11.4 Mass–Energy Equivalence

In Section 11.3, you learned how to adjust the definition of momentum using the mathematics of special relativity so that the observations between inertial frames of reference make sense at speeds near c. Now we will examine the concepts of work and energy conservation.

In the Newtonian definition of work, the mass of an object remains constant, and any energy transferred to increase its kinetic energy results in an increase in speed only. You would expect this relation to remain valid for a continuous net force. However, this does not happen, as scientists discovered in experiments performed in the early twentieth century. Only when special relativity was applied to the problem, so that acceleration and force were proportional with a factor of $\sqrt{1 - \frac{v^2}{c^2}}$, could the results of these experiments be reconciled with the laws of physics.

When you apply this result to the classical work–kinetic energy equation, an additional term results from special relativity. This additional term forms what is probably the most famous equation in physics: $E = mc^2$. Perhaps the most familiar application of this equation is in nuclear physics. During nuclear fusion, protons and neutrons combine in the interior of the Sun to form helium nuclei, as well as the nuclei of other elements (**Figure 1**). The difference in the masses of the reactants and products of fusion, multiplied by c^2, equals the released energy.

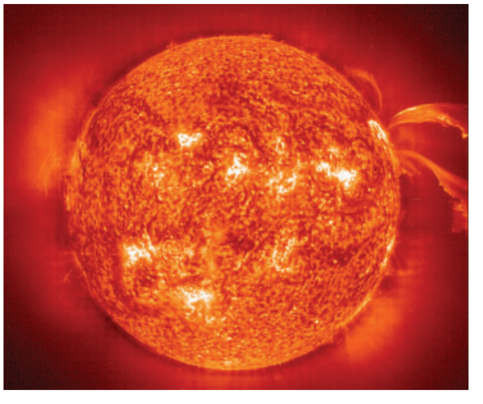

Figure 1 The Sun's energy comes from the fusion of hydrogen nuclei and neutrons to produce helium nuclei and energy. This process is a direct result of mass converting to energy, as predicted in the special theory of relativity.

Mass–Energy Equivalence

Recall from the discussion of relativistic momentum (Section 11.3) that the relativistic mass of an object varies with the inertial reference frames in which the mass and an observer are located. From the relativistic momentum equation, it follows that the equation for relativistic mass is

$$m_{\text{relativistic}} = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where m is the rest mass of the object.

Using the equations of special relativity, Einstein concluded that the total energy, E_{total}, for an object with rest mass m moving with speed v is equal to

$$E_{\text{total}} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

When the object is at rest ($v = 0$), the equation simplifies to

$$E_{rest} = mc^2$$

This is Einstein's famous $E = mc^2$ equation. It means that the rest mass of an object and its energy are equivalent. This energy is called the object's rest energy. The **rest energy**, E_{rest}, is the amount of energy an object at rest has with respect to an observer, and it does not change.

When an object is in motion, its total energy is larger than its rest energy. The **relativistic kinetic energy**, E_k, is the difference between the total energy of the object and its rest energy:

rest energy (E_{rest}) the amount of energy an object at rest has with respect to an observer

relativistic kinetic energy (E_k) the energy of an object in excess of its rest energy

$$E_k = E_{total} - E_{rest}$$

$$E_k = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2$$

Remember that, because of the relativistic treatment, kinetic energy can become very large, but the object's speed never quite reaches the speed of light (**Figure 2**). From the total energy expression, you can see that as the speed increases, the object responds as if it had a mass larger than its mass at low speeds.

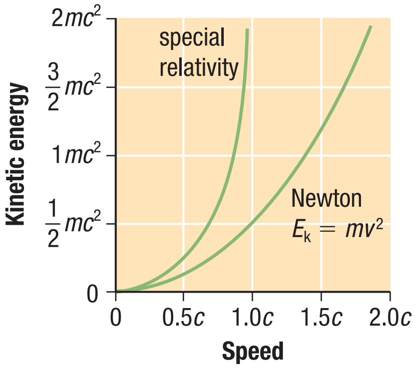

Figure 2 Even when an object has a very large kinetic energy, its speed never reaches the speed of light.

