10.3 The Diffraction Grating

diffraction grating a device with a large number of equally spaced parallel slits that produces interference patterns

It is difficult to measure the wavelength of light accurately using the interference pattern from either a double slit or a single slit. The interference pattern may be dull or the resolution fuzzy. To solve these problems, most researchers use a **diffraction grating**, which is a device that has an array of many parallel slits. White light is a mixture of light of different wavelengths, so as it passes through the slits in the diffraction grating, the waves originating from the slits interfere. The interference produces light of various wavelengths that travel along different paths (**Figure 1**). This creates an effect similar to passing white light through a prism.

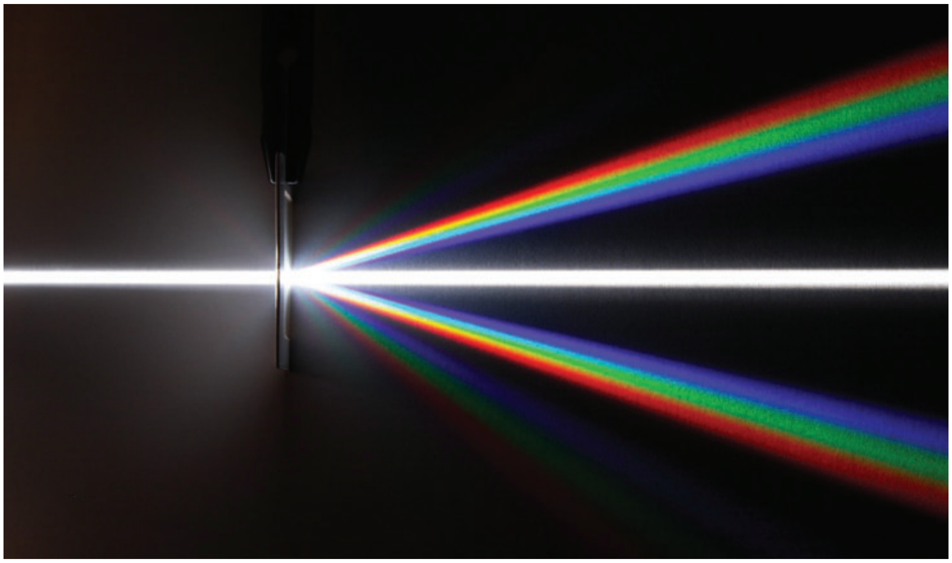

Figure 1 A diffraction grating splits white light into light of different wavelengths that travel along different paths. The effect shown here is due to interference, not dispersion.

Diffraction Gratings

There are two types of diffraction gratings: transmission gratings, which transmit light, and reflection gratings, which reflect light. CDs and DVDs are common examples of a reflection grating. When illuminated, they both produce iridescent reflections. A transmission grating, however, usually has an anti-reflection coating. Transmission gratings are typically used in spectroscopy. Most of the discussion here concerns transmission gratings, but the same concepts apply to both types.

The process of manufacturing an effective transmission grating involves precision machinery. One method uses a diamond tip to etch closely spaced parallel lines on the grating surface. The lines are opaque, and the transparent spaces between them serve as the slits. Newer photographic methods use interference from lasers to produce the pattern on photographic film. Then, the film is processed to produce the parallel lines. A typical diffraction grating might have 10 000 lines per centimetre. CAREER LINK

In Chapter 9, you learned that when coherent monochromatic light passes through a double slit, it produces a pattern of alternating bright and dark fringes on a screen located far from the slits. The bright fringes (maxima) occur in directions for which the path length from slits to screen are whole-number multiples of the wavelength of light used. In these directions, light waves arrive in phase and interfere constructively.

Three equally spaced slits would produce the same type of interference pattern for the same reasons. Consider what happens when light of wavelength λ passes through a large number, N, of equally spaced slits, as in a diffraction grating and as shown in **Figure 2**. The spacing between the slits is represented by w.

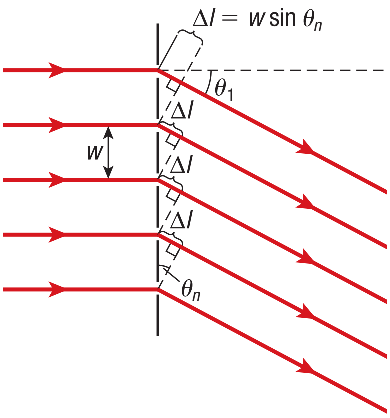

Figure 2 Light waves pass through a diffraction grating to produce a constructive interference pattern.

