12.3

Wave Properties of Classical Particles

Research into the quantum world has led to many discoveries in science and technology. In previous sections, you read about how photons have both wave and particle properties. The same is true of other particles. For example, electrons do not behave as particles only, but as both particles and waves. In this section, we will examine the results of one of the most important experiments in quantum physics: the electron double-slit experiment. This experiment produced the clearest evidence of how the properties of waves and particles are both present in the quantum world.

Wave-like Properties of Classical Particles

Earlier you read about the wave-like properties of electrons. The concept that the properties of both classical waves and classical particles are present at the same time—wave–particle duality—is essential for understanding the world of electrons, atoms, and molecules.

By the early 1920s, the photon theory of light was well established. However, physicists were still struggling with how to describe particles such as electrons in the quantum world. In 1924, Louis de Broglie first suggested that all classical particles have wave-like properties. At the time, experimental evidence of interference with electrons had not yet been discovered. That did not stop de Broglie, however, who developed his theory to agree with the behaviour of photons.

In Section 12.2, you read that a photon has a momentum given by

$$p_{\text{photon}} = \frac{h}{\lambda}$$

de Broglie turned this result around and hypothesized that a particle with momentum p has a wavelength of

$$\lambda = \frac{h}{p}$$

de Broglie wavelength the wavelength associated with the motion of a particle possessing momentum of magnitude p

matter wave the wave-like behaviour of particles with mass

This quantity—the wavelength associated with the motion of a particle with momentum p—is the **de Broglie wavelength**. If a particle has a wavelength, the particle should exhibit interference just as waves do. The test of de Broglie's hypothesis was to look for interference involving classical particles. Such an experiment is easiest if the wavelength is long, and according to the de Broglie wavelength equation, a long wavelength means that the value for momentum has to be small. For a classical particle, the momentum is mv. Therefore, a particle with a very small mass is required, and the lightest known particle at that time was the electron. As a result, the first observation of **matter waves**, the wave-like behaviour of massive particles, came from an experiment done with electrons.

The Electron Double-Slit Experiment

As you learned in Sections 9.5 and 12.1, a simple physics experiment to illustrate the wave nature of light is the double-slit experiment. In this experiment, a screen, called screen 1, with two slits is placed at a distance from a point source of light, and a second screen, called screen 2, is placed behind screen 1. When light is directed at screen 1, many interference fringes appear across screen 2, instead of just two bars of light directly in line with the light and the slits. Why does that happen? The slits diffract the light, and the fringes mark the interference of the light waves.

In 1927, physicists Clinton Davisson and Lester Germer performed an experiment in which they aimed a beam of electrons at a crystal target (**Figure 1**). The atoms in the target were spaced at regular intervals, acting as a series of slits for the electrons. Just as with the diffraction of light, the Davisson–Germer experiment exhibits interference when the wavelength of the electrons is similar to the spacing between the atoms in the crystal. The diffraction technique used in the Davisson–Germer experiment is still used today as a way to measure molecule spacing within a crystal. WEB LINK

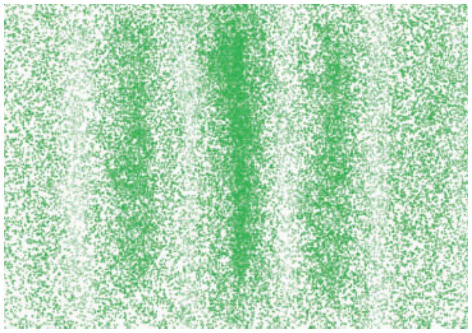

Figure 1 The atoms in a crystal act as slits when electrons or high-energy photons are directed at the crystal. When electrons exit the crystal, they form an interference pattern.

In the following Tutorial, you will quantitatively examine the wave-like properties of electrons.

Tutorial 1 Determining the Wavelength of an Electron

Davisson and Germer's demonstration of interference with electrons showed conclusively that electrons have wave-like properties. Careful measurements of the interference pattern allowed them to calculate the de Broglie wavelength of the electron. This Sample Problem shows how to calculate the de Broglie wavelength.

