11.3 Length Contraction, Simultaneity, and Relativistic Momentum

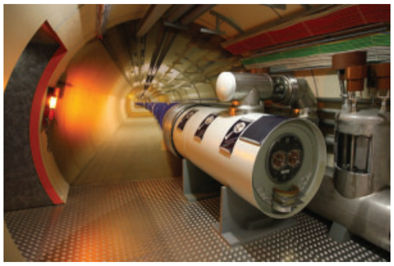

Figure 1 Particle accelerators, such as the Large Hadron Collider, increase particle speeds to nearly the speed of light. The resulting increase in the inertia of the particles means that strong magnetic fields are needed to bend the trajectories of the particles into a circle.

Time dilation is only one of the consequences of Einstein's postulates. There are also consequences that deal with space, such as the contraction, or compression, of length. In this section, we will explore this concept using a thought experiment.

As well as changing our understanding of time and length, special relativity also changes our understanding of momentum, energy, and mass. Despite the shortcomings of some of the older ways of thinking about these concepts, they are still useful in many situations. Science is a process; sometimes there are incremental changes, and sometimes new information forces a complete rethinking of what we know.

For example, you will read below that the momentum of a moving object in special relativity is different from the momentum in Newtonian mechanics. An understanding of both of the ways of thinking about momentum, however, is useful for solving different types of problems. The equations of special relativity also reveal that mass and energy are equivalent. This insight can lead to the discovery of new physical processes, such as nuclear fission, that can convert mass to mechanical energy. CAREER LINK

The effects of relativity become very important in particle accelerators (**Figure 1**). Particle accelerators accelerate subatomic particles to nearly the speed of light. To describe the particles correctly at such high speeds, we must use special relativity.

Length Contraction

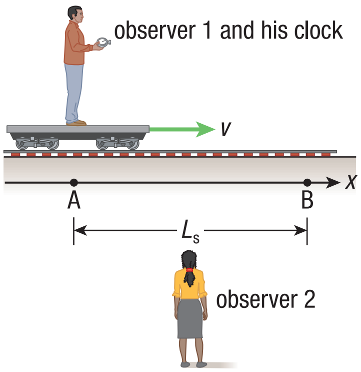

Figure 2 Observer 1 measures the distance between points A and B by using a clock to measure the time, Δt_s, it takes him to travel between the two points, together with his known speed, v.

proper length (L_s) the length of an object or distance between two points as measured by an observer who is stationary relative to the object or distance

In the past two sections, you learned how special relativity contradicts the concept of absolute time in Newtonian mechanics. Measurements of time intervals are relative, in that they can be different for different observers. However, time is just one aspect of a reference frame; reference frames also involve measurements of position and length. How are these measurements affected by relativity?

Recall the example of observer 1 on the railway car and observer 2 on the ground near the tracks. Consider how observers 1 and 2 might each measure a particular length or distance (**Figure 2**). Suppose observer 2 marks two locations, A and B, on the ground. She then measures these locations to be a distance L_s apart on the x-axis. Observer 1 travels in the positive x-direction at a constant speed v, and as he passes point A he reads his clock. Observer 1 reads his clock again when he passes point B and calls the difference between the two readings Δt_s. This is the proper time interval because observer 1 measures the start and finish times at the same location (the centre of his railway car) with the same clock.

Like proper time, **proper length** is a measurement made by an observer who is stationary relative to the object being measured. Just as we denote the proper time by Δt_s, we will denote the proper length by L_s. An observer at rest relative to the object measures the length as proper length.

Recall from Section 11.2 that when observer 2 measures with her clock the time it takes for observer 1 to travel from A to B, the value she determines for Δt_m is given by the time dilation equation:

$$\Delta t_m = \frac{\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Multiplying both sides of this equation by v gives

$$v\Delta t_m = \frac{v\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$

For observer 2, the time that observer 1 travels multiplied by v is simply the distance between A and B, or L_s:

$$v\Delta t_m = L_s$$

Similarly, the distance measured by observer 1 is the speed, v, times the proper time measured in his reference frame, so

$$v\Delta t_s = L_m$$

Substituting these last two equations into the time dilation equation gives the following result:

$$L_s = \frac{L_m}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$L_m = L_s\sqrt{1 - \frac{v^2}{c^2}}$$

