Section 1.1: Motion and Motion Graphs

Tutorial 1 Practice, page 10

1. (a) Given: $\Delta d_1 = 1.2$ km; $\Delta t_1 = 24$ min; $\Delta d_2 = 1.2$ km; $\Delta t_2 = 24$ min

Required: v_{av}

Analysis: Calculate the total distance travelled, $\Delta d = \Delta d_1 + \Delta d_2$, and the total time taken, $\Delta t = \Delta t_1 + \Delta t_2$. Then, use the equation $v_{av} = \dfrac{\Delta d}{\Delta t}$ to calculate the average speed.

$$\Delta t_1 = 24\ \cancel{\text{min}} \times \frac{1\ \text{h}}{60\ \cancel{\text{min}}}$$
$$\Delta t_1 = 0.40\ \text{h}$$

$$\Delta t_2 = 0.40\ \text{h}$$

Solution:
$$\begin{aligned}\Delta d &= \Delta d_1 + \Delta d_2 \\ &= 1.2\ \text{km} + 1.2\ \text{km} \\ \Delta d &= 2.4\ \text{km}\end{aligned}$$

$$\begin{aligned}\Delta t &= \Delta t_1 + \Delta t_2 \\ &= 0.40\ \text{h} + 0.40\ \text{h} \\ \Delta t &= 0.80\ \text{h}\end{aligned}$$

$$\begin{aligned}v_{av} &= \frac{\Delta d}{\Delta t} \\ &= \frac{2.4\ \text{km}}{0.80\ \text{h}} \\ v_{av} &= 3.0\ \text{km/h}\end{aligned}$$

Statement: The average speed for the entire route is 3.0 km/h.

(b) Given: $\Delta \vec{d} = 1.2$ km [E]; $\Delta t = 24$ min

Required: $\vec{v}_{av}$

Analysis: The average velocity is the ratio of the displacement and the time taken, $\vec{v}_{av} = \dfrac{\Delta \vec{d}}{\Delta t}$.

$$\Delta t = 24\ \cancel{\text{min}} \times \frac{1\ \text{h}}{60\ \cancel{\text{min}}}$$
$$\Delta t = 0.40\ \text{h}$$

Solution:
$$\begin{aligned}\vec{v}_{av} &= \frac{\Delta \vec{d}}{\Delta t} \\ &= \frac{1.2\ \text{km [E]}}{0.40\ \text{h}} \\ \vec{v}_{av} &= 3.0\ \text{km/h [E]}\end{aligned}$$

Statement: The average velocity from the house to the farthest position from the house is 3.0 km/h [E].

(c) Given: $\Delta\vec{d}_1 = 1.2$ km [E]; $\Delta t_1 = 0.40$ h; $\Delta\vec{d}_2 = 1.2$ km [W]; $\Delta t_2 = 0.40$ h

Required: $\vec{v}_{\text{av}}$

Analysis: Since the two displacements are equal in magnitude and opposite in direction, the total displacement during the walk is 0.0 km. So, the average velocity is 0.0 km/h.
Statement: The average velocity is 0.0 km/h.
(d) Answers may vary. Sample answer: The average velocities in parts (b) and (c) are different. Average velocity depends on displacement, which is a vector quantity. Since the displacements are not the same, the average velocities are not the same.
2. Given: $\vec{v} = 27$ m/s [forward]; $\Delta t = 0.32$ s

Required: $\Delta\vec{d}$

Analysis: Rearrange $\vec{v}_{\text{av}} = \dfrac{\Delta\vec{d}}{\Delta t}$ to solve for $\Delta\vec{d}$: $\Delta\vec{d} = \vec{v}_{\text{av}}\Delta t$.

Solution:
$$\begin{aligned}\Delta\vec{d} &= \vec{v}_{\text{av}}\Delta t\\ &= (27\ \text{m}/\cancel{\text{s}}\ [\text{forward}])(0.32\ \cancel{\text{s}})\\ \Delta\vec{d} &= 8.6\ \text{m [forward]}\end{aligned}$$

Statement: The bus moves 8.6 m [forward] before the driver reacts.
3. Given: $\Delta d = 200$ laps, at 4.02 km/lap; $\Delta t = 6.69$ h
Required: v_{av} in km/h

Analysis: Calculate the distance covered (in kilometres) and then the average speed, $v_{\text{av}} = \dfrac{\Delta d}{\Delta t}$.

