11.2 Time Dilation

Figure 1 This artist's depiction conveys the idea that time behaves very differently for an observer at rest compared to an observer moving at close to the speed of light.

The implications that arise from Einstein's postulates and the constant speed of light for all inertial frames are not obvious. While Einstein showed that physical laws in inertial frames behave in understandable and expected ways, he also showed, using another thought experiment, that time behaves in ways that are unexpected and counter-intuitive for a stationary observer watching another observer who is moving at a speed close to the speed of light. Although the distorted clock in **Figure 1** is art, special relativity's predictions about time in an inertial frame of reference are no less bizarre.

In this section, you will see the development of a model for the behaviour of time in different frames of reference. This model, called **time dilation**, explains the slowing down of time in one reference frame moving relative to an observer in another reference frame. This treatment will start in terms of a thought experiment and then expand through a simple algebraic derivation. You will then learn about the physical significance of time dilation, as well as the experimental evidence supporting the results predicted by special relativity.

time dilation the slowing down of time in one reference frame moving relative to an observer in another reference frame

Time Dilation

Einstein's two postulates seem straightforward. The first postulate—that the laws of physics must be the same in all inertial reference frames—agrees with Newton's laws, so it does not seem that this postulate can lead to anything new for mechanics. The second postulate concerns the speed of light, and it is not obvious what it will mean for objects other than light. Einstein, however, showed that these two postulates together lead to a surprising result concerning the very nature of time. He did so by carefully considering how time in inertial frames is measured.

Einstein analyzed the operation of the simple clock in **Figure 2** in a thought experiment. This clock keeps time in a frame that, for the purpose of this thought experiment, is at rest. The clock measures time using a pulse of light that travels back and forth between two mirrors. A distance d separates the mirrors, and light travels between them at speed c. The time required for a light pulse to make one round trip through the clock is thus $\frac{2d}{c}$. That is the time required for the clock to "tick" once.

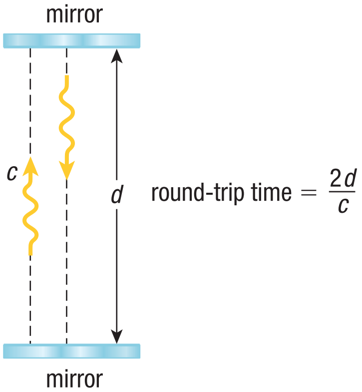

Figure 2 Each round trip of a light pulse between the mirrors corresponds to one tick of the light clock.

Now the light clock moves with constant horizontal speed v relative to a clock at rest. For this thought experiment, a railway car capable of moving at high speeds along a long, straight track provides this motion. How does this motion affect the operation of the clock? See **Figure 3**. For observer 1, who rides with the clock on the car, the return path of the light pulse, and thus one tick of the clock, appears as it does for any observer at rest with respect to the clock. The pulse simply travels up and down between the two mirrors. The motion of the car has no effect on the measurement of the speed of light for observer 1, in accordance with the postulates of special relativity. The separation of the mirrors is still d, so the round-trip time is still $\frac{2d}{c}$.

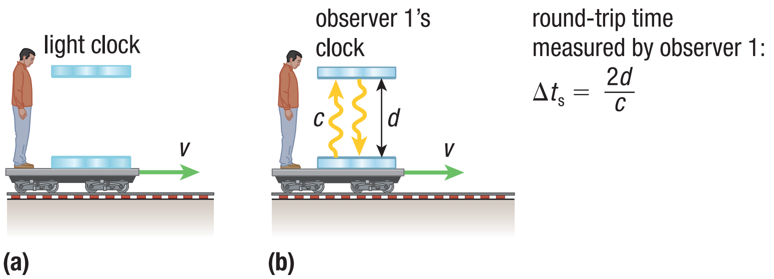

Figure 3 (a) A light clock is travelling with observer 1 on his railway car. (b) Light pulses travel back and forth in the clock. Each tick of the clock takes a time $\Delta t_s = \frac{2d}{c}$. According to observer 1, the operation of the clock is the same whether or not the railway car is moving.