Note that, because Δt_m is different from Δt_s due to time dilation, the lengths measured by the two observers will also be different. The length, L_m, measured by observer 1 is shorter than the length, L_s, measured by observer 2. This effect, called **length contraction**, or compression, is the shortening of distances in an inertial frame of reference moving relative to an observer in another inertial frame of reference. Contraction occurs along the direction of motion. Length contraction is the spatial counterpart to time dilation.

length contraction the shortening of length or distance in an inertial frame of reference moving relative to an observer in another inertial frame of reference

Consider the same situation, but now points A and B are the two ends of a metre stick. Observer 2 and the metre stick are in motion, while observer 1 is at rest (**Figure 3**). The metre stick is at rest relative to observer 2, so she measures the length of the metre stick and determines that $L_s = 1$ m, exactly. Observer 1 measures the length of the metre stick as it moves past him; he measures a length L_m that is shorter than L_s.

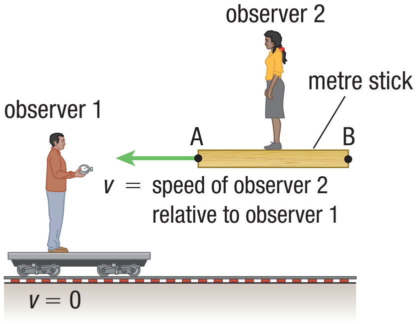

Figure 3 Observer 1 is at rest and observer 2, along with the metre stick, is in a reference frame moving with speed v relative to observer 1. Observer 1 observes that the moving metre stick is shorter than the length measured by observer 2.

Another way of saying this is that a metre stick moving relative to a stationary observer becomes shortened. The proper length, L_s, is the length measured by an observer at rest relative to the metre stick. It follows, then, that the length L_m, which is measured by the other observer and is always shorter than the proper length, is the **relativistic length**.

relativistic length (L_m) the length of an object or the distance between two points as measured by an observer moving with respect to the object or distance

Length contraction is described by the following equation:

$$\frac{L_m}{L_s} = \sqrt{1 - \frac{v^2}{c^2}}$$

The graph of $\frac{L_m}{L_s}$ versus the ratio $\frac{v}{c}$ in **Figure 4** shows that, for speeds that are small compared to c, the fraction $\frac{L_m}{L_s}$ is nearly 1. At speeds where v approaches c, the fraction $\frac{L_m}{L_s}$ approaches zero.

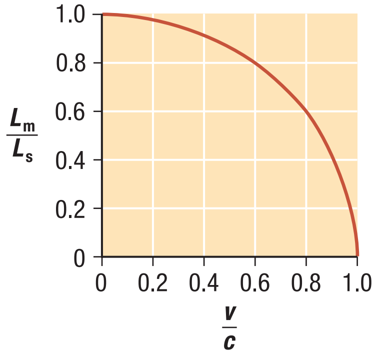

Figure 4 For typical terrestrial speeds, $\frac{v}{c}$ is very small and $L_m \approx L_s$.

In the previous example, the metre stick is in one frame of reference and the clock is in the other, so each observer must use a different relativistic property to obtain correct measurements. Observer 1 uses proper time to measure the length L_m, which he sees contracted. Observer 2 observes the proper length in her frame but must use time dilation for the time Δt_m observed in the frame of observer 1.

Length contraction occurs along the direction of motion (in these examples, along the x-axis). The following Tutorial models how to solve problems in which length is contracted.

Tutorial 1 Solving Problems Related to Length Contraction

The following Sample Problem illustrates how length is contracted for an observer in a moving frame of reference.

Sample Problem 1: Calculating Length Contraction

An observer on Earth measures the length of a spacecraft travelling at a speed of 0.700c to be 78.0 m long. Determine the proper length of the spacecraft.