For an angle $\theta = 0°$, all of the waves arrive in phase, with a maximum in intensity. This is the **zero-order maximum**, which is the same for all wavelengths of light.

zero-order maximum the location of maximum intensity in the diffraction pattern at $\theta = 0°$

The next maximum occurs at an angle θ_1, where the path length difference between successive slits is exactly λ. The waves are again in phase when they reach the screen. If you apply trigonometry to Figure 2, you will see that the path length difference, Δl, is equal to $w \sin\theta_1$. The first maximum of intensity, called the **first-order maximum**, occurs when this path is equal to one wavelength, according to the equation

first-order maximum the first maximum of intensity on either side of the zero-order maximum in an interference pattern from a diffraction grating

$$\lambda = w \sin\theta_1$$

This condition is exactly the same as the condition for the first maximum for the double slit. The result is constructive interference at an angle θ from each slit and a bright maximum in that direction.

For each whole number m, the angle θ_m must satisfy the following condition:

$$m\lambda = w \sin\theta_m$$

where $m = 0, 1, \ldots.$

This corresponds to a path that differs by a whole-number multiple of a wavelength. Again, the waves arrive in phase at this angle and interfere constructively, resulting in a maximum called the mth-order maximum. Here, m is called the **order number**. At angles between maxima, the waves from each slit differ in phase and interfere to produce relatively wide dark areas on the screen. The overall result is a pattern of extremely narrow maxima.

order number the value of m for a given maximum in a diffraction-grating interference pattern; sequentially numbers the maxima on either side of the zero-order maximum

Figure 3 compares the interference patterns for different numbers of slits. The same equation describes where the maxima occur for a single given slit separation w, so the maxima are all at the same angle. As the number of slits increases, each maximum becomes narrower. Since a typical diffraction grating has thousands of slits, the maxima it produces are in precisely defined directions. In addition, since the separation between slits is typically quite small, the maxima are widely separated from each other.

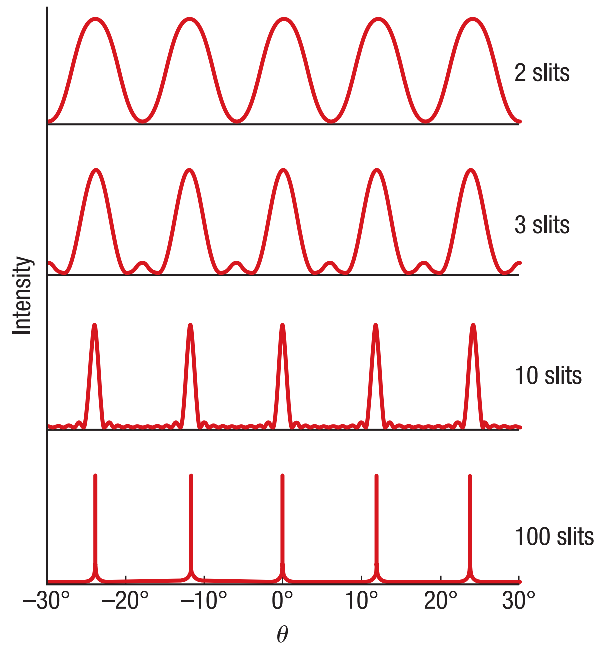

Figure 3 As the number of slits increases, the maxima become narrower and more sharply peaked. The resulting patterns of bright and dark lines are called diffraction fringes.

Using Gratings as a Spectrometer

When light of different wavelengths is incident on a diffraction grating, each wavelength produces diffraction peaks in different directions. This makes a diffraction grating a powerful tool for separating light of different wavelengths.

Figure 4 shows what happens when a mixture of green and red light passes through a diffraction grating and then onto a screen behind it. The resulting spots on the screen are circular because the deflected beam is circular (Figure 4(b)).

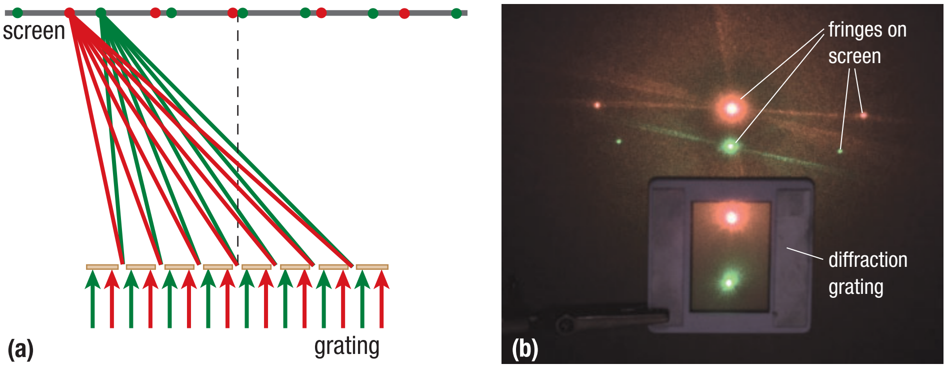

Figure 4 (a) Red and green light diffract at different angles. (b) The resulting interference pattern is seen on the screen.

Figure 4(a) also shows that the angles at which maxima occur have a simple and precise relation to the wavelength of the light. This behaviour of the diffraction grating makes it a powerful tool for precisely measuring wavelengths of light.