Sample Problem 1: Calculating the de Broglie Wavelength of an Electron

In their studies of interference with electrons, Davisson and Germer used electrons with a kinetic energy of approximately 50 eV, which equals 8.0×10^{-18} J. The mass of an electron is 9.11×10^{-31} kg.

(a) Calculate the de Broglie wavelength of these electrons in metres and nanometres.

(b) The spacing between atoms in a typical crystal is about 0.3 nm. How does this spacing compare with the wavelength of the electrons used by Davisson and Germer?

Solution

(a) **Given:** $m = 9.11 \times 10^{-31}$ kg; $E_k = 50$ eV $= 8.0 \times 10^{-18}$ J; $h = 6.63 \times 10^{-34}$ J·s

Required: λ

Analysis: Using the equation for kinetic energy, $E_k = \frac{1}{2}mv^2$, solve for the speed. Use the equation for momentum, $p = mv$, and the quantum relation $\lambda = \frac{h}{p}$ to calculate the de Broglie wavelength.

Solution: $E_k = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{2E_k}{m}}$$

$$= \sqrt{\frac{2(8.0 \times 10^{-18}\ \text{J})}{9.11 \times 10^{-31}\ \text{kg}}}$$

$$v = 4.19 \times 10^{6}\ \text{m/s}$$

$$\lambda = \frac{h}{p}$$

$$= \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-34}\ \text{J·s}}{(9.11 \times 10^{-31}\ \text{kg})(4.19 \times 10^{6}\ \text{m/s})}$$

$$\lambda = 1.74 \times 10^{-10}\ \text{m}$$

Statement: The wavelength of the electrons in the Davisson–Germer experiment was about 1.74×10^{-10} m, or 0.174 nm.

(b) This wavelength is less than, but similar to, the spacing between atoms in a crystal solid.

Practice

1. Calculate the de Broglie wavelength for an electron with momentum 1.8×10^{-25} kg·m/s. K/U T/I [ans: 3.7×10^{-9} m]
2. Calculate the de Broglie wavelength for a proton moving with a speed of 3.4×10^5 m/s. The mass of a proton is 1.7×10^{-27} kg. K/U T/I [ans: 1.1×10^{-12} m]
3. Calculate the de Broglie wavelength for a 140 g baseball moving at 140 km/h. T/I A [ans: 1.2×10^{-34} m]
4. Compare your answer to Question 3 with the diameter of a proton, which is about 10^{-15} m. What does the baseball's de Broglie wavelength mean? K/U T/I A

It is possible to determine the de Broglie wavelength of larger objects, such as baseballs. However, the momentum of large objects tends to be so large that it implies an incredibly small wavelength. That is why we are unable to see the interference of these objects.

Interpreting the Double-Slit Experiment

The results of double-slit experiments using photons, electrons, or any other small particle are surprising and difficult to understand. The picture of nature at very small scales differs greatly from the classical picture at large scales. Electrons arrive at a screen in single, particle-like amounts, but the spot on the screen where they arrive is determined by wave-like interference behaviour.

The equations of quantum mechanics make precise predictions about the results of a double-slit experiment. However, they do not give a clear description of what happens to photons, electrons, or other particles as they pass through the slits and travel to the screen. As a result of this uncertainty, researchers have proposed different interpretations of what happens in the quantum world. The debate still continues over which interpretation gives the best description. There are researchers, including some at Canada's Perimeter Institute for Theoretical Physics, who support each of these interpretations. CAREER LINK

COLLAPSE INTERPRETATION

In the collapse interpretation, the electron may behave sometimes as a wave and sometimes as a particle, but always one or the other. The laws that determine the motion of an electron differ in either case. The electron leaves its source behaving as a particle, but then it spreads out and travels as a wave until it is measured at the screen (**Figure 2**). When you measure the location of the electron, it somehow collapses back into a particle. It then arrives at one location on the screen, as a particle would. The collapse interpretation claims that an electron physically changes from a particle to a wave and back again. These two behaviours and the physical laws that go with them alternate in a way that is not predicted by quantum mechanics.