Given: $L_m = 78.0$ m; $v = 0.700c$

Required: L_s

Analysis: $\frac{L_m}{L_s} = \sqrt{1 - \frac{v^2}{c^2}}$

$$L_s = \frac{L_m}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Solution: $L_s = \frac{L_m}{\sqrt{1 - \frac{v^2}{c^2}}}$

$$= \frac{78.0 \text{ m}}{\sqrt{1 - \frac{(0.700c)^2}{c^2}}}$$

$$= \frac{78.0 \text{ m}}{\sqrt{1 - 0.700^2}}$$

$$L_s = 109 \text{ m}$$

Statement: The proper length of the spacecraft is 109 m.

Practice

1. An object at rest is 5.0 m long, but when it drives past a stationary observer, the observer measures it to be only 4.5 m long. Determine how fast the object is moving. T/I [ans: 0.44c, or 1.3×10^8 m/s]
2. A spacecraft passes you at a speed of 0.80c. The proper length of the spacecraft is 120 m. Determine the length that you measure as it passes you. T/I [ans: 72 m]
3. (a) A car with proper length 2.5 m moves past you at speed v, and you measure its length to be 2.2 m. Determine the car's speed. [ans: 1.4×10^8 m/s]

   (b) A rocket with a proper length of 33 m moves past you at speed v, and you measure its length to be 26 m. Determine the rocket's speed. T/I [ans: 1.8×10^8 m/s]

Muons and Evidence for Length Contraction and Time Dilation

The decay of unstable elementary particles called muons demonstrates how length contraction and time dilation complement each other. Muons are particles that are about 207 times as massive as electrons, travel at speeds of about 0.99c, and decay in 2.2 ms for an observer at rest relative to the muons.

One source of muons is the cosmic radiation that collides with atoms in Earth's upper atmosphere. In Newtonian mechanics, most of these muons should decay after travelling about 660 m into the atmosphere. Yet experimental evidence shows that a large number of muons decay after travelling 4800 m—over seven times as far.

Why does this happen? The only known explanation comes from special relativity. Consider Earth as the stationary frame of reference. As observed from Earth, these muons undergo time dilation. They also undergo length contraction, but they are so small to begin with that this is a minor effect. Due to time dilation at very high speeds, the muons' "clocks" run slower relative to Earth clocks, so their lifetimes as measured on Earth increase by a factor of seven. This allows them to travel the greater distance.

What does this physical situation look like in the muon's frame of reference? An observer moving with the muon would notice a contracted Earth rushing toward the muon. More importantly, the distance from the upper atmosphere to Earth's surface would appear to be about one-seventh its normal thickness. Therefore, while the muons decay in their own frame of reference in just 2.2 ms, the 4800 m distance they must travel shortens in their frame of reference to 660 m.

With this example, you start to see the interrelationship between space and time through the complementary effects of length contraction in one reference frame and time dilation in the other. We will examine the inseparability of space and time later in this section.

Investigation 11.2.1

Analyzing Relativistic Data
(page 604)
Now that you have learned how to calculate length contraction, perform the portion of Investigation 11.2.1 that uses the length contraction equation.

Relativity of Simultaneity

Suppose you are driving down the street, and you see two different traffic lights change colour at the same time. This is an example of **simultaneity**—the occurrence of two or more events at the same time. Your everyday experiences and intuition suggest that the notion of simultaneity is *absolute*; that is, two events are either simultaneous or not simultaneous for all observers. However, determining whether or not two events are simultaneous involves the measurement of time. Your study of time dilation in this chapter has already shown that different observers do not always agree on measurements involving clocks, time intervals, and lengths. What does special relativity imply for our perception of simultaneity?

simultaneity the occurrence of two or more events at the same time

If two events appear to be simultaneous to one observer, will other observers also find these events to be simultaneous? Consider the situation shown in **Figure 5**. Observer 1 is standing in the middle of his railway car, moving with a speed v relative to observer 2, when two lightning bolts strike the ends of the car. The lightning bolts leave burn marks on the ground (points A and B), which indicate the locations of the two ends of the car when the bolts struck. We now ask, "Did the two lightning bolts strike simultaneously?"

Observer 2 is midway between the burn marks at A and B. The light pulses from the lightning bolts reach her at the same time (Figure 5(c)). Observer 2 concludes that, because she is midway between points A and B and the light pulses reach her at the same time, the lightning bolts struck the railway car at the same time. Therefore, the bolts are simultaneous, as viewed by observer 2.