As in the case of relativistic momentum, the mass here is the object's rest mass (or proper mass). Unlike the various types of potential energy, where a force is acting on an object without moving it, rest energy is a property of matter itself. Just as theories about space and time shifted to a unified space-time theory, so too did theories about mass and energy shift to a unified mass–energy theory. Consequently, the conservation of energy principle is now the principle of **conservation of mass–energy**, which states that the rest energy is equal to rest mass times the speed of light squared.

conservation of mass–energy the principle that rest mass and energy are equivalent

Since the speed of light is a very large number, the magnitude of the rest energy can be very large, even when the rest mass is small. In fact, a rest mass of only 1 kg corresponds to a tremendous amount of rest energy:

$$\begin{aligned} E_{rest} &= mc^2 \\ &= (1\text{ kg})(3.0 \times 10^8\text{ m/s})^2 \\ E_{rest} &= 9 \times 10^{16}\text{ J} \end{aligned}$$

To put this number into perspective, the total daily energy consumption for Canada in 2008 was 2.4×10^{16} J, which is less than a third of the amount of energy available from a rest mass of 1 kg. You can now understand why scientists and engineers search for ways to convert mass into energy. The most efficient nuclear fission reactors convert less than 1 % of the rest mass of nuclear fuel to energy, yet their energy output is enormous. Since the late 1940s, physicists have looked at ways to produce controlled nuclear fusion reactions (**Figure 3**), similar to the reactions that produce energy in the Sun and other stars. This process also uses the conversion of mass to energy, but uses hydrogen (the most common element in the universe) as fuel. One of the advantages of the fusion process is fewer radioactive waste products. However, researchers have not yet overcome the challenge of confining hydrogen at the temperatures and pressures needed for fusion to take place.

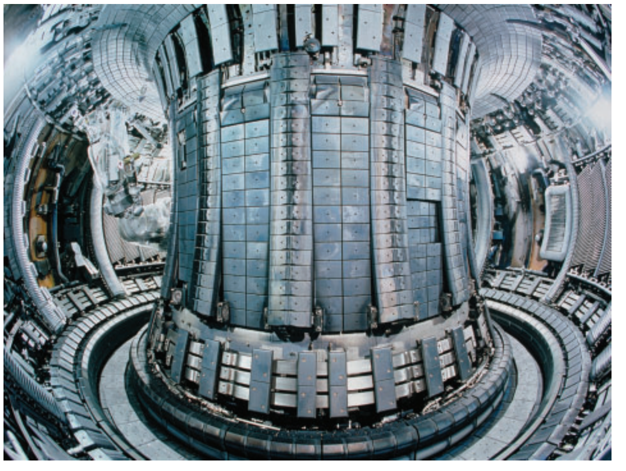

Figure 3 Nuclear fusion reactions have been studied and partially achieved in devices such as this tokamak, which uses magnetic fields to create conditions similar to those within the Sun.

UNIT TASK **BOOKMARK**

You can apply what you have learned about mass–energy conversion to the Unit Task on page 666.

While many scientists think that mass–energy conversion occurs in most processes that release energy, such as chemical reactions, the change in mass that occurs is extremely small. For instance, the energy released by the combustion of 1.0 kg of coal is equivalent to a mass of about 3.6×10^{-10} kg. This amount is too small to detect, even with the best electronic balance. It is only at the nuclear level or smaller that the mass–energy conversion becomes both significant and measurable. The Large Hadron Collider discussed at the beginning of this unit operates at energies where this mass–energy conversion is important. CAREER LINK

The following Tutorial will demonstrate how to solve problems involving mass–energy equivalence.

Tutorial 1 Solving Problems Related to Mass–Energy Equivalence

Sample Problem 1 involves mass–energy conversion. Sample Problem 2 models how to calculate total energy, kinetic energy, and rest energy using units of electron-volts instead of joules.

Sample Problem 1: Calculating Energy Equivalence

The average home in Canada uses 3.6×10^{10} J of energy per day. Imagine that a cabbage with a rest mass of 0.750 kg could be completely converted to another form of energy (although in a nuclear reaction only a fraction of the mass is converted into electrical energy).

(a) Calculate how much energy is released by the cabbage.

(b) Determine the number of days this cabbage could supply energy for an average home in Canada.

