{ m [W]}$ and $\Delta d_y = 1.97\text{ m [N]}$.

4. Given: $\Delta\vec{d} = 25.0\text{ m [E 30.0° N]}$

Required: Δd_x; Δd_y

Analysis: Draw the displacement vector, and then use trigonometry to determine the components. Use east and north as positive.

Solution:



$$\begin{aligned}\Delta d_x &= +\left|\Delta\vec{d}\right|\cos\theta \\ &= +(25.0\text{ m})(\cos 30.0°) \\ \Delta d_x &= +21.6\text{ m}\end{aligned}$$

$$\begin{aligned}\Delta d_y &= +\left|\Delta\vec{d}\right|\sin\theta \\ &= +(25.0\text{ m})(\sin 30.0°) \\ \Delta d_y &= +12.5\text{ m}\end{aligned}$$

Statement: The components of the displacement are $\Delta d_x = 21.6\text{ m [E]}$ and $\Delta d_y = 12.5\text{ m [N]}$.

5. Given: $\Delta d_x = 54 \text{ m [E]}$; $\Delta d_y = 24 \text{ m [N]}$

Required: $|\Delta\vec{d}|$

Analysis: Use the Pythagorean theorem to determine the length of the displacement vector. Use east and north as positive.

Solution:
$$\begin{aligned}|\Delta\vec{d}| &= \sqrt{\Delta d_x^2 + \Delta d_y^2}\\ &= \sqrt{(54 \text{ m})^2 + (24 \text{ m})^2}\\ |\Delta\vec{d}| &= 59 \text{ m}\end{aligned}$$

Statement: The length of the displacement vector is 59 m.

(b) Required: θ

Analysis: Use the tangent ratio to calculate θ.

Solution:
$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{|\Delta\vec{d}_y|}{|\Delta\vec{d}_x|}\right)\\ &= \tan^{-1}\left(\frac{24\ \cancel{\text{m}}}{54\ \cancel{\text{m}}}\right)\\ \theta &= 24^\circ\end{aligned}$$

Statement: The vector points [E 24° N].

6. Given: $\Delta\vec{d}_1 = 15.0 \text{ km [W]}$; $\Delta\vec{d}_2 = 45.0 \text{ km [}S\text{]}$; $\Delta\vec{d}_3 = 32 \text{ km [N } 25^\circ \text{ W]}$

Required: $\Delta\vec{d}$

Analysis: $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2 + \Delta\vec{d}_3$. Determine the *x*- and *y*-components of the given displacement vectors. Add these *x*- and *y*-components to calculate the *x*- and *y*-components of the total displacement. Finally, use the Pythagorean theorem and tangent ratio to determine the total displacement vector. Use east and north as positive.

Solution: For the first vector,

$$\Delta d_{1x} = -15.0 \text{ km}$$

$$\Delta d_{1y} = 0 \text{ km}$$

For the second vector,

$$\Delta d_{2x} = 0 \text{ km}$$

$$\Delta d_{2y} = -45.0 \text{ km}$$

For the third vector,

$$\begin{aligned}\Delta d_{3x} &= -|\Delta\vec{d}_3|\sin\theta\\ &= -(32 \text{ km})(\sin 25^\circ)\\ \Delta d_{3x} &= -13.52 \text{ km (two extra digits carried)}\end{aligned}$$

$$\begin{aligned}\Delta d_{3y} &= +|\Delta\vec{d}_3|\cos\theta\\ &= +(32 \text{ km})(\cos 25^\circ)\\ \Delta d_{3y} &= +29.00 \text{ km (two extra digits carried)}\end{aligned}$$

Add the horizontal components.

$$\Delta d_x = \Delta d_{1x} + \Delta d_{2x} + \Delta d_{3x}$$
$$= (-15.0 \text{ km}) + 0 \text{ km} + (-13.52 \text{ km})$$
$$\Delta d_x = -28.52 \text{ km}$$

Add the vertical components.

$$\Delta d_y = \Delta d_{1y} + \Delta d_{2y} + \Delta d_{3y}$$
$$= 0 \text{ km} + (-45.0 \text{ km}) + 29.00 \text{ km}$$
$$\Delta d_y = -16.00 \text{ km}$$

Combine the total displacement components to determine the total displacement.

$$|\Delta \vec{d}| = \sqrt{\Delta d_x^2 + \Delta d_y^2}$$
$$= \sqrt{(28.52 \text{ km})^2 + (16.00 \text{ km})^2}$$
$$|\Delta \vec{d}| = 33 \text{ km}$$
$$\theta = \tan^{-1}\left(\frac{|\Delta \vec{d}_y|}{|\Delta \vec{d}_x|}\right)$$
$$= \tan^{-1}\left(\frac{16.00 \cancel{\text{ km}}}{28.52 \cancel{\text{ km}}}\right)$$
$$\theta = 29°$$

Statement: The total displacement of the driver is 33 km [W 29° S].