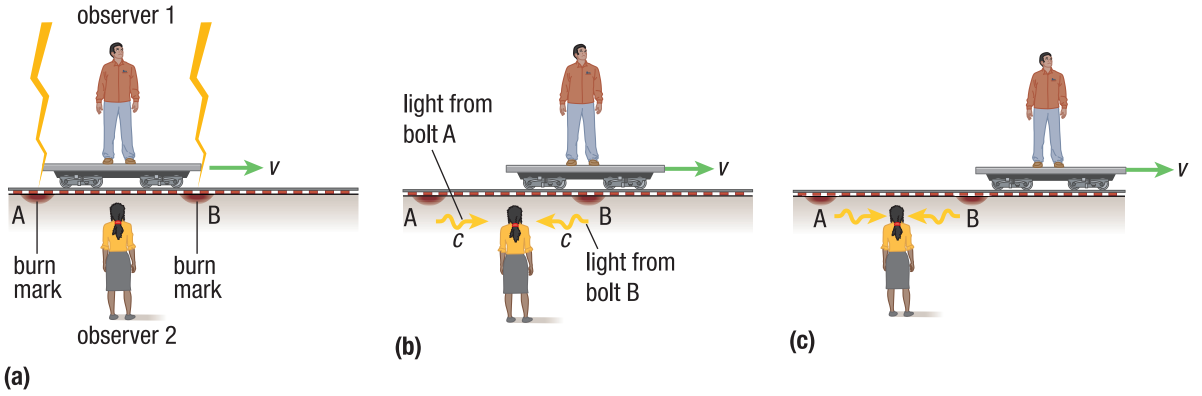

Figure 5 In a thought experiment to study simultaneity, (a) two lightning bolts strike the ends of the moving railway car, leaving burn marks on the ground. (b) According to observer 2, the lightning bolts are simultaneous. She comes to this conclusion because she is midway between the two burn marks. (c) The light pulses from the two bolts also reach observer 2 at the same time.

What does observer 1 see? He stands in the middle of his railway car, so, like observer 2, he is also midway between the places where the lightning bolts strike. Hence, if the two events are simultaneous as viewed by observer 1, the light pulses should reach him at the same time. Do they? Observer 2 can answer this question.

She observes that the railway car moves to the right, and because observer 1 is moving, the flash at B will reach him before the flash from A. Even with the distortions of time and space that arise from relativity, events do not occur out of sequence. So observer 1 will see the lightning strike at B before the lightning strike at A. The speeds of the light pulses from A and B are the same (a consequence of Einstein's postulates), and the distances that the pulses travel are the same. Therefore, observer 1 must conclude that the light pulses were not emitted at the same time.

The two lightning bolts in Figure 5 are therefore simultaneous for one observer (observer 2) but not for another observer (observer 1). Yet both observers are correct in their own reference frames, even if their observations are different. No reference frame is preferred. The observation of simultaneity can be different in different reference frames.

Relativistic analysis of simultaneity can help clarify apparent paradoxes. For example, in a simple time dilation experiment, one clock (C) travels between two clocks (A and B) that are stationary and synchronized with respect to each other. A stationary observer with respect to A and B notes that 10 s elapse on C as it moves from A to B, while 20 s elapse on A and B. A person holding C, however, will see A and B running slow by the same factor of $\frac{10}{20}$, or $\frac{1}{2}$. She will see only $\frac{1}{2}$ of 10 s, or 5 s, elapse on A and B as she moves from A to B.

How is this possible? Where did the "extra" 15 s go? The answer is that while clocks A and B are synchronized in their reference frame, they are not synchronized in the reference frame of C. At any instant of time for the moving observer, a snapshot of A and B will show that B is 15 s ahead of clock A. As the moving observer moves from A to B, A will go from 0 s to 5 s, and B will go from 15 s to 20 s.

Relativity of simultaneity is necessary to make sense of the reciprocity of time dilation, as well as the reciprocity of length contraction. Together, these concepts ensure that no reference frame is preferred. Relativity of simultaneity also works to ensure that the existence of a universal speed limit, c, does not lead to logical problems, as illustrated by the lightning example above. All of these concepts work together to ensure that special relativity makes sense.