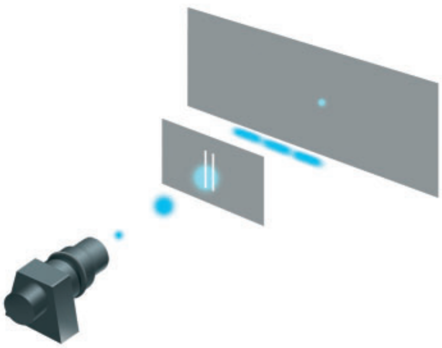

Figure 2 In the collapse interpretation, each electron in a double-slit experiment travels as a spread-out wave.

PILOT WAVE INTERPRETATION

In the pilot wave interpretation, the electron is just a simple particle whose motion is described by a single law. The pilot wave interpretation avoids any unexplained collapse, but the motion of the electron depends on a mysterious pilot wave (**Figure 3**). To obtain the interference pattern in the double-slit experiment, the behaviour of the pilot wave must depend on everything everywhere in the universe, including future events. For example, the pilot wave "knows" whether one or two slits are open, and whether or not a detector is turned on at the screen.

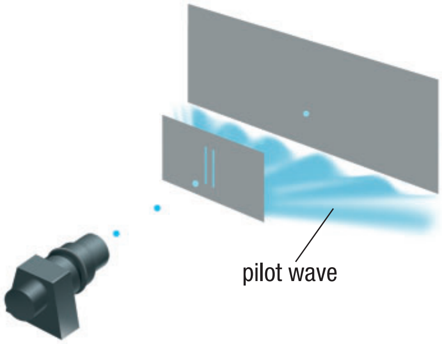

Figure 3 In the pilot wave interpretation, the electrons in a double-slit experiment are particles whose motion depends on a pilot wave.

MANY WORLDS INTERPRETATION

In the many worlds interpretation, electrons are simple particles, as in the pilot wave interpretation. The many worlds interpretation does not introduce a wave-like spreading or collapsing of the electron, though. Nor does it use a mysterious pilot wave. Instead, according to the many worlds interpretation, the universe constantly splits into many versions of itself. Each version exists as a separate parallel universe that cannot interact with any other version. A parallel universe exists for each of the electron's possible states. When the electron reaches the slits, the entire universe splits into two. In one version of the universe, the electron passes through the left slit, and in the other version it passes through the right slit. If the many worlds interpretation is true, then our universe consists of a vast number of parallel worlds, some holding versions of every one of us.

COPENHAGEN INTERPRETATION

The Copenhagen interpretation deals directly with the results of measurements made on physical objects. This interpretation of quantum mechanics was developed in the 1920s by Niels Bohr and his colleagues, primarily at the University of Copenhagen. The interpretation discusses only what you can actually do or observe. It does not use theoretical ideas such as undetectable collapses, invisible pilot waves, and multiple parallel universes. The Copenhagen view interprets the physical laws in terms of information about actual measurements made on a quantum-mechanical system. The Copenhagen interpretation basically says that certain questions do not have answers, such as what electrons are "doing" as they travel to the detection screen. You can only ask what the results will be if you do a certain experiment. This view was the dominant one for most of the twentieth century.

The debate over the different interpretations continues. Although the interpretations have some strange and useful features, they all make the same predictions for experimental results. Some researchers take the view that this fact means that the debate will never end, or that the debate does not need to be settled. They focus on using the predictions of quantum mechanics to explore new features of the quantum world and new technological uses.

The Wave Function: A Mathematical Description of Wave–Particle Duality

The equations of quantum physics describe wave–particle duality and the behaviour of photons, electrons, and other quantum objects with a mathematical tool called a wave function. A wave function gives the probability for a particle to take any possible path, or for the particle to show up at any possible location on the detection screen in the double-slit experiment. All the interpretations of quantum mechanics must include some discussion of the wave function. However, we will not include any wave functions in this discussion because they are beyond the scope of this book.

You can calculate the wave function using an equation developed by Erwin Schrödinger. He was one of the inventors of quantum theory and in 1933 received the Nobel Prize in Physics for this work. In quantum mechanics, researchers use the Schrödinger equation to determine the wave function and how it varies with time. This is in much the same way that physicists use Newton's laws of mechanics to determine the motion of an object.