Solution:
$$\begin{aligned}\Delta d &= 200\ \cancel{\text{laps}} \times \frac{4.02\ \text{km}}{1\ \cancel{\text{lap}}}\\ \Delta d &= 804\ \text{km}\end{aligned}$$

$$\begin{aligned}v_{\text{av}} &= \frac{\Delta d}{\Delta t}\\ &= \frac{804\ \text{km}}{6.69\ \text{h}}\\ v_{\text{av}} &= 1.20 \times 10^2\ \text{km/h}\end{aligned}$$

Statement: The average speed of a driver who completes 200 laps in 6.69 h is 1.20×10^2 km/h.
4. (a) Given: $\Delta d_1 = 140$ m; $\Delta t_1 = 55$ s; $\Delta d_2 = 45$ m; $\Delta t_2 = 21$ s

Required: $\vec{v}_{\text{av}}$

Analysis: Calculate the total distance walked, $\Delta d = \Delta d_1 + \Delta d_2$, and the time taken, $\Delta t = \Delta t_1 + \Delta t_2$. Then, calculate the average speed using $v_{\text{av}} = \dfrac{\Delta d}{\Delta t}$.

Solution:
$$\begin{aligned}\Delta d &= \Delta d_1 + \Delta d_2\\ &= 140\ \text{m} + 45\ \text{m}\\ \Delta d &= 185\ \text{m}\end{aligned}$$

$$\begin{aligned}\Delta t &= \Delta t_1 + \Delta t_2\\ &= 55\ \text{s} + 21\ \text{s}\\ \Delta t &= 86\ \text{s}\end{aligned}$$

$$\begin{aligned}v_{\text{av}} &= \frac{\Delta d}{\Delta t}\\ &= \frac{185\ \text{m}}{76\ \text{s}}\\ v_{\text{av}} &= 2.4\ \text{m/s}\end{aligned}$$

Statement: The student's average speed is 2.4 m/s.

(b) Given: $\Delta\vec{d}_1 = 140$ m [E]; $\Delta t_1 = 55$ s; $\Delta\vec{d}_2 = 45$ m [W]; $\Delta t_2 = 21$ s

Required: $\vec{v}_{av}$

Analysis: Calculate the total displacement, $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2$. Then, calculate the average velocity using $\vec{v}_{av} = \dfrac{\Delta\vec{d}}{\Delta t}$. Use east as positive.

Solution:

$$\begin{aligned}\Delta\vec{d} &= \Delta\vec{d}_1 + \Delta\vec{d}_2\\ &= 140\text{ m} + (-45\text{ m})\\ &= +95\text{ m}\\ \Delta\vec{d} &= 95\text{ m [E]}\end{aligned}$$

$$\begin{aligned}\vec{v}_{av} &= \frac{\Delta\vec{d}}{\Delta t}\\ &= \frac{95\text{ m [E]}}{76\text{ s}}\\ \vec{v}_{av} &= 1.2\text{ m/s [E]}\end{aligned}$$

Statement: The student's average velocity is 1.2 m/s [E].

5. (a) Given: $\Delta\vec{d}_1 = 62$ km [S]; $\Delta\vec{d}_2 = 78$ m [N]; $v_{av} = 55$ km/h

Required: $\vec{v}_{av}$

Analysis: Use the total distance, $\Delta d = \Delta d_1 + \Delta d_2$, and average speed, $v_{av} = \dfrac{\Delta d}{\Delta t}$, to determine the time taken for the trip, Δt. Then, determine the total displacement, $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2$. Then calculate the average velocity using $\vec{v}_{av} = \dfrac{\Delta\vec{d}}{\Delta t}$. Use north as positive.

Solution:

$$\begin{aligned}\Delta d &= \Delta d_1 + \Delta d_2\\ &= 62\text{ km} + 78\text{ km}\\ &= 140\text{ km}\\ \Delta d &= 140\text{ km}\end{aligned}$$

$$\begin{aligned}v_{av} &= \frac{\Delta d}{\Delta t}\\ \Delta t &= \frac{\Delta d}{v_{av}}\\ &= \frac{140\ \cancel{\text{km}}}{55\ \cancel{\text{km}}/\text{h}}\\ \Delta t &= 2.545\text{ h (two extra digits carried)}\end{aligned}$$

$$\begin{aligned}\Delta\vec{d} &= \Delta\vec{d}_1 + \Delta\vec{d}_2\\ &= (-62\text{ km}) + (+78\text{ km})\\ &= +16\text{ km}\\ \Delta\vec{d} &= 16\text{ m [N]}\end{aligned}$$

$$\begin{aligned}\vec{v}_{av} &= \frac{\Delta\vec{d}}{\Delta t}\\ &= \frac{16\text{ km [N]}}{2.545\text{ h}}\\ \vec{v}_{av} &= 6.3\text{ km/h [N]}\end{aligned}$$

Statement: The truck's average velocity is 6.3 km/h [N].

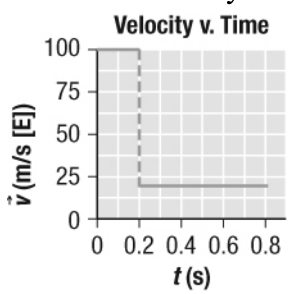
(b) Answers may vary. Sample answer: The truck turned around at one point during its trip. At the end, it was only 16 km away from where it started. As a result, the magnitude of the average velocity is quite low compared to the average speed.