If the previous sentences sound familiar, the reason is that they are a restatement of Einstein's postulates for relativity. Originally, Einstein wanted to call his model "the special theory of invariance" because it was of utmost importance to him that all laws of physics be invariant between inertial frames. Led by this principle, methodical logic, and early experimental evidence, he was able to conclude that the speed of light is invariant between and within inertial frames. To achieve this conclusion, he had to abandon the traditional views of time (in which time measurements were the same for all observers) and space (in which all spatial measurements were the same for all observers). Einstein's postulates reveal that the speed of light is independent of the speed not only of the source (just like all waves) but also of the observer.

The Twin Paradox

With our new understanding of space and time as properties that are both affected by motion comes the challenge of keeping track of what happens in each reference frame. Problems requiring special relativity become difficult to conceptualize without the classical notions of simultaneous events and absolute space and time. One of the most famous examples of this type of problem is the **twin paradox**—a thought experiment in which a traveller in one frame of reference returns from a voyage to learn that time has dilated in his reference frame, but not in the reference frame of his Earth-bound twin.

twin paradox a thought experiment in which a traveller in one frame of reference returns from a voyage to learn that time had passed more slowly in his spacecraft relative to the passage of time on Earth, despite the seemingly symmetric predictions of special relativity

Consider an astronaut who travels to the Sirius star system, which is 8.6 light years (ly) from Earth (**Figure 6**). His spacecraft is capable of a maximum speed of 0.90c, which means that he can reach the Sirius system in about 9.6 years. A round trip will take him just over 19 years. While on his mission, a crew of scientists on Earth, one of whom just happens to be the astronaut's twin sister, tracks the astronaut's health. The scientists' observations of the astronaut's biological and physical clocks indicate that he is aging more slowly than he would have done on Earth (although within his own frame of reference, he is, of course, unaware of any change in the flow of time). During the 19-year round trip, the crew notes that he ages only 8.3 years.

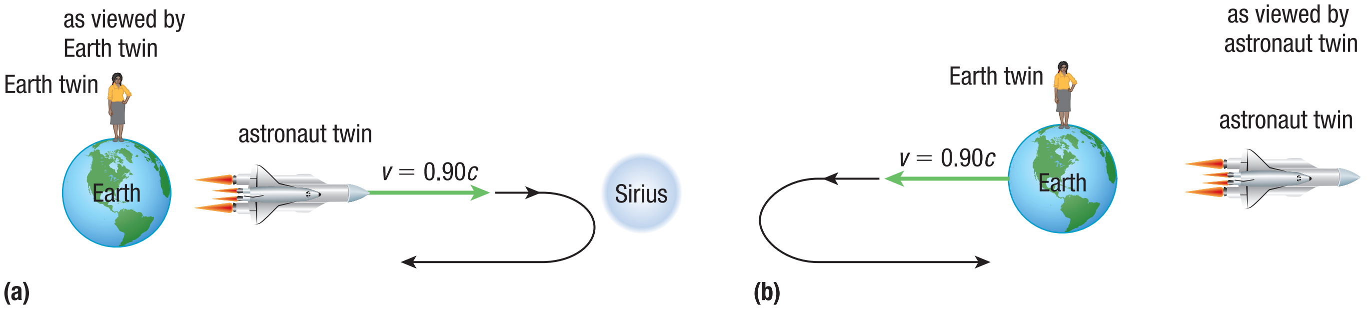

Figure 6 (a) An astronaut travels to the Sirius star system and then returns while his twin sister on Earth monitors his trip. (b) As viewed by the astronaut (in his reference frame), his Earth twin and planet Earth take a journey in the direction opposite to that in (a).

From the astronaut's frame of reference, Earth recedes from him at a rate of 0.90c. He therefore expects everyone on Earth, including his sister, to age only 8.3 years, while he ages 19 years. Imagine his surprise, then, when upon his return, his sister's analysis is correct and his analysis is wrong—she aged 19 years compared to his 8.3 years.