Solution

(a) **Given:** $m = 0.750$ kg; $c = 3.0 \times 10^8$ m/s

Required: E_{rest}

Analysis: $E_{rest} = mc^2$

Solution:

$$E_{rest} = mc^2$$
$$= (0.750 \text{ kg})(3.0 \times 10^8 \text{ m/s})^2$$
$$E_{rest} = 6.75 \times 10^{16} \text{ J (one extra digit carried)}$$

Statement: The energy released by the conversion of the cabbage's rest mass would be 6.8×10^{16} J.

(b) **Given:** $E_{rest} = 6.75 \times 10^{16}$ J; $E_{home} = 3.6 \times 10^{10}$ J per day usage

Required: days of energy provided by cabbage, d

Analysis: $d = \dfrac{E_{rest}}{E_{home}}$

Solution:

$$d = \frac{6.75 \times 10^{16} \text{ J}}{3.6 \times 10^{10} \dfrac{\text{J}}{\text{day}}}$$
$$d = 1.9 \times 10^6 \text{ days}$$

Statement: If a cabbage could be completely converted to energy, it would provide enough energy to power an average home in Canada for 1.9×10^6 days, or over 5000 years.

Sample Problem 2: Calculating an Electron's Energy

For subatomic particles with extremely small masses, the standard SI units of kilograms and joules are not convenient. Therefore, physicists use the electron-volt (eV) as the unit of energy. The electron-volt is defined as the work done on an electron by 1 V of electric potential. The conversion factor between electron-volts and joules is 1 eV $= 1.60 \times 10^{-19}$ J. Mass–energy equivalence allows us to express the mass of a subatomic particle in electron-volts.

An electron has a speed of $0.900c$ in a laboratory, and the rest mass of an electron is 9.11×10^{-31} kg. With respect to the laboratory's frame of reference, calculate the electron's rest energy, total energy, and kinetic energy in electron-volts.

Given: $m = 9.11 \times 10^{-31}$ kg; $v = 0.900c$; $c = 3.0 \times 10^8$ m/s; 1 eV $= 1.60 \times 10^{-19}$ J

Required: E_{rest}; E_{total}; E_k

Analysis: Use the equation $E_{rest} = mc^2$ to determine the rest energy of the electron. Include the conversion factor for electron-volts. Use the equation $E_{total} = \dfrac{mc^2}{\sqrt{1 - \dfrac{v^2}{c^2}}}$ to calculate the total relativistic energy of the electron. To determine the kinetic energy of the electron, use the equation $E_k = E_{total} - E_{rest}$.

Solution:

$$E_{\text{rest}} = mc^2$$

$$= (9.11 \times 10^{-31}\ \text{kg})(3.0 \times 10^8\ \text{m/s})^2 \left(\frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right)$$

$$= 5.12 \times 10^5\ \text{eV}$$

$$E_{\text{rest}} = 0.512\ \text{MeV (one extra digit carried)}$$

$$E_{\text{total}} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{(9.11 \times 10^{-31}\ \text{kg})(3.0 \times 10^8\ \text{m/s})^2}{\sqrt{1 - \frac{(0.900c)^2}{c^2}}} \left(\frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right)$$

$$= 1.17 \times 10^6\ \text{eV}$$

$$E_{\text{toal}} = 1.17\ \text{MeV (one extra digit carried)}$$

$$E_k = E_{\text{total}} - E_{\text{rest}}$$

$$= 1.17\ \text{MeV} - 0.512\ \text{MeV}$$

$$E_k = 0.66\ \text{MeV}$$

Statement: The rest energy of an electron is 0.51 MeV. The total relativistic energy of an electron moving at 0.900*c* is 1.2 MeV. The kinetic energy of an electron moving at 0.900*c* is 0.66 MeV.

Practice

1. A cellphone has a rest energy of 2.25×10^{16} J. Calculate its rest mass. T/I A [ans: 0.25 kg]
2. A proton moves with a speed of 0.800*c* through a particle accelerator. In the accelerator's frame of reference, calculate (a) the total and (b) the kinetic energies of the proton in megaelectron-volts. T/I A [ans: (a) 1.57×10^3 MeV; (b) 6.26×10^2 MeV]
3. Nuclear power stations in Canada use a type of reactor invented in Canada called a Canada Deuterium Uranium (CANDU) reactor. A typical CANDU fuel bundle has a mass of 23 kg. T/I A
   (a) Calculate the amount of electrical energy produced by a single CANDU fuel bundle, assuming that the entire bundle is converted to electrical energy. [ans: 2.1×10^{18} J]
   (b) An average Canadian home uses 3.6×10^{10} J of energy per day. Determine the number of days that a single CANDU fuel bundle could theoretically provide energy to a home. [ans: 5.8×10^7 days]
4. An asteroid with a mass of 2500 kg has a relativistic kinetic energy of 1.5×10^{20} J. Calculate the speed of the asteroid. T/I A [ans: 0.80*c*]