Tutorial 2 Practice, page 13

1. (a) Velocity is slope on a position–time graph. Graph (c) has increasing slope, showing that the velocity is increasing.
(b) Graph (b) has decreasing slope, showing that the velocity is decreasing.
2. Answers may vary. Sample answers:
(a) Initially, the velocity is large and constant in the east direction. Then, it is lower and constant in the east direction. Calculate the slope of each section of the position–time graph using data from the graph.

t **(s)**	0.0	0.2	0.8
$\vec{d}$ **(m [E])**	0.0	20	30
$\vec{v}$ **(m/h [E])**	—	$\frac{\Delta \vec{d}}{\Delta t} = \frac{20-0.0}{0.2-0.0} = 100$	$\frac{\Delta \vec{d}}{\Delta t} = \frac{30-20}{0.8-0.2} = 17$

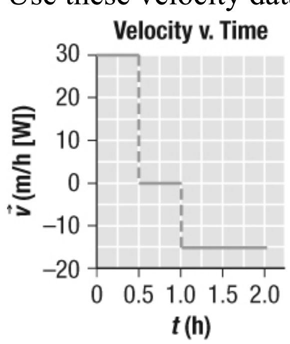
Use these velocity data to draw a velocity–time graph.



(b) Initially the velocity is large and constant in a west direction, then it is zero and finally it is low and constant in an east direction. Calculate the slope of each section of the position–time graph using data from the graph.

t **(h)**	0.0	0.5	1.0	2.0
$\vec{d}$ **(m [W])**	0.0	15	15	0.0
$\vec{v}$ **(m/h [W])**	—	$\frac{\Delta \vec{d}}{\Delta t} = \frac{15-0.0}{0.5-0.0} = 30$	$\frac{\Delta \vec{d}}{\Delta t} = \frac{15-15}{1.0-0.5} = 0$	$\frac{\Delta \vec{d}}{\Delta t} = \frac{0.0-15}{2.0-1.0} = -15$

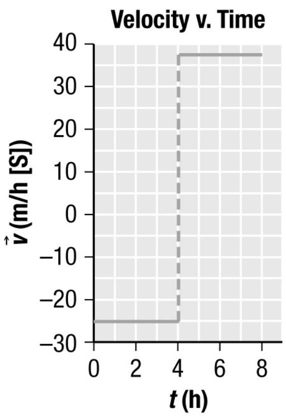
Use these velocity data to draw a velocity–time graph.



(c) Initially the velocity is low and constant in a north direction, and then it is large and constant in a south direction. Calculate the slope of each section of the displacement–time graph using data from the graph.

t **(h)**	0.0	4.0	8.0
$\vec{d}$ **(m [S])**	50	–50	100
$\vec{v}$ **(m/h [S])**	—	$\frac{\Delta\vec{d}}{\Delta t}=\frac{(-50)-50}{4.0-0.0}=-25$	$\frac{\Delta\vec{d}}{\Delta t}=\frac{100-(-50)}{8.0-4.0}=37.5$

Use these velocity data to draw a velocity–time graph.



Tutorial 3 Practice, page 15

1. Answers may vary. Sample answer:

(a) The velocity–time graph is a straight line showing that the car's velocity is changing at a constant rate. So, the car is moving with constant acceleration.

(b) The car starts from rest at $t = 0$ s. It moves north with increasing speed and constant acceleration. The car is moving at 12 m/s after 6.0 s.

(c) Given: velocity–time graph

Required: $\vec{a}$

Analysis: Read the coordinates of two points on the graph. Use these points to calculate the slope of the line, $\vec{a}=\frac{\Delta\vec{v}}{\Delta t}$. As in the graph, use north as positive.

Solution: Two clear points are (0.0 s, 0 m/s) and (6.0 s, 12 m/s).

$$\begin{aligned}\vec{a}&=\frac{\Delta\vec{v}}{\Delta t}\\&=\frac{12\text{ m/s}-0\text{ m/s}}{6.0\text{ s}-0.0\text{ s}}\\&=\frac{12\text{ m/s}}{6.0\text{ s}}\\&=+2.0\text{ m/s}^2\end{aligned}$$

$\vec{a}=2.0\text{ m/s}^2$ [N]

Statement: The acceleration of the car is 2.0 m/s^2 [N].