What happened here? Did the special theory of relativity fail, or is the error in the interpretation of relativistic effects? To understand the situation correctly, you need to consider that the astronaut moved in a frame of reference that was not truly inertial. So far, all the examples in this chapter have involved observers moving with constant velocities. In the case of the astronaut, however, he had to accelerate to change direction during the trip. The situation is, therefore, not symmetrical between the astronaut and his non-accelerating sister, and he cannot draw the same conclusions as his sister. Put another way, his frame of reference is not equivalent to that of his sister.

Nevertheless, there is a relativistic observation that the astronaut can use that completely reconciles the imagined paradox. Recall the interpretation of muon decay in the atmosphere and how the observations in each frame complement each other through the interconnection of space and time. **Space-time** is the four-dimensional coordinate system in which the three space coordinates (x, y, and z) are combined with a fourth dimension—time. From the astronaut's frame of reference, the universe undergoes length contraction. The distance that he must travel each way is, from his point of view, not 8.6 ly, but 3.7 ly. With a total distance of 7.4 ly and a speed that is most of the time $0.90c$, the astronaut sees the mission take only 8.3 years—the same amount by which his twin has determined he will age during the voyage. So he returns to find that his sister has aged 19 years while he has aged only 8.3 years.

space-time a four-dimensional coordinate system in which the three space coordinates are combined with time, a fourth dimension

Relativistic Momentum

As you learned in Chapter 5, an object with mass m moving with speed v has a momentum equal to the product of the two values. Momentum is conserved when there are no external forces acting on the object. This is an essential concept in physics. In fact, the conservation of momentum is one of the fundamental conservation rules in physics and is believed to be satisfied by all laws of physics, including the theory of special relativity.

When an object's speed is small compared with the speed of light, the object's momentum can be determined using the Newtonian formula for momentum, $p = mv$. However, as v approaches the speed of light, we have to take special relativity into account. Newtonian momentum gives a linear relation between p and v. By contrast, in special relativity the **relativistic momentum**—the momentum of objects moving at speeds near the speed of light—becomes extremely large as the object's speed approaches c. **Figure 7** graphically compares the momentum in Newtonian physics and special relativity.

relativistic momentum the momentum of objects moving at speeds near the speed of light

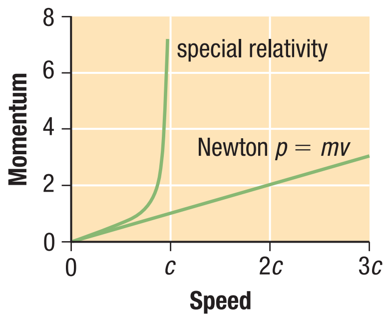

Figure 7 Newtonian mechanics predicts that momentum increases linearly with speed, while special relativity predicts that momentum approaches infinity at speeds close to c.

The effects of time dilation and length contraction are not included in the Newtonian momentum used in classical mechanics. To account for the relativistic effects on the momentum of objects moving near the speed of light, Einstein showed that proper time should be used to calculate momentum. This amounts to using a clock that travels along with the object. At the same time, an observer who watches the object move with speed v should take the measurement of length. The proper time is given by the expression $\Delta t_s = \Delta t_m \sqrt{1 - \frac{v^2}{c^2}}$, where Δt_m is the dilated time. Speed v is $\frac{\Delta x}{\Delta t}$, so when we replace t with Δt_s in the equation for Newtonian momentum, the resulting equation is

$$p = m\frac{\Delta x}{\Delta t_s}$$

$$p = m\frac{\Delta x}{\Delta t_m \sqrt{1 - \frac{v^2}{c^2}}}$$

where v is the speed of the object as viewed by the observer in the stationary reference frame. Since $v = \frac{\Delta x}{\Delta t_m}$, we get

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This is the equation for relativistic momentum, which is used to calculate the momentum of objects moving at close to the speed of light. The momentum of an object differs noticeably from the predictions of Newtonian mechanics for speeds greater than about $0.1c$.