The applications of mass–energy equivalence and other concepts from special relativity have had a deep effect on technology, society, and the environment. Conversion of mass into energy through nuclear fission, for example, led to nuclear power and nuclear weapons, and scientists hope that experiments on nuclear fusion will lead to a source of clean, efficient energy.

11.4 Review

Summary

- The rest energy of an object with mass m is the energy of an object that is not in motion. It is a measure of the energy that is intrinsically contained in the matter that makes up the object and is given by the equation $E_{\text{rest}} = mc^2$.
- The total relativistic energy of an object with rest mass m is $E_{\text{total}} = \dfrac{mc^2}{\sqrt{1 - \dfrac{v^2}{c^2}}}$.

  This expression is equal to the sum of the kinetic energy and the rest energy of the object: $E_{\text{total}} = E_{\text{rest}} + E_{\text{k}}$.
- Special relativity shows that mass and energy are equivalent and thus establishes the principle of conservation of mass–energy.
- The energy produced from nuclear reactions has many applications, including nuclear power and nuclear weapons.

Questions

1. A 1 kg object is travelling at 0.95c. Calculate the object's relativistic mass. K/U T/I
2. In your own words, explain how the equation $E_{\text{rest}} = mc^2$ leads to the conservation of mass–energy. K/U T/I C
3. The chemical energy released by 1.00 kg of the explosive TNT equals about 4.20×10^6 J. Determine the rest mass, when converted, that produces a rest energy identical to the amount of energy released by TNT. T/I A
4. The anti-proton is a type of antimatter and is the antimatter "cousin" of the proton. The two particles have the same rest mass, which is 1.67×10^{-27} kg. It is possible for a proton and an anti-proton to collide and annihilate each other, producing pure energy in the form of gamma radiation. Calculate the energy that a proton and an anti-proton release when they annihilate each other. Assume that the two particles are at rest just before the annihilation. T/I A
5. In a physics laboratory, a subatomic particle has a rest energy of 1.28 MeV and a total relativistic energy of 1.72 MeV with respect to the laboratory's frame of reference. T/I A
   (a) Calculate the particle's rest mass.
   (b) Calculate the particle's kinetic energy in the laboratory's frame.
   (c) Determine the particle's speed in the laboratory.
6. The Moon has a rest mass of 7.35×10^{22} kg and an average orbital speed of 1.02×10^3 m/s. Calculate the amount of mass that would need to be converted to energy to accelerate the Moon from rest to its final orbital speed. K/U A
7. Tritium is a heavy form of hydrogen, with a nucleus consisting of one proton and two neutrons. The rest energy of a tritium nucleus is 2809.4 MeV. The formation of a tritium nucleus releases energy in the forms of increased kinetic energy of the nucleus and gamma radiation. Determine the amount of energy released when a proton with rest energy 938.3 MeV combines with two neutrons, each with a rest energy of 939.6 MeV, to form a tritium nucleus. T/I A
8. The kinetic energy of a particle is equal to its rest energy. Calculate the speed of the particle. K/U T/I
9. A spacecraft with a mass of 2500 kg has a relativistic kinetic energy of 2.0×10^{19} J. Determine the speed of the spacecraft. K/U A
10. Using the measured speed and rest mass of the electron (Newtonian mechanics), an electron in a television picture tube has a classical kinetic energy of 30.0 keV. Determine the actual kinetic energy of the electron, in kiloelectron-volts, by using the total relativistic energy and the rest energy. K/U T/I A
11. A car requires 1.0×10^8 J of energy to drive 30.0 km. Calculate how many kilometres you could hypothetically drive using the energy contained in the rest mass of 100.0 mg of fuel. K/U
12. A piece of uranium with a mass of 100.000 kg is placed in a reactor. Four years later, a mass of 99.312 kg is left over. Determine the energy that was released during this time. For this question, use the speed of light to six significant digits, which is $2.997\ 99 \times 10^8$ m/s. T/I A