2. Answers may vary. Sample answer:
(a) Given: velocity–time graph
Required: $\vec{a}_{av}$
Analysis: Read the coordinates of the initial and final points on the graph. Use these points to calculate the slope of the secant, $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$. As in the graph, use forward as positive.
Solution: The points are (0 s, 0 m/s) and (6 s, 35 m/s).

$$\begin{aligned}\vec{a}_{av} &= \frac{\Delta\vec{v}}{\Delta t}\\ &= \frac{35\ \text{m/s} - 0\ \text{m/s}}{6\ \text{s} - 0\ \text{s}}\\ &= +6\ \text{m/s}^2\end{aligned}$$

$$\vec{a}_{av} = 6\ \text{m/s}^2\ [\text{forward}]$$

Statement: The average acceleration of the car for the entire trip is 6 m/s^2 [forward].
(b) Given: velocity–time graph
Required: $\vec{a}$ at $t = 3$ s and at $t = 5$ s.
Analysis: The instantaneous acceleration at a given time is the slope of the tangent to the velocity–time graph at that time. Sketch tangent lines at the required times, read the coordinates of two points on each tangent line, and then calculate the slopes.
Solution:
For the tangent at 3 s, two points are (1.5 s, 0 m/s) and (6 s, 25 m/s).

$$\begin{aligned}\vec{a} &= \frac{\Delta\vec{v}}{\Delta t}\\ &= \frac{25\ \text{m/s} - 0\ \text{m/s}}{6\ \text{s} - 1.5\ \text{s}}\\ &= \frac{25\ \text{m/s}}{4.5\ \text{s}}\end{aligned}$$

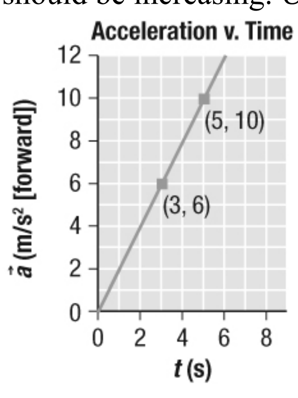
$$\vec{a} = 6\ \text{m/s}^2\ [\text{forward}]$$

For the tangent at 5 s, two points are (2.5 s, 0 m/s) and (6 s, 34 m/s).

$$\begin{aligned}\vec{a} &= \frac{\Delta\vec{v}}{\Delta t}\\ &= \frac{34\ \text{m/s} - 0\ \text{m/s}}{6\ \text{s} - 2.5\ \text{s}}\\ &= \frac{34\ \text{m/s}}{3.5\ \text{s}}\end{aligned}$$

$$\vec{a} = 10\ \text{m/s}^2\ [\text{forward}]$$

Statement: The instantaneous acceleration is 6 m/s^2 [forward] at 3 s and 10 m/s^2 [forward] at 5 s.
(c) The slope of the velocity–time graph increases gradually, so the acceleration–time graph should be increasing. Use the values of instantaneous acceleration from part (b) to draw the graph.



3. Answers may vary. Sample answer:
(a) Given: velocity–time graph
Required: $\vec{a}_{\text{av}}$
Analysis: Read the coordinates of initial and final points on the graph. Use these points to calculate the slope of the secant, $a_{\text{av}} = \dfrac{\Delta v}{\Delta t}$. As in the graph, use forward as positive.
Solution: The points are (0 s, 50 m/s) and (7 s, 0 m/s).

$$\begin{aligned} a_{\text{av}} &= \frac{\Delta v}{\Delta t} \\ &= \frac{0\text{ m/s} - 50\text{ m/s}}{7\text{ s} - 0\text{ s}} \\ &= \frac{-50\text{ m/s}}{7\text{ s}} \\ a &= -7\text{ m/s}^2 \end{aligned}$$

Statement: The average acceleration of the car for the entire trip is 7 m/s^2 [backward].
(b) Given: velocity–time graph
Required: $\vec{a}$ at $t = 2$ s, $t = 4$ s, and $t = 6$ s
Analysis: The instantaneous acceleration at a given time is the slope of the tangent to the velocity–time graph at that time. Sketch tangent lines at the required times, read the coordinates of two points on each tangent line, and then calculate the slopes.
Solution:
For the tangent at 2 s, two points are (1 s, 50 m/s) and (3 s, 42 m/s).

$$\begin{aligned} \vec{a} &= \frac{\Delta \vec{v}}{\Delta t} \\ &= \frac{42\text{ m/s} - 50\text{ m/s}}{3\text{ s} - 1\text{ s}} \\ &= \frac{-8\text{ m/s}}{2\text{ s}} \\ \vec{a} &= -4\text{ m/s}^2 \end{aligned}$$

For the tangent at 4 s, two points are (3 s, 42 m/s) and (5 s, 26 m/s).