An important feature of the equation for relativistic momentum is the rest mass, m. **Rest mass** is the mass of the object as measured at rest with respect to the observer. This value is sometimes called the proper mass. The rest mass is an invariant value in special relativity; that is, it does not change at different speeds. However, at speeds close to the speed of light, the measured mass of an object will differ from the rest mass. This measured mass, or **relativistic mass**, is observed in a frame moving at speed v with respect to the observer.

The application of forces increases an object's momentum. So, after a large force is applied or a collision occurs, the object's momentum becomes very large. However, even when the momentum is very large, the object's speed never quite reaches the speed of light. In the following Tutorial, you will compare the values obtained using the classical momentum equation and the relativistic momentum equation.

rest mass the mass of an object measured at rest with respect to the observer; also called the proper mass

relativistic mass the mass of an object measured by an observer moving with speed v with respect to the object

Tutorial 2 Calculating Relativistic Momentum

The following Sample Problem illustrates the difference between classical and relativistic momentum.

Sample Problem 1: Comparing Classical and Relativistic Momentum

In experiments to study the properties of subatomic particles, physicists routinely accelerate electrons to speeds close to the speed of light. An electron has a mass of 9.11×10^{-31} kg and moves with a speed of $0.99c$.

(a) Calculate the electron's momentum using the non-relativistic equation.

(b) Calculate the electron's relativistic momentum. Compare the relativistic momentum and the non-relativistic momentum.

Solution

(a) **Given:** $m = 9.11 \times 10^{-31}$ kg; $v = 0.99c$; $c = 3.0 \times 10^8$ m/s

Required: $p_{\text{classical}}$

Analysis: $p_{\text{classical}} = mv$

Solution:
$$\begin{aligned} p_{\text{classical}} &= mv \\ &= m(0.99c) \\ &= (9.11 \times 10^{-31}\ \text{kg})(0.99)(3.0 \times 10^8\ \text{m/s}) \\ p_{\text{classical}} &= 2.7 \times 10^{-22}\ \text{kg}\cdot\text{m/s} \end{aligned}$$

Statement: The non-relativistic momentum of the electron is 2.7×10^{-22} kg·m/s.

(b) **Given:** $m = 9.11 \times 10^{-31}$ kg; $v = 0.99c$; $c = 3.0 \times 10^8$ m/s

Required: $p_{\text{relativistic}}$

Analysis: $p_{\text{relativistic}} = \dfrac{mv}{\sqrt{1 - \dfrac{v^2}{c^2}}}$

Solution:

$$p_{\text{relativistic}} = \frac{mv}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

$$= \frac{m(0.99c)}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

$$= \frac{(9.11 \times 10^{-31}\ \text{kg})(0.99)(3.00 \times 10^8\ \text{m/s})}{\sqrt{1 - \dfrac{(0.99c)^2}{c^2}}}$$

$$p_{\text{relativistic}} = 1.9 \times 10^{-21}\ \text{kg}\cdot\text{m/s}$$

Statement: The relativistic momentum of the electron is 1.9×10^{-21} kg·m/s, which is about seven times as great as the momentum predicted by the classical definition.

Practice

1. A proton with a mass of 1.67×10^{-27} kg moves in a particle accelerator at $0.85c$. Calculate the proton's
   (a) non-relativistic momentum [ans: 4.3×10^{-19} kg·m/s]
   (b) relativistic momentum T/I [ans: 8.1×10^{-19} kg·m/s]
2. Suppose a 100.0 g projectile is launched with a speed $0.30c$ relative to Earth. Determine its relativistic momentum with respect to Earth. T/I [ans: 9.4×10^6 kg·m/s]
3. A proton moves at $0.750c$ relative to an inertial system in a lab. Given that the proton's mass is 1.67×10^{-27} kg, determine its relativistic momentum in the lab's frame of reference. T/I [ans: 5.68×10^{-19} kg·m/s]
4. A cube of iridium has the following dimensions: 0.100 m × 0.100 m × 0.100 m. Suppose the cube is moving at $0.950c$, in the direction of the y-axis. The density of iridium is 2.26×10^4 kg/m^3 when measured at rest. K/U T/I A
   (a) Which of the three directions, x, y, or z, is affected by the motion? [ans: y]
   (b) Calculate the relativistic volume of the cube. [ans: 3.12×10^{-4} m^3]
   (c) Calculate the relativistic momentum of the cube. [ans: 2.06×10^{10} kg·m/s]

Investigation 11.2.1

Analyzing Relativistic Data (page 604)

Now that you have learned about relativistic momentum, perform the portion of Investigation 11.2.1 that uses the relativistic momentum equation.