In many situations, the solutions of the Schrödinger equation are mathematically similar to standing waves. As an example, consider an electron confined to a particular region of space, as shown in **Figure 4(a)**. In classical terms, you can think of this region of space as an extremely deep canyon or box from which the electron cannot escape. A classical particle moving around inside the box would simply travel back and forth, bouncing from one wall to another. The wave function for a particle–wave (such as an electron) inside this box is described by standing waves, similar to those you would see on a string.

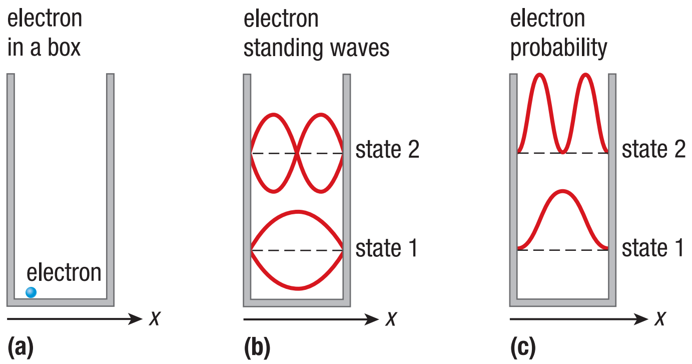

Figure 4 (a) A thought experiment of an electron that is trapped in a box. (b) The electron wave probability function forms a standing particle–wave similar to the standing waves on a string fastened to the walls of the box. The electron wavelength must therefore "fit" into the box as it would for a standing wave. (c) The quantum-mechanical probabilities of finding the electron at different locations in the box correspond to each of the two wave functions in (b).

Figure 4(b) shows two possible wave function solutions corresponding to electrons with different kinetic energies. The wavelengths of these standing waves are different, since the wavelength of an electron depends on its kinetic energy. In the case of mechanical waves in classical mechanics, these two solutions correspond to two standing waves with different wavelengths.

After determining the wave function for a particular situation, such as for the electron in Figure 4(b), you can try to calculate the position and speed of the electron. However, the results do not give a simple single value for x. Instead, the wave function allows the calculation of the probability of finding the electron at different locations in space. The probability for each of the wave functions in Figure 4(b) is shown in **Figure 4(c)** for the electron in a box. The probability of finding the electron at certain values of x is large in some regions and small in others. This corresponds to the anti-nodes and nodes of a standing wave. The probability distribution (how the probability varies with position within the box) is different for each wave function.

The Heisenberg Uncertainty Principle

German physicist Werner Heisenberg's early studies of the meaning of quantum mechanics led him to discover a limitation on measurements of quantum systems. The **Heisenberg uncertainty principle** says that there is a limit to how accurately simultaneous measurements of the position and momentum of a quantum object can be. If you measure the position of a quantum object with great accuracy, then you can only measure its momentum with little accuracy. In addition, the act of measuring the system itself disturbs the system. While this disturbance is not significant in the macroscopic world, the effects are obvious in the quantum world. The mathematical expression of the Heisenberg uncertainty principle is

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

where h is Planck's constant.

Heisenberg uncertainty principle a mathematical statement that says that if Δx is the uncertainty in a particle's position, and Δp is the uncertainty in its momentum, then $\Delta x \, \Delta p \geq \frac{h}{4\pi}$, where h is Planck's constant

Imagine that you set up a detector to measure electron diffraction through a single slit. The width of the slit equals the uncertainty in the position of the electron, Δx, as it passes through the first screen. If you decrease the width of the slit, you decrease the amount of uncertainty in the electron's position. As a consequence, according to Heisenberg's uncertainty principle, the uncertainty in the momentum of the electron, Δp, must increase. The direction in which the electron moves as it leaves the slit becomes more uncertain. As a result, the electron may hit the second screen in a wider range of locations, and the peaks of the diffraction pattern may become wider and less focused (**Figure 5**).