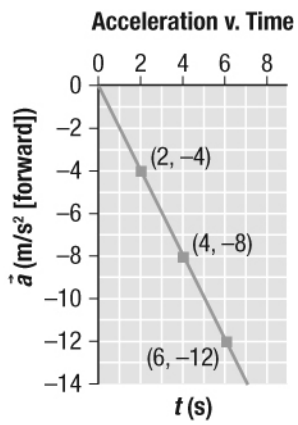
$$\begin{aligned} \vec{a} &= \frac{\Delta \vec{v}}{\Delta t} \\ &= \frac{26\text{ m/s} - 42\text{ m/s}}{5\text{ s} - 3\text{ s}} \\ &= \frac{-16\text{ m/s}}{2\text{ s}} \\ \vec{a} &= -8\text{ m/s}^2 \end{aligned}$$

For the tangent at 6 s, two points are (5 s, 26 m/s) and (7 s, 2 m/s).

$$\begin{aligned} \vec{a} &= \frac{\Delta \vec{v}}{\Delta t} \\ &= \frac{2\text{ m/s} - 26\text{ m/s}}{7\text{ s} - 5\text{ s}} \\ &= \frac{-24\text{ m/s}}{2\text{ s}} \\ \vec{a} &= -12\text{ m/s}^2 \end{aligned}$$

Statement: The instantaneous acceleration is 4 m/s^2 [backward] at 2 s, 8 m/s^2 [backward] at 4 s, and 12 m/s^2 [backward] at 6 s.

(c) The slope of the velocity–time graph decreases gradually, so the acceleration–time graph should be decreasing, possibly linearly. Use the values of instantaneous acceleration from part (b) to draw the graph.



Section 1.1 Questions, page 16

1. (a) Given: $\Delta d_1 = 22$ m; $\Delta d_2 = 11$ m

Required: total distance, Δd

Analysis: $\Delta d = \Delta d_1 + \Delta d_2$

Solution:
$$\begin{aligned}\Delta d &= \Delta d_1 + \Delta d_2\\ &= 22\text{ m} + 11\text{ m}\\ \Delta d &= 33\text{ m}\end{aligned}$$

Statement: The total distance travelled by the cardinal is 33 m.

(b) Given: $\Delta t_1 = 2.9$ s; $\Delta t_2 = 1.5$ s

Required: v_{av}

Analysis: $v_{\text{av}} = \dfrac{\Delta d}{\Delta t}$. To determine the total time add the partial times, $\Delta t = \Delta t_1 + \Delta t_2$.

Solution:
$$\begin{aligned}\Delta t &= \Delta t_1 + \Delta t_2\\ &= 2.9\text{ s} + 1.5\text{ s}\\ \Delta t &= 4.4\text{ s}\end{aligned}$$

$$\begin{aligned}v_{\text{av}} &= \frac{\Delta d}{\Delta t}\\ &= \frac{33\text{ m}}{4.4\text{ s}}\\ v_{\text{av}} &= 7.5\text{ m/s}\end{aligned}$$

Statement: The average speed of the cardinal is 7.5 m/s.

(c) Given: $\Delta\vec{d}_1 = 22\text{ m [E]}$; $\Delta\vec{d}_2 = 11\text{ m [N]}$

Required: $\vec{v}_{\text{av}}$

Analysis: The average velocity is the ratio of the total displacement to the total time, $\vec{v}_{\text{av}} = \dfrac{\Delta\vec{d}}{\Delta t}$.

To determine the total displacement, calculate the vector sum of the partial displacements, $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2$.

Solution: Determine the magnitude of the displacement.

$$\begin{aligned}\left|\Delta\vec{d}\right|^2 &= \left|\Delta\vec{d}_1\right|^2 + \left|\Delta\vec{d}_2\right|^2 \\ &= (22\text{ m})^2 + (11\text{ m})^2 \\ &= 605\text{ m}^2 \\ \left|\Delta\vec{d}\right| &= 24.60\text{ m (two extra digits carried)}\end{aligned}$$

Determine the angle θ.

$$\begin{aligned}\tan\theta &= \frac{\left|\Delta\vec{d}_2\right|}{\left|\Delta\vec{d}_1\right|} \\ &= \frac{11\,\cancel{\text{m}}}{22\,\cancel{\text{m}}} \\ \tan\theta &= 0.50 \\ \theta &= \tan^{-1}(0.50) \\ \theta &= 27^\circ\end{aligned}$$

Determine the average velocity.

$$\begin{aligned}\vec{v}_{\text{av}} &= \frac{\Delta\vec{d}}{\Delta t} \\ &= \frac{24.60\text{ m [E 27° N]}}{4.4\text{ s}} \\ \vec{v}_{\text{av}} &= 5.6\text{ m/s [E 27° N]}\end{aligned}$$

Statement: The cardinal's average velocity is 5.6 m/s [E 27° N].

2. (a) Given: $v_{\text{av}} = 15.0\text{ km/h}$

Required: v_{av} in m/s

Analysis: Convert units using 1 km = 1000 m, and 1 h = 3600 s.