The Universal Speed Limit

Throughout this chapter, you have seen the development of the various components of special relativity—time dilation, length contraction, simultaneity, and relativistic momentum. Each of these concepts involves the expression $\sqrt{1 - \dfrac{v^2}{c^2}}$, which is a mathematical consequence of the postulates of special relativity. All physical laws remain invariant between inertial frames of reference with a relative velocity, and the speed of light remains the same in all frames of reference, regardless of whether the frame, light source, or observer is moving.

After using thought experiments to discover time dilation, length contraction, simultaneity, and relativistic momentum, Einstein also realized that, as a consequence of relativity, the speed c is a unique speed that plays a unique role in the universe. Although Einstein originally concentrated on the behaviour of light, researchers now understand that the speed c is special in its own right, independent of the properties of light waves. The universe truly does have an ultimate speed limit.

11.3 Review

Summary

- Proper length, L_s, is the length of an object as measured by an observer who is at rest with respect to the object. Relativistic length, L_m, is the length of the object as measured by an observer not at rest with respect to the object.
- The equation for length contraction is $L_m = L_s\sqrt{1 - \frac{v^2}{c^2}}$. For v greater than zero, $L_m < L_s$. Contraction occurs along the direction of motion.
- For two observers in motion relative to each other, events that appear simultaneous for one observer are not simultaneous for the other observer. However, in both cases, events appear to both observers in the order that they occur. The observers perceive the time between the two events differently.
- The twin paradox describes a thought experiment in which a moving observer ages more slowly than his or her "twin," despite the reciprocity of time dilation because the reference frame of the moving observer is not inertial.
- The equation for relativistic momentum is $p = \dfrac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$. Relativistic momentum increases as the speed increases and is limited by the speed of light.
- The rest mass of an object is the mass of the object as measured by an observer at rest with respect to the object.
- No object with a rest mass greater than zero can move as fast as, or faster than, the speed of light.

Questions

1. Spacecraft 1 passes spacecraft 2 at $0.755c$ relative to spacecraft 2. An astronaut on spacecraft 1 measures the length of spacecraft 2 to be 475 m. Calculate the proper length of spacecraft 2. T/I
2. An astronaut moving at $0.55c$ with respect to Earth measures the distance to a nearby star as 8.0 ly. Another astronaut makes the same voyage at $0.85c$ with respect to Earth. Calculate the distance the second astronaut measures between Earth and the star. T/I
3. François is travelling at a speed of $0.95c$ on a railway car, which François has measured as 25 m in length. Soledad, who is located on the ground near the railway tracks, arranges for two small explosions to occur on the ground next to the ends of the railway car. According to Soledad, the two explosions occur simultaneously, and she uses the burn marks on the ground to measure the length of François's railway car. According to François, do the two explosions occur simultaneously? If not, then according to François, which explosion occurs first? K/U T/I
4. In experiments, physicists routinely accelerate protons to speeds quite close to the speed of light. The mass of a proton is 1.67×10^{-27} kg, and the proton is moving with a speed of $0.99c$. T/I A
   (a) Calculate the proton's momentum, according to Newton's definition.
   (b) Calculate the proton's relativistic momentum.
   (c) Determine the ratio of the relativistic momentum to the Newtonian momentum.
5. The relativistic momentum of a particle of rest mass m and speed v is equal to $5mv$. Calculate the speed of the particle. T/I A
6. An electron with a speed of $0.999c$ has a momentum that is equal to the momentum of a ship with a mass of 4.38×10^7 kg moving at a certain speed. Determine the speed of the ship. T/I
7. If you were travelling on a spacecraft at $0.99c$ relative to Earth, would you feel compressed in the direction of travel? Explain your answer. K/U
8. Why do we not notice the effects of length contraction in our everyday lives? For example, why do cars not appear shorter when they drive past us at high speeds? K/U