The term Δt represents the time interval for one tick of the clock. When an observer is at rest (stationary) with respect to the clock, we write Δt_s for the time the observer measures for one tick. We write Δt_m for the time interval measured by an observer who sees the clock moving relative to her.

If Δt_s is the time required for the clock to make one tick as measured by observer 1 (who is stationary relative to the clock), then

$$\Delta t_s = \frac{2d}{c}$$

Observer 2 sees the clock moving while standing on the ground, as shown in **Figure 4**. She also measures the same speed of light as observer 1, but for her the light pulse moves a greater distance. This situation is analogous to the observer on a railway car tossing a ball vertically in the air (Figure 2 in Section 11.1). To the observer on the ground, the ball traces out a parabolic arc because of the two-dimensional motion provided by the horizontally moving car and the vertically displaced ball. In the case of the pulse in the light clock, observer 2 sees the light move at speed c, but along a path that has a horizontal deflection dependent on the speed, v, of the railway car.

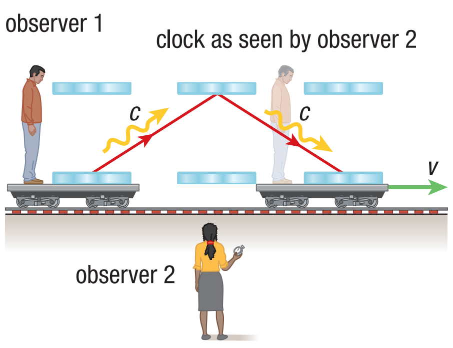

Figure 4 Observer 2, who is at rest on the ground, views the motion of the light pulses in the clock and sees the light pulse move a greater distance.

The distance that the light travels is longer for observer 2 than for observer 1, but the speed of light is the same for both observers. So, for observer 2, the time taken for the light to complete one tick (Δt_m) as it travels between the mirrors will be longer than for observer 1. The mathematical expression for Δt_m in terms of v, c, and d can be derived using geometry and the Pythagorean theorem.

In Figure 4, observer 2 sees the light pulse travel at speed c. As it travels, the light pulse covers a total vertical distance of $2d$ and a total horizontal distance of $v\Delta t_m$. The path of the light pulse forms the hypotenuse, z, of the two back-to-back right triangles in **Figure 5**.

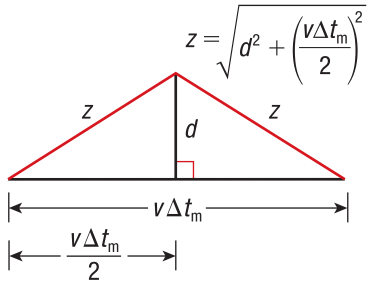

Figure 5 According to observer 2, the round-trip travel distance for a light pulse is $2z$, where $z = \sqrt{d^2 + \left(\frac{v\Delta t_m}{2}\right)^2}$, which is longer than the round-trip distance $2d$ seen by observer 1.

Applying the Pythagorean theorem to each triangle,

$$z^2 = d^2 + \left(\frac{v\Delta t_m}{2}\right)^2 \qquad \text{(Equation 1)}$$

We know that $z = c\Delta t_m$. Since z is half the total round-trip distance, and replacing v in Figure 5 with c, the speed of light, we get

$$z = \frac{c\Delta t_m}{2}$$

Squaring z,

$$z^2 = \frac{c^2(\Delta t_m)^2}{4}$$

Substitute z^2 into Equation 1:

$$\frac{c^2(\Delta t_m)^2}{4} = d^2 + \frac{v^2(\Delta t_m)^2}{4}$$

Now solve for Δt_m:

$$(\Delta t_m)^2 = \frac{4d^2}{c^2} + \frac{v^2}{c^2}(\Delta t_m)^2$$

$$(\Delta t_m)^2\left(1 - \frac{v^2}{c^2}\right) = \frac{4d^2}{c^2}$$

$$(\Delta t_m)^2 = \frac{\frac{4d^2}{c^2}}{\left(1 - \frac{v^2}{c^2}\right)}$$

Taking the square root of both sides and expressing the equation in terms of Δt_s, where $\Delta t_s = \frac{2d}{c}$, leads to