Solution:
$$\begin{aligned}v_{\text{av}} &= 15.0\ \frac{\text{km}}{\text{h}} \\ &= 15.0\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\text{ m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3600\text{ s}} \\ &= 4.167\text{ m/s (one extra digit carried)} \\ v_{\text{av}} &= 4.17\text{ m/s}\end{aligned}$$

Statement: The average speed is 4.17 m/s.

(b) Given: $\Delta d = 2.90\ \text{km};\ v_{\text{av}} = 4.167\ \text{m/s}$
Required: Δt
Analysis: Use the relation $v_{\text{av}} = \dfrac{\Delta d}{\Delta t};\ \Delta t = \dfrac{\Delta d}{v_{\text{av}}}$.

Solution:
$$\begin{aligned}\Delta t &= \frac{\Delta d}{v_{\text{av}}}\\ &= \frac{2.90\ \cancel{\text{km}}}{4.167\ \cancel{\text{m}}/\text{s}} \times \frac{1000\ \cancel{\text{m}}}{1\ \cancel{\text{km}}}\\ \Delta t &= 696\ \text{s}\end{aligned}$$

Statement: It takes 696 s to complete one lap.
3. (a) Given: $\Delta d = 16\ \text{m};\ \Delta t = 2.1\ \text{s}$
Required: average speed, v_{av}
Analysis: $v_{\text{av}} = \dfrac{\Delta d}{\Delta t}$

Solution:
$$\begin{aligned}v_{\text{av}} &= \frac{\Delta d}{\Delta t}\\ &= \frac{16\ \text{m}}{2.1\ \text{s}}\\ v_{\text{av}} &= 7.6\ \text{m/s}\end{aligned}$$

Statement: The average speed of the skater is 7.6 m/s.
(b) Given: diameter, $D = 16\ \text{m}$
Required: time for one lap, Δt
Analysis: Calculate the circumference of the pond, $\Delta d = \pi D$. Use $v_{\text{av}} = \dfrac{\Delta d}{\Delta t}$ to determine the time, Δt, for one lap.
Solution:
$$\begin{aligned}\Delta d &= \pi D\\ &= \pi(16\ \text{m})\\ \Delta d &= 50.26\ \text{m (two extra digits carried)}\end{aligned}$$

$$\begin{aligned}v_{\text{av}} &= \frac{\Delta d}{\Delta t}\\ \Delta t &= \frac{\Delta d}{v_{\text{av}}}\\ &= \frac{50.26\ \cancel{\text{m}}}{7.619\ \cancel{\text{m}}/\text{s}}\\ \Delta t &= 6.6\ \text{s}\end{aligned}$$

Statement: It takes the skater 6.6 s to go around the edge of the pond.

4. (a) Given: $\Delta d = 450\text{ km} = 4.5 \times 10^4\text{ m}$; $\Delta t = 45\text{ min}$

Required: v_{av}, in m/s

Analysis: $v_{\text{av}} = \dfrac{\Delta d}{\Delta t}$;

$$\Delta t = 45\ \cancel{\text{min}} \times \frac{60\text{ s}}{1\ \cancel{\text{min}}}$$

$$\Delta t = 2.7 \times 10^3\text{ s}$$

Solution:
$$\begin{aligned} v_{\text{av}} &= \frac{\Delta d}{\Delta t} \\ &= \frac{4.5 \times 10^4\text{ m}}{2.7 \times 10^3\text{ s}} \\ v_{\text{av}} &= 170\text{ m/s} \end{aligned}$$

Statement: The airplane's average speed is 170 m/s.

(b) Given: $\Delta\vec{d} = 450\text{ km [E 15° N]}$

Required: $\vec{v}_{\text{av}}$, in m/s

Analysis: The airplane travels in one direction for the whole trip, so the average velocity is a vector equal in magnitude to the average speed.

Solution: The airplane's average velocity is 170 m/s $[\text{E } 15° \text{ N}]$.

5. Given: $\vec{v}_{\text{i}} = 0\text{ m/s}$; $\vec{v}_{\text{f}} = 96\text{ km/h [W]}$; $\Delta t = 4.1\text{ s}$

Required: $\vec{a}$

Analysis: Since the rocket starts from rest, its change in velocity is equal to its final velocity: $\Delta\vec{v} = \vec{v}_{\text{f}}$. Its acceleration is the ratio of its change in velocity to the time interval taken: $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$.

$$\vec{v}_{\text{f}} = \frac{96\ \cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\text{ m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\text{ s}}$$

$\vec{v}_{\text{f}} = 26.67\text{ m/s [W]}$ (one extra digit carried)

Solution:
$$\begin{aligned} \vec{a} &= \frac{\Delta\vec{v}}{\Delta t} \\ &= \frac{26.67\text{ m/}\cancel{\text{s}}\text{ [W]}}{4.1\ \cancel{\text{s}}} \\ \vec{a} &= 6.5\text{ m/s}^2\text{ [W]} \end{aligned}$$

Statement: The rocket accelerates at $6.5\text{ m/s}^2\text{ [W]}$.