7. Given: $\Delta \vec{d}_1 = 2.5$ m [W 30.0° S]; $\Delta \vec{d}_2 = 3.6$ m [*S*]; $\Delta \vec{d}_3 = 4.9$ m [E 38.0° S]

Required: $\Delta \vec{d}$

Analysis: $\Delta \vec{d} = \Delta \vec{d}_1 + \Delta \vec{d}_2 + \Delta \vec{d}_3$. Determine the *x*- and *y*-components of the given displacement vectors. Add these *x*- and *y*-components to calculate the *x*- and *y*-components of the total displacement. Finally, use the Pythagorean theorem and tangent ratio to determine the total displacement vector. Use east and north as positive.

Solution: For the first vector,

$$\Delta d_{1x} = -|\Delta \vec{d}_1|\cos\theta$$
$$= -(2.5 \text{ m})(\cos 30.0°)$$
$$\Delta d_{1x} = -2.165 \text{ m (two extra digits carried)}$$
$$\Delta d_{1y} = -|\Delta \vec{d}_1|\sin\theta$$
$$= -(2.5 \text{ m})(\sin 30.0°)$$
$$\Delta d_{1y} = -1.250 \text{ m (two extra digits carried)}$$

For the second vector,

$$\Delta d_{2x} = 0 \text{ m}$$
$$\Delta d_{2y} = -3.6 \text{ m}$$

For the third vector,

$$\Delta d_{3x} = +\left|\Delta \vec{d}_3\right|\cos\theta$$
$$= +(4.9\text{ m})(\cos 38.0°)$$
$$\Delta d_{3x} = +3.861\text{ m (two extra digits carried)}$$
$$\Delta d_{3y} = -\left|\Delta \vec{d}_3\right|\sin\theta$$
$$= -(4.9\text{ m})(\sin 38.0°)$$
$$\Delta d_{3y} = -3.017\text{ m (two extra digits carried)}$$

Add the horizontal components.

$$\Delta d_x = \Delta d_{1x} + \Delta d_{2x} + \Delta d_{3x}$$
$$= (-2.165\text{ m}) + 0\text{ m} + 3.861\text{ m}$$
$$\Delta d_x = 1.696\text{ m}$$

Add the vertical components.

$$\Delta d_y = \Delta d_{1y} + \Delta d_{2y} + \Delta d_{3y}$$
$$= -1.250\text{ m} + (-3.6\text{ m}) + (-3.017\text{ m})$$
$$\Delta d_y = -7.867\text{ m}$$

Combine the components to determine the total displacement vector.

$$\left|\Delta \vec{d}\right| = \sqrt{\Delta d_x^2 + \Delta d_y^2}$$
$$= \sqrt{(1.696\text{ m})^2 + (7.867\text{ m})^2}$$
$$\left|\Delta \vec{d}\right| = 8.0\text{ m}$$
$$\theta = \tan^{-1}\left(\frac{\left|\Delta \vec{d}_y\right|}{\left|\Delta \vec{d}_x\right|}\right)$$
$$= \tan^{-1}\left(\frac{7.867\cancel{\text{ m}}}{1.696\cancel{\text{ m}}}\right)$$
$$\theta = 78°$$

Statement: The total displacement is 8.0 m [E 78° S].

8. (a) Given: $\Delta \vec{d}_1 = 2.70$ km [E 25.0° N]; $\Delta \vec{d}_2 = 4.80$ km [E 45.0° S]

Required: Δd_x; Δd_y

Analysis: $\Delta \vec{d} = \Delta \vec{d}_1 + \Delta \vec{d}_2$. Determine the *x*- and *y*-components of the given displacement vectors. Add these *x*- and *y*-components to calculate the *x*- and *y*-components of the total displacement. Use east and north as positive.