$$\Delta t_m = \frac{\frac{2d}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\Delta t_m = \frac{\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Recall that Δt_m is the time interval measured by an observer who sees the clock moving relative to herself, and Δt_s is the time interval for an observer who is stationary with respect to the moving clock. In other words, the equation indicates that these times are different for each observer. These times are **relativistic times**, which means that time changes relative to an observer. The time interval required for the pulses of light to travel between the two mirrors depends on the relative motion between the observers. This is one of Einstein's key insights: time is not absolute.

relativistic time time that is not absolute, but changes relative to the observer

Now consider the implications of the last equation in more detail. The clock in Figures 3 and 4 is at rest relative to observer 1, and observer 1 measures a time Δt_s for each tick. The same clock is moving with speed v relative to observer 2, and according to the equation she measures a longer time Δt_m for each tick. This result is not limited to light clocks. Postulate 1 of special relativity states that all the laws of physics must be the same in all inertial reference frames. We could use a light clock to time any process in any reference frame. Since the equation holds for light clocks, it must therefore apply to any process, including biological processes.

Divide both sides of the previous equation by Δt_s. Then, we find that the ratio of Δt_m (the time measured by observer 2) to Δt_s (the time measured by observer 1) is

$$\frac{\Delta t_m}{\Delta t_s} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This equation describes the phenomenon of time dilation. Looking at the above equation, if v were greater than c, then the term under the square root sign would be negative. Since the square root of a negative number is undefined, v can never be greater than the speed of light, c. So the right side of the equation is always greater than 1. Hence, the ratio $\frac{\Delta t_m}{\Delta t_s}$ is greater than 1, which means that observer 2 measures a longer time for the clock than observer 1 does. In other words, according to observer 2, a moving clock will take longer for each tick. Therefore, special relativity predicts that moving clocks run more slowly from the point of view of an observer at rest.

This result seems very strange because your everyday experience tells you that a clock (such as your wristwatch) travelling in a car gives the same time as an identical clock at rest. If the equation for time dilation is true (and experiments have conclusively shown that it is), why have you not noticed time dilation before now? The graph of $\frac{\Delta t_m}{\Delta t_s} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ shown in **Figure 6** indicates the answer as a function of the speed, v, of the clock. At ordinary terrestrial speeds, v is much smaller than the speed of light c, and v^2 is even smaller than c^2. Therefore, the ratio $\frac{\Delta t_m}{\Delta t_s}$ is very close to 1 for speeds less than $0.1c$.

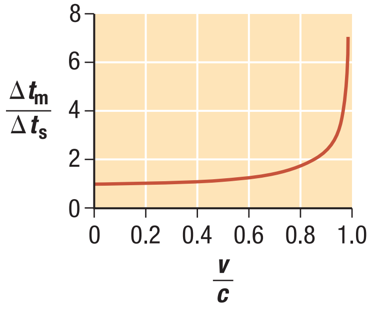

Figure 6 For typical terrestrial speeds, $\frac{v}{c}$ is very small and $\Delta t_m \approx \Delta t_s$.

For example, when $v = 50$ m/s, the ratio is

$$\frac{\Delta t_m}{\Delta t_s} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{1}{\sqrt{1 - \frac{(50 \text{ m/s})^2}{(3.0 \times 10^8 \text{ m/s})^2}}}$$

$$\frac{\Delta t_m}{\Delta t_s} = 1.000\,000\,000\,000\,014$$

The result is extremely close to 1, so for typical terrestrial speeds, the difference between Δt_s and Δt_m is negligible.