6. Given: $\vec{a}_{av} = 1.37 \times 10^3 \text{ m/s}^2$ [W]; $\Delta t = 3.12 \times 10^{-2}$ s; $\vec{v}_f = 0$ m/s

Required: velocity of the ball before hitting the wall, $\vec{v}_i$

Analysis: The ball slows down when it hits the wall. Since the acceleration points west, the initial velocity must point east. Use $\vec{a}_{av} = \dfrac{\Delta \vec{v}}{\Delta t}$ to determine the change in velocity and the initial velocity.

Solution: $\vec{a} = \dfrac{\Delta \vec{v}}{\Delta t}$

$$\begin{aligned}\Delta \vec{v} &= \vec{a}_{av}\Delta t \\ &= (1.37 \times 10^3 \text{ m/s}^{\cancel{2}} \text{ [W]})(3.12 \times 10^{-2} \cancel{\text{s}}) \\ \Delta \vec{v} &= 42.744 \text{ m/s [W] (two extra digits carried)}\end{aligned}$$

$$\begin{aligned}\Delta \vec{v} &= \vec{v}_f - \vec{v}_i \\ \vec{v}_i &= \vec{v}_f - \Delta \vec{v} \\ &= 0 \text{ m/s} - 42.744 \text{ m/s [W]} \\ \vec{v}_i &= 42.7 \text{ m/s [E]}\end{aligned}$$

Statement: The ball hits the wall with velocity 42.7 m/s [E].

7. Given: $\vec{v}_i = 0$ m/s; $\vec{v}_f = 9.3$ m/s [forward]; $\Delta t = 3.9$ s

Required: average acceleration of the runner, $\vec{a}_{av}$

Analysis: $\vec{a}_{av} = \dfrac{\Delta \vec{v}}{\Delta t}$

Solution: $\vec{a}_{av} = \dfrac{\Delta \vec{v}}{\Delta t}$

$$\begin{aligned}&= \frac{9.3 \text{ m/s [forward]} - 0 \text{ m/s}}{3.9 \text{ s}} \\ \vec{a}_{av} &= 2.4 \text{ m/s}^2 \text{ [forward]}\end{aligned}$$

Statement: The runner's average acceleration is 2.4 m/s^2 [forward].

8. Answers may vary slightly based on student reading of the graph data. Sample answer:

(a) Given: position–time graph

Required: $\vec{v}_{av}$ for the entire trip

Analysis: Read the coordinates of initial and final points on the graph. Use these points to calculate the slope of the secant, $v_{av} = \dfrac{\Delta d}{\Delta t}$. As in the graph, use east as positive.

Solution: The points are (0 s, 0 m) and (14 s, 400 m).

$$v_{av} = \frac{\Delta d}{\Delta t}$$
$$= \frac{400\text{ m} - 0\text{ m}}{14\text{ s} - 0\text{ s}}$$
$$= \frac{400\text{ m}}{14\text{ s}}$$
$$v_{av} = 29\text{ m/s}$$

Statement: The average velocity of the car for the entire trip is 29 m/s^2 [E].

(b) Given: position–time graph

Required: $\vec{v}_{av}$ from t = 4 s to t = 14 s

Analysis: Use the graph to determine the position at t = 4 s and at t = 14 s. Use these points to calculate the slope of the secant, $v_{av} = \frac{\Delta d}{\Delta t}$. As in the graph, use east as positive.

Solution: The points are (4 s, 30 m) and (14 s, 400 m).

$$v_{av} = \frac{\Delta d}{\Delta t}$$
$$= \frac{400\text{ m} - 30\text{ m}}{14\text{ s} - 4\text{ s}}$$
$$= \frac{370\text{ m}}{10\text{ s}}$$
$$v_{av} = 37\text{ m/s}$$

Statement: The average velocity of the car for the last 10 s of the trip is 37 m/s [E]. This is higher than for the trip as a whole because the car is accelerating—the car was moving faster on average toward the end of the trip.

(c) Given: position–time graph

Required: $\vec{v}$ at $t = 4.0$ s, $t = 8.0$ s, and at $t = 12.0$ s

Analysis: The instantaneous velocity at a given time is the slope of the tangent to the position–time graph at that time. Sketch in tangent lines at the required times, read the coordinates of two points on each tangent line and then calculate the slopes.

Solution:

For the tangent at 4.0 s, two points are (2.0 s, 0.0 m/s) and (6.0 s, 68 m/s).

$$v = \frac{\Delta d}{\Delta t}$$
$$= \frac{68\text{ m} - 0\text{ m}}{6.0\text{ s} - 2.0\text{ s}}$$
$$= \frac{68\text{ m}}{4.0\text{ s}}$$
$$v = 17\text{ m/s}$$

For the tangent at 8.0 s, two points are (6.0 s, 70 m/s) and (10 s, 200 m/s).