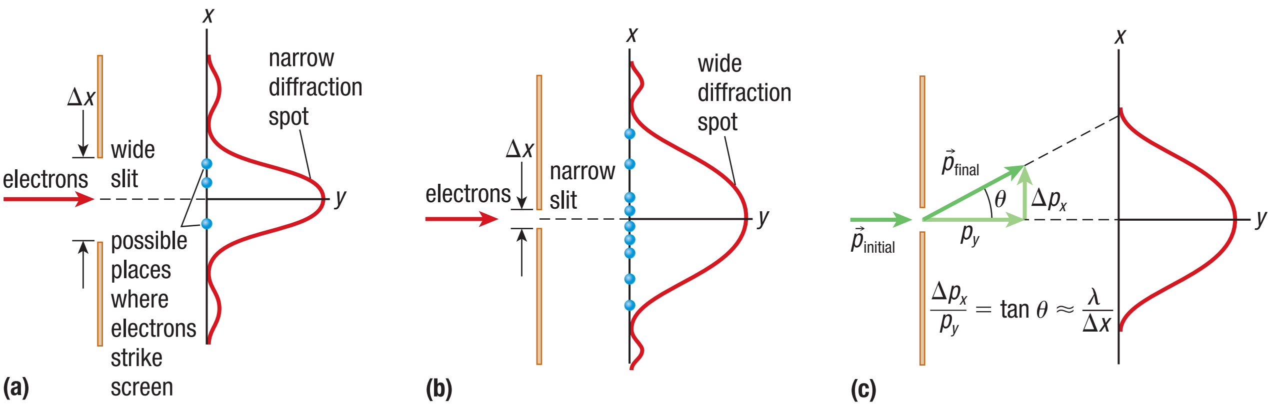

Figure 5 (a) When an electron passes through a wide slit, the diffraction spot on the screen is narrow. (b) When the slit is made narrower, the diffraction spot becomes wider. (c) The spreading of an electron wave as it passes through a slit gives a way to relate the uncertainty in the electron's position, Δx, to the uncertainty in its momentum, Δp.

Applications of the Quantum World

The double-slit interpretations and Heisenberg's ideas may leave you feeling that science cannot adequately explain quantum physics. The debate over the interpretations of what happens to electrons after they exit the double slits illustrates the idea that science does not always answer every question. Scientific understanding progresses tentatively and is always evolving and partly uncertain.

Quantum mechanics does, however, make quite accurate predictions about the statistics of observed results, such as the interference patterns made by many particles in a double-slit interference experiment. It simply says that the world is unpredictable for single events, such as a single electron passing through a double-slit setup. This unpredictability is in contrast to the fully predictable world of classical physics. That unpredictability has not slowed the advance of engineering designs and technologies using the principles of quantum physics.

UNIT TASK BOOKMARK

As you work on the Unit Task on page 666, apply what you have learned about technological applications of quantum theory.

Figure 6 This electron microscope image is a foot of a housefly. The white hairs under the claws allow the fly to cling to smooth surfaces.

Applications of wave–particle duality have led to the invention of LEDs, solar cells, and electron microscopes, and to advances in computers, the Internet, and nanotechnologies. In the case of the electron microscope, electron waves create high-resolution images of extremely small objects in a variety of fields (**Figure 6**). The resolution of a microscope depends on the wavelength of the waves used in its beam. In an electron microscope, a beam of electrons is aimed at an object using magnetic and electric fields. The wavelength of the beam is reduced to the de Broglie wavelength of the electrons, which is much smaller than the wavelengths in visible light.

Electronic devices that contain transistors are possible because engineers use the wave model of electrons to design the transistors. You are using the quantum nature of electrons when you talk on a cellphone, search the Internet on a computer, or listen to music on an MP3 player.

Medical imaging is another important application of quantum mechanics. Positron emission tomography (PET) scans allow doctors to produce images of biochemical processes in our bodies using the particle nature of light. Patients swallow a substance that emits positrons, or antimatter electrons, into the body. When positrons collide with an electron, two photons are released. The photons are detected by a device that can create highly detailed images. The use of photons in this way has enabled medical practitioners to observe brain function (**Figure 7**). CAREER LINK

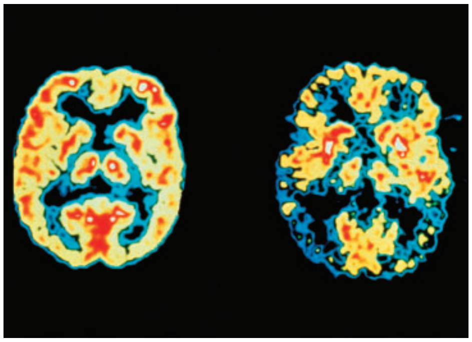

Figure 7 These false-colour PET scans show the brain of a normal patient on the left and the brain of a patient with Alzheimer's disease on the right. The red and yellow represent areas of high brain activity, and the blue and black represent areas of low brain activity.