The time interval for a particular clock (or process) as measured by an observer who is stationary relative to that clock is called the **proper time**, Δt_s. The word "proper" does not mean that measurements of time in other frames are incorrect. Proper time is always measured by an observer at rest relative to the clock or any observed process being studied. Therefore, while observer 1 is moving on his railway car in Figure 4, the clock is moving along with him. Therefore, he is at rest relative to this clock and he measures the clock's proper time. On the other hand, observer 2 sees the clock moving relative to her, so she does not measure the proper time. The time interval measured by an observer who is in relative motion with respect to a clock or process Δt_m is always longer than the proper time of that clock or process.

proper time (Δt_s) the time interval measured by an observer at rest with respect to a clock

When an observer is at rest relative to a clock or process, the start and end of the process occur at the same location for this observer. For the light clock in Figure 4, observer 1 might be standing next to the bottom mirror, so from his viewpoint the light pulse starts and ends at the same location. By comparison, for observer 2 in Figure 4, the light pulse begins at the bottom mirror when the clock is at the left and returns to this mirror when the clock is in a different location relative to this observer. Observer 2 therefore measures a longer time interval, Δt_m. The proper time is always the shortest possible time that can be measured for a process, by any observer.

Time dilation is just one consequence of special relativity. In Section 11.3, you will learn that changes in time between frames of reference are accompanied by changes in length along the direction of motion. Therefore, it is not correct to think of time dilation as if it is an isolated effect. When you treat space and time as being interconnected, you will find it is easier to understand some of the contradictions of special relativity.

Tutorial 1 illustrates time dilation in a frame of reference moving close to the speed of light with respect to an observer in another frame of reference.

Tutorial 1 Determining Time Dilation

Sample Problem 1: Time Dilation for an Astronaut

On Earth, an astronaut has a pulse of 75.0 beats/min. He travels into space in a spacecraft capable of reaching very high speeds.

(a) Determine the astronaut's pulse with respect to a clock on Earth when the spacecraft travels at a speed of $0.10c$.

(b) Determine the astronaut's pulse with respect to a clock on Earth when the spacecraft travels at a speed of $0.90c$.

Solution

(a) **Given:** $\frac{1}{\Delta t_s} = 75.0$ beats/min; $v = 0.10c$

Required: $\frac{1}{\Delta t_m}$

Analysis: To calculate the time of one beat, take the reciprocal of the pulse rate. The process at rest in the moving reference frame is the time of one beat, which is the proper time, Δt_s. The time interval of the pulse as observed from Earth (Δt_m) must be longer than the proper time because the astronaut (who is basically the same as the stationary clock in his reference frame) moves with respect to Earth. Use $\Delta t_m = \frac{\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$ to calculate Δt_m.

Solution:

$$\Delta t_m = \frac{\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{\left(\frac{1}{75.0 \text{ beats/min}}\right)}{\sqrt{1 - \frac{(0.10c)^2}{c^2}}}$$

$$= \frac{1.33 \times 10^{-2} \text{ min/beat}}{\sqrt{1 - 0.10^2}}$$

$$= \frac{1.33 \times 10^{-2} \text{ min/beat}}{\sqrt{0.99}}$$

$\Delta t_m = 1.34 \times 10^{-2}$ min/beat (one extra digit carried)

Converting this result to a pulse rate yields

$$\text{pulse} = \frac{1}{\Delta t_m}$$

$$= \frac{1}{1.34 \times 10^{-2} \text{ min/beat}}$$

$$= 74.6 \text{ beats/min}$$

pulse = 75 beats/min

Statement: The observed pulse of the astronaut is 75 beats/min to two significant digits; however, the exact rate is slightly slower than his pulse in his own frame of reference (the spacecraft), or when he was on Earth and not moving with respect to Earth.

(b) **Given:** $\frac{1}{\Delta t_s} = 75.0$ beats/min; $v = 0.90c$

Required: $\frac{1}{\Delta t_m}$

Analysis: Follow the same steps as in (a).