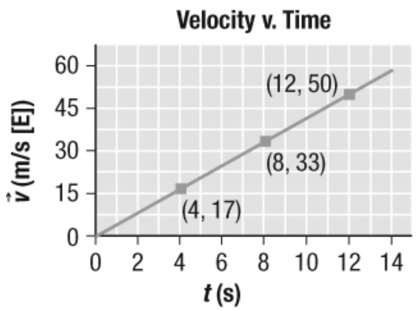
$$v = \frac{\Delta d}{\Delta t}$$
$$= \frac{200\text{ m} - 70\text{ m}}{10.0\text{ s} - 6.0\text{ s}}$$
$$= \frac{130\text{ m}}{4.0\text{ s}}$$
$$v = 33\text{ m/s}$$

For the tangent at 12 s, two points are (10.0 s, 200 m/s) and (14.0 s, 400 m/s).

$$\begin{aligned} v &= \frac{\Delta d}{\Delta t} \\ &= \frac{400\text{ m} - 200\text{ m}}{14.0\text{ s} - 10.0\text{ s}} \\ &= \frac{200\text{ m}}{4.0\text{ s}} \\ v &= 50\text{ m/s} \end{aligned}$$

Statement: The instantaneous velocities are approximately 17 m/s [E] at 4.0 s, 33 m/s [E] at 8.0 s, and 50 m/s [E] at 12.0 s.

(d) The slope of the position–time graph gradually increases throughout the trip showing that the velocity is increasing to the east. The velocity–time graph should show this gradual increase; it may be linear. Draw a velocity–time graph using the values of instantaneous velocity. Draw a smooth line for a reasonable trend.



9. Answers may vary slightly based on student reading of the graph data. Sample answer:

(a) Given: velocity–time graph

Required: $\vec{a}_{av}$

Analysis: Read the coordinates of the initial and final points on the graph. Use these points to calculate the slope of the secant, $a_{av} = \frac{\Delta v}{\Delta t}$. As in the graph, use forward as positive.

Solution: The points are (0 s, 50 m/s) and (10 s, 0 m/s).

$$\begin{aligned} a_{av} &= \frac{\Delta v}{\Delta t} \\ &= \frac{0\text{ m/s} - 50\text{ m/s}}{10\text{ s} - 0\text{ s}} \\ &= \frac{-50\text{ m/s}}{10\text{ s}} \\ a_{av} &= -5\text{ m/s}^2 \end{aligned}$$

Statement: The average acceleration of the car for the entire trip is 5 m/s^2 [backward].

(b) Given: velocity–time graph
Required: $\vec{a}$ at $t = 3$ s, $t = 6$ s and at $t = 9$ s
Analysis: The instantaneous acceleration at a given time is the slope of the tangent to the velocity–time graph at that time. Sketch in tangent lines at the required times, read the coordinates of two points on each tangent line and then calculate the slopes.
Solution:
For the tangent at 3 s, two points are (2 s, 50 m/s) and (4 s, 44 m/s).

$$\begin{aligned}\vec{a} &= \frac{\Delta\vec{v}}{\Delta t}\\ &= \frac{44\ \text{m/s} - 50\ \text{m/s}}{4\ \text{s} - 2\ \text{s}}\\ &= \frac{-6\ \text{m/s}}{2\ \text{s}}\\ \vec{a} &= -3\ \text{m/s}^2\end{aligned}$$

For the tangent at 6 s, two points are (5 s, 36 m/s) and (7 s, 24 m/s).

$$\begin{aligned}\vec{a} &= \frac{\Delta\vec{v}}{\Delta t}\\ &= \frac{24\ \text{m/s} - 36\ \text{m/s}}{7\ \text{s} - 5\ \text{s}}\\ &= \frac{-12\ \text{m/s}}{2\ \text{s}}\\ \vec{a} &= -6\ \text{m/s}^2\end{aligned}$$

For the tangent at 9 s, two points are (8 s, 18 m/s) and (10 s, 0 m/s).

$$\begin{aligned}a &= \frac{\Delta v}{\Delta t}\\ &= \frac{0\ \text{m/s} - 18\ \text{m/s}}{10\ \text{s} - 8\ \text{s}}\\ &= \frac{-18\ \text{m/s}}{2\ \text{s}}\\ a &= -9\ \text{m/s}^2\end{aligned}$$

Statement: The instantaneous accelerations are 3 m/s^2 [backward] at 3 s, 6 m/s^2 [backward] at 6 s, and 9 m/s^2 [backward] at 9 s.
(c) The slope of the velocity–time graph decreases gradually, so the acceleration–time graph should be decreasing, possibly linearly. Use the values of instantaneous acceleration from part (b) to draw the graph.