Research This

Exploring Quantum Computers

Skills: Researching, Analyzing, Evaluating, Communicating — SKILLS HANDBOOK A4.1

A quantum computer is to a traditional digital computer what "a laser is to a light bulb," according to Seth Lloyd, a quantum engineer at the Massachusetts Institute of Technology. Unlike a digital computer, which uses bits and bytes, a quantum computer uses qubits and quantum properties to manipulate data. In this activity, you will research quantum computing and its potential applications.

1. Research quantum computers, and find out how they are designed.
2. Examine some of the problems facing scientists as they attempt to create workable quantum computers.
3. Identify some possible applications of quantum computing.

A. Describe the major differences in design between quantum computers and digital computers. T/I C

B. Describe some of the issues facing scientists attempting to create quantum computers. T/I C

C. Suggest some possible applications of quantum computing. T/I C

WEB LINK

12.3 Review

Summary

- Louis de Broglie hypothesized that electrons possess both particle-like and wave-like properties, including a wavelength related to their momentum as $\lambda = \frac{h}{p}$, where h is Planck's constant. The wavelength is called the de Broglie wavelength.
- Experiments by Davisson and Germer confirmed that electrons exhibit the wave-like property of interference with a wavelength given by de Broglie's wavelength.
- The equations of quantum mechanics do not explain what happens to a single electron during a double-slit experiment. Different interpretations of the equations attempt to address the question. The collapse interpretation says that the electron physically changes from a particle to a wave and back again. The pilot wave interpretation avoids any unexplained collapse, but states that the motion of the electron depends on a pilot wave. In the many worlds interpretation, the electron is a real particle that exists in several parallel universes. The Copenhagen interpretation states that we cannot comment on the nature of the electron between measurements.
- A wave function gives the probability for any quantum object to take any possible path, or for the object to be at any possible location on the detection screen in the double-slit experiment.
- The Heisenberg uncertainty principle says that we cannot simultaneously determine the position and momentum of a quantum object with great accuracy.
- Wave–particle duality has many technological applications, such as electron microscopy and PET scans.

Questions

1. An electron has a de Broglie wavelength of 150 nm. Determine its speed. K/U T/I
2. The mass of a proton is 1800 times the mass of an electron. An electron and a proton have the same de Broglie wavelength. Determine the ratio of their energies. K/U T/I
3. Determine the de Broglie wavelength of a 1000.0 kg car that (a) has a speed of 100.0 km/h, (b) has a speed of 10.0×10^3 km/h, and (c) is at rest. K/U T/I
4. Compare how particles are viewed in classical physics and in quantum mechanics. K/U C
5. List one example of experimental evidence for (a) wave-like properties of matter and (b) particle-like properties of electromagnetic radiation. K/U C A
6. Are wave functions real? Explain your answer. K/U T/I
7. Why do the different interpretations of quantum mechanics exist? Which interpretation do you think is most likely? Explain your answer. K/U T/I C A
8. Taking the Heisenberg uncertainty principle into account, explain whether it is possible to take exact measurements of an electron when it is at rest. K/U C
9. The late Canadian Nobel laureate Willard Boyle (**Figure 8**) developed an important application of quantum mechanics called a charge-coupled device (CCD). Research Boyle and CCDs, and prepare a summary that includes biographical information, the technology and the physics behind CCDs, the development of the technology, and how CCDs contribute to physics and society. Present your research in the form of a research paper, a website, or a short video documentary. T/I C A

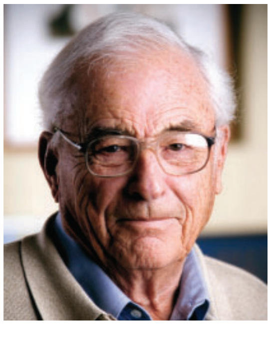

Figure 8

WEB LINK