Solution:

$$\Delta t_m = \frac{\Delta t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{\left(\frac{1}{75.0 \text{ beats/min}}\right)}{\sqrt{1 - \frac{(0.90c)^2}{c^2}}}$$

$$= \frac{1.33 \times 10^{-2} \text{ min/beat}}{\sqrt{1 - 0.90^2}}$$

$$= \frac{1.33 \times 10^{-2} \text{ min/beat}}{\sqrt{0.19}}$$

$\Delta t_m = 3.05 \times 10^{-2}$ min/beat (one extra digit carried)

Converting this result to a pulse rate yields

$$\text{pulse} = \frac{1}{\Delta t_m}$$

$$= \frac{1}{3.05 \times 10^{-2} \text{ min/beat}}$$

pulse = 33 beats/min

Statement: The observed pulse of the astronaut is 33 beats/min, which is much slower than his pulse in his own frame of reference (the spacecraft). This occurs because the speed of the spacecraft is close to the speed of light, which causes the time dilation to be large.

Practice

1. Determine how much longer a 1.00 s proper time interval appears to a stationary observer when a clock is moving with a speed of $0.60c$. T/I [ans: 0.25 s]
2. A beam of particles travels at a speed of 2.4×10^8 m/s. Scientists in the laboratory measure the average lifetime of the particle in the beam as 3.7×10^{-6} s. Calculate the average lifetime of the particles when they are at rest. K/U T/I [ans: 2.2×10^{-6} s]
3. An 8.0 s interval as measured on a moving spacecraft is measured as 10.0 s on Earth. Calculate how fast, relative to Earth, the spacecraft is moving. T/I A [ans: 1.8×10^8 m/s]
4. A spacecraft has a speed of $0.700c$ with respect to Earth. The crew of the spacecraft observes two events on Earth. According to the spacecraft's clocks, the time between the events is 30.0 h. K/U T/I A
   (a) Calculate the proper time, in hours, between the two events. [ans: 21.4 h]
   (b) What time interval does the crew measure when their craft travels at $0.950c$? [ans: 68.6 h]
5. Two astronauts travel to the Moon at a speed of 1.1×10^4 m/s. Their clock is accurate enough to detect time dilation. T/I C A
   (a) Determine the ratio of Δt_m to Δt_s to nine decimal places. [ans: 1.000 000 001]
   (b) What does your answer to (a) mean?

Investigation 11.2.1

Analyzing Relativistic Data (page 604)

Now that you have learned how to calculate time dilation, perform the part of Investigation 11.2.1 that uses the time dilation equation.

Verification of Time Dilation

In the early twentieth century, very few experiments could confirm, much less test directly, the predicted results of special relativity. However, since 1905, scientists have performed and repeated many experiments that have confirmed time dilation. Two of these experiments are discussed briefly here.

CLOCKS AND PASSENGER JETS

In the early 1970s, a series of experiments using atomic clocks took place. Atomic clocks use the vibrations of cesium atoms to measure time intervals very precisely. Four of these clocks were placed on separate jet aircraft and flown around the world twice. The purpose of this experiment, called the Hafele–Keating experiment after the two physicists who designed it, was to see if clocks moving at different speeds with respect to the centre of Earth run slower relative to a clock recording proper time. CAREER LINK

One complication posed by the experiment is that Earth rotates. Thus, a clock on Earth's surface is also a moving clock. To correct for this, one clock was placed on a plane moving westward against the rotation of Earth. This plane had the slowest speed and served as the clock to record the proper time for the system of clocks. A clock on Earth's surface was the next fastest-moving clock, and the clock on the plane flying east was moving fastest. In the final analysis of the data, the scientists made various corrections for gravitational effects as well as for the fact that none of the clocks were in a truly inertial frame.

The results showed that after two trips around the planet, the clock on Earth's surface ran 273 billionths of a second (or 273 ns) slower than the westbound clock (proper time), and that the eastbound clock ran 59 ns slower than the Earth clock, or 332 ns slower than the westbound clock. The error in these measurements was about 25 ns. Later repetitions of the experiment improved the accuracy, and all have been consistent with the predicted time dilation.

Figure 7 Location-based games use GPS systems, which rely on relativistic corrections to operate with continuous high accuracy.

RELATIVITY AND GPS

Location-based games are video games that use the player's location as part of the software that the game uses (**Figure 7**). A typical form of this game is a treasure hunt, called geo-caching, that does not use a map. Rather, it uses a GPS (global positioning satellite) system to indicate exactly where the player is and when the player is close to certain items, such as "buried gold." How is the computer able to know exactly where you are?