Solution: For the first vector,

$$\Delta d_{1x} = +\left|\Delta \vec{d}_1\right|\cos\theta$$
$$= +(2.70\text{ km})(\cos 25.0°)$$
$$\Delta d_{1x} = +2.4470\text{ km (two extra digits carried)}$$

$$\Delta d_{1y} = +\left|\Delta \vec{d}_1\right| \sin\theta$$
$$= +(2.70 \text{ km})(\sin 25.0°)$$
$$\Delta d_{1y} = +1.1411 \text{ km (two extra digits carried)}$$

For the second vector,

$$\Delta d_{2x} = +\left|\Delta \vec{d}_2\right| \cos\theta$$
$$= +(4.80 \text{ km})(\cos 45.0°)$$
$$\Delta d_{2x} = +3.3941 \text{ km (two extra digits carried)}$$
$$\Delta d_{2y} = -\left|\Delta \vec{d}_2\right| \sin\theta$$
$$= -(4.80 \text{ km})(\sin 45.0°)$$
$$\Delta d_{2y} = -3.3941 \text{ km (two extra digits carried)}$$

Add the horizontal components.

$$\Delta d_x = \Delta d_{1x} + \Delta d_{2x}$$
$$= +2.4470 \text{ km} + 3.3941 \text{ km}$$
$$= +5.841 \text{ km}$$
$$\Delta d_x = +5.84 \text{ km}$$

Add the vertical components.

$$\Delta d_y = \Delta d_{1y} + \Delta d_{2y}$$
$$= +1.1411 \text{ km} + (-3.3941 \text{ km})$$
$$= -2.2530 \text{ km}$$
$$\Delta d_y = -2.25 \text{ km}$$

Statement: The components of the boat's displacement are $\Delta d_x = 5.84$ km [E] and $\Delta d_y = 2.25$ km [S].

(b) Required: $\Delta \vec{d}$

Analysis: Use the Pythagorean theorem and tangent ratio to determine the total displacement vector.

$$\left|\Delta \vec{d}\right| = \sqrt{\Delta d_x^2 + \Delta d_y^2}$$
$$= \sqrt{(5.8411 \text{ km})^2 + (2.2530 \text{ km})^2}$$
$$\left|\Delta \vec{d}\right| = 6.26 \text{ km}$$

$$\theta = \tan^{-1}\left(\frac{\left|\Delta d_y\right|}{\left|\Delta d_x\right|}\right)$$
$$= \tan^{-1}\left(\frac{2.2530 \cancel{\text{ km}}}{5.8411 \cancel{\text{ km}}}\right)$$
$$\theta = 21.1°$$

Statement: The total displacement of the boat is 6.26 km [E 21.1° S].

9. Given: $\Delta\vec{d}_1 = 1512.0$ km [W 19.30° N]; $\Delta\vec{d}_2 = 571.0$ km [W 4.35° N];
$\Delta\vec{d}_3 = 253.1$ km [W 39.39° N]

Required: $\Delta\vec{d}$

Analysis: $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2 + \Delta\vec{d}_3$. Determine the *x*- and *y*-components of the given displacement vectors. Add these *x*- and *y*-components to calculate the *x*- and *y*-components of the total displacement. Finally, use the Pythagorean theorem and tangent ratio to determine the total displacement vector. Notice that all of the displacements point somewhat west and north. Use west and north as positive.

Solution: For the first vector,

$$\Delta d_{1x} = \left|\Delta\vec{d}_1\right|\cos\theta$$
$$= (1512.0 \text{ km})(\cos 19.30°)$$
$$\Delta d_{1x} = 1427.027 \text{ km (two extra digits carried)}$$
$$\Delta d_{1y} = \left|\Delta\vec{d}_1\right|\sin\theta$$
$$= (1512.0 \text{ km})(\sin 19.30°)$$
$$\Delta d_{1y} = 499.738 \text{ km (two extra digits carried)}$$

For the second vector,

$$\Delta d_{2x} = \left|\Delta\vec{d}_2\right|\cos\theta$$
$$= (571.0 \text{ km})(\cos 4.35°)$$
$$\Delta d_{2x} = 569.355 \text{ km (two extra digits carried)}$$
$$\Delta d_{2y} = \left|\Delta\vec{d}_2\right|\sin\theta$$
$$= (571.0 \text{ km})(\sin 4.35°)$$
$$\Delta d_{2y} = 43.310 \text{ km (two extra digits carried)}$$

For the third vector,

$$\Delta d_{3x} = \left|\Delta\vec{d}_3\right|\cos\theta$$
$$= (253.1 \text{ km})(\cos 39.39°)$$
$$\Delta d_{3x} = 195.607 \text{ km (two extra digits carried)}$$
$$\Delta d_{3y} = \left|\Delta\vec{d}_3\right|\sin\theta$$
$$= (253.1 \text{ km})(\sin 39.39°)$$
$$\Delta d_{3y} = 160.616 \text{ km (two extra digits carried)}$$

Add the horizontal components.

$$\Delta d_x = \Delta d_{1x} + \Delta d_{2x} + \Delta d_{3x}$$
$$= 1427.027 \text{ km} + 569.355 \text{ km} + 195.607 \text{ km}$$
$$\Delta d_x = 2191.989 \text{ km}$$

Add the vertical components.

$$\Delta d_y = \Delta d_{1y} + \Delta d_{2y} + \Delta d_{3y}$$
$$= 499.738 \text{ km} + 43.310 \text{ km} + 160.616 \text{ km}$$
$$\Delta d_y = 703.664 \text{ km}$$

Combine the components to determine the total displacement vector.

$$\left|\Delta \vec{d}\right| = \sqrt{\Delta d_x^2 + \Delta d_y^2}$$
$$= \sqrt{(2191.989 \text{ km})^2 + (703.664 \text{ m})^2}$$
$$= 2302.164 \text{ km}$$
$$\left|\Delta \vec{d}\right| = 2.30 \times 10^3 \text{ km}$$

$$\theta = \tan^{-1}\left(\frac{\left|\Delta d_y\right|}{\left|\Delta d_x\right|}\right)$$
$$= \tan^{-1}\left(\frac{703.664 \ \cancel{\text{km}}}{2191.989 \ \cancel{\text{km}}}\right)$$
$$\theta = 17.8°$$

Statement: The total displacement is 2.30×10^3 km [W 17.8° N].

10. Given: $\Delta \vec{d}_1 = 25$ km [N]; $\Delta \vec{d}_\text{T} = 62$ km [N 38° W]

Required: $\Delta \vec{d}_2$

Analysis: $\Delta \vec{d}_\text{T} = \Delta \vec{d}_1 + \Delta \vec{d}_2$. Determine the *x*- and *y*-components of the given displacement vectors. Subtract these *x*- and *y*-components to calculate the *x*- and *y*-components of the second displacement. Finally, use the Pythagorean theorem and tangent ratio to determine the second displacement vector. Use east and north as positive.

Solution: For the first vector:

$$\Delta d_{1x} = 0 \text{ km}$$
$$\Delta d_{1y} = 25 \text{ km}$$

For the total vector,

$$\Delta d_x = -\left|\Delta \vec{d}\right| \sin\theta$$
$$= -(62 \text{ km})(\sin 38°)$$
$$\Delta d_x = -38.171 \text{ km (two extra digits carried)}$$
$$\Delta d_y = \left|\Delta \vec{d}\right| \cos\theta$$
$$= (62 \text{ km})(\cos 38°)$$
$$\Delta d_y = 48.857 \text{ km (two extra digits carried)}$$

Subtract the horizontal components.

$$\begin{aligned}\Delta d_{2x} &= \Delta d_x - \Delta d_{1x}\\ &= (-38.171\text{ km}) + (0\text{ km})\\ \Delta d_{2x} &= -38.171\text{ km}\end{aligned}$$

Subtract the vertical components.

$$\begin{aligned}\Delta d_{2y} &= \Delta d_y - \Delta d_{1y}\\ &= (48.857\text{ km}) + (25\text{ km})\\ \Delta d_{2y} &= 23.857\text{ km}\end{aligned}$$

Combine the displacement components of the second vector to determine the second displacement.

$$\begin{aligned}\left|\Delta \vec{d}_2\right| &= \sqrt{\Delta d_{2x}^2 + \Delta d_{2y}^2}\\ &= \sqrt{(-38.171\text{ km})^2 + (23.857\text{ km})^2}\\ \left|\Delta \vec{d}_2\right| &= 45\text{ km}\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{\left|\Delta \vec{d}_{2y}\right|}{\left|\Delta \vec{d}_{2x}\right|}\right)\\ &= \tan^{-1}\left(\frac{23.857\ \cancel{\text{km}}}{38.171\ \cancel{\text{km}}}\right)\\ \theta &= 32^\circ\end{aligned}$$

Statement: The second displacement is 45 km [W 32° N].

11. Answers may vary. Sample answer: The length of the total displacement vector is determined by how the two displacement vectors line up.

If the second vector points west, parallel to the first vector, then the total displacement is as large as possible: 450 km [W] + 220 km [W] = 670 km [W]

If the second vector points east, opposite to the first vector, then the total displacement is as small as possible: 450 km [W] + 220 km [E] = 230 km [W]

If the second vector points in any other direction, the magnitude of the total displacement is between 230 km and 670 km.

12. Answers may vary. Sample answer: Vector addition is commutative. This means that it does not matter what order you use when you add two vectors. You can put the tail of the second to the tip of the first, or you can put the tail of the first to the tip of the second. You get the same total vector either way.