Satellites orbiting Earth send electromagnetic signals outward. A GPS computer on the ground determines from the signal the position of the satellite and the time it takes for the signal to arrive from the satellite. The computer gathers this data from three or more satellites and uses the speed of light (which for all satellites is the same) to determine its location on Earth. This simple procedure gives fairly good results. However, the speed of the signal is the speed of light, a very large number. So the error from just three measurements can be as much as a third of a kilometre and is usually only accurate to about 15 m. Adding a fourth satellite's signal as a time correction for all the GPS satellite signals improves accuracy to within 10 m or better. CAREER LINK

Relativity affects the long-term accuracy of the GPS system. Gravity affects the rate at which a clock runs (as described by the theory of general relativity) and must be corrected for. Another correction takes the fast relative motions of the satellites themselves into account. This matters because each satellite moves at nearly 3900 m/s with respect to Earth, causing time dilation. Without correction, these two kinds of relativistic effects cause the GPS system to lose accuracy by up to 11 km/day. WEB LINK

11.2 Review

Summary

- An observer in an inertial frame of reference will see the time in another inertial frame of reference as running slower.
- The equation for time dilation is $\frac{\Delta t_m}{\Delta t_s} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$, where v is the relative speed of an object (or observer) with respect to an observer, c is the speed of light, and Δt_s is the proper time of the object, as measured by an observer at rest with respect to the object. For v greater than zero, $\Delta t_m > \Delta t_s$.
- The value $\sqrt{1 - \frac{v^2}{c^2}}$ is undefined for v greater than c. No object can move at a speed greater than or equal to c.
- Time dilation is a natural result of the two postulates of special relativity and the realization that the speed of light is the same for all observers.
- Numerous experiments have provided evidence of time dilation, including the Hafele–Keating experiment using passenger jets and atomic clocks.
- Time dilation, along with general relativity corrections, has to be taken into account to maintain the accuracy of GPS systems.

Questions

1. Refer to the thought experiment in Figure 4 on page 581. Explain what would have to be true about observer 2 for her to measure the same time on the light clock that observer 1 measures. K/U T/I
2. A process takes place in a given amount of time. K/U
   (a) Does the process seem to take longer for an observer moving relative to the process, or for an observer at rest with respect to the process?
   (b) Which observer measures the proper time of the process?
3. Two identical clocks are synchronized. One clock stays on Earth, and the other clock orbits Earth for one year, as measured by the clock on Earth. After the year elapses, the orbiting clock returns to Earth for comparison with the stationary clock. K/U
   (a) Do the clocks remain synchronized?
   (b) Will the clock that was in orbit run slower after it returns?
   (c) Will the clock that was in orbit have the same time as the clock that stayed on Earth or a different time?
   (d) Does the clock that stayed on Earth have the wrong time? Explain.
   (e) Does the clock that was in orbit have the wrong time? Explain.
4. Suppose an atomic clock is placed on a jet flying westward around Earth at a constant altitude. The jet lands at the same airport from which it departed 8.64×10^4 s earlier. A similar clock at the airport was synchronized with respect to the clock on the plane before the plane took off. Determine which clock ran slower (ignore the various forces in the non-inertial frames of the two clocks). Explain your answer. K/U T/I C
5. Why do you think the accuracy of a GPS system depends on correcting satellite clocks for special relativity? K/U T/I C A
6. Roger is travelling with a speed of 0.85c relative to Mia. Roger travels for 30 s as measured on his watch. K/U T/I A
   (a) Determine who measures the proper time for Roger's trip, Roger or Mia. Explain your answer.
   (b) Calculate the elapsed time on Mia's watch during this motion.
7. An astronaut travels at a speed of 0.95c away from Earth. The astronaut sends a light signal back to Earth every 1.0 s, as measured by her clock. An observer on Earth notes that these signals arrive at intervals equal to Δt_m. Calculate the value of Δt_m. T/I A