The spectrometer shown in **Figure 5** is a device for measuring wavelengths of light. Light from a source passes through the slit and into the collimator. A collimator is a system of mirrors or lenses that produces parallel wave fronts. The light then passes through a diffraction grating and onto a telescope. The telescope produces an image of the slit that appears as a line formed from the given wavelength of light. The observer positions the telescope so that the crosshairs mounted in it fall on the slit image. The observer then reads the angle from the scale below the telescope. Since the number of lines per centimetre and therefore the line spacing for the diffraction grating being used is known, the wavelength of light can be calculated from the measured angle. Astronomers use spectrometers to identify elements in space and on other planets. CAREER LINK

Investigation 10.3.1

CD and DVD Storage Capacity (page 547)

In Investigation 10.3.1, you will use a CD and a DVD as a diffraction grating and determine the groove spacing of each. You will use your data to assess which disc can hold more data.

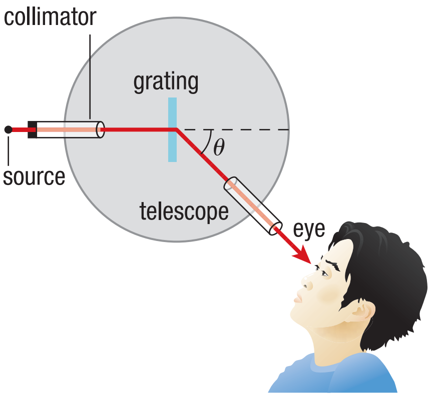

Figure 5 You can use a grating mounted in a spectrometer to measure the angles of interference maxima.

The Tutorial on the next page models how to locate and number the maxima produced by a diffraction grating.

Tutorial 1 Locating Maxima on a Diffraction Grating

The following Sample Problems show how to determine the location and number of maxima produced by a given diffraction grating.

Sample Problem 1: Determining the Maxima for a Diffraction Grating

Light with a wavelength of 540 nm is incident on a diffraction grating that has 8500 lines/cm. Calculate the angles of the maxima.

Given: $\lambda = 540 \text{ nm} = 5.4 \times 10^{-7}$ m; $N = 8500$ lines/cm

Required: the angle, θ_m, giving the locations of the mth-order maxima for $m = 1, 2, \ldots$, etc.

Analysis: The equation $w = \frac{1}{N}$ can be used to calculate the slit separation from the number of lines. Then use the equation $m\lambda = w \sin\theta_m$ to locate the maximum for each value of m.

Solution: $w = \frac{1}{N}$

$$= \frac{1}{8500 \text{ lines/cm}} \times \frac{1 \text{ m}}{100 \text{ cm}}$$

$$w = 1.176 \times 10^{-6} \text{ m (two extra digits carried)}$$

For the first-order maximum, $m = 1$:

$$\begin{aligned} m\lambda &= w \sin\theta_1 \\ (1)(5.4 \times 10^{-7} \text{ m}) &= (1.176 \times 10^{-6} \text{ m}) \sin\theta_1 \\ \sin\theta_1 &= 0.4591 \\ \theta_1 &= 27^\circ \end{aligned}$$

For the second-order maximum, $m = 2$:

$$\begin{aligned} m\lambda &= w \sin\theta_2 \\ (2)(5.4 \times 10^{-7} \text{ m}) &= (1.176 \times 10^{-6} \text{ m}) \sin\theta_2 \\ \sin\theta_2 &= 0.9180 \\ \theta_2 &= 67^\circ \end{aligned}$$

Performing the same calculation for $m = 3$ to determine the third-order maximum leads to 1.377 as the value required for $\sin\theta_3$. Since the sine of an angle can never be greater than 1, no third-order maximum exists.

Statement: The first-order maximum is at 27°, the second-order maximum is at 67°, and no other maximum is possible.

Sample Problem 2: Calculating Angles of Diffraction in a Diffraction Grating

Light emitted by a particular source is incident on a diffraction grating with 9000 lines/cm and produces a first-order maximum at 32.0°. Determine the wavelength of the light.

Given: $N = 9000$ lines/cm; $\theta_1 = 32.0^\circ$; $m = 1$

Required: λ

Analysis: Use $w = \frac{1}{N}$ to determine the slit separation, w. Then use the equation $m\lambda = w \sin\theta_m$ to determine the wavelength from w and the given angle, θ_1.

Solution: $w = \frac{1}{9000 \text{ lines/cm}} \times \frac{1 \text{ m}}{100 \text{ cm}}$

$$w = 1.111 \times 10^{-6} \text{ m (one extra digit carried)}$$

For the first-order maximum, $m = 1$:

$$\begin{aligned} \lambda &= \frac{w \sin\theta_1}{m} \\ &= (1.111 \times 10^{-6} \text{ m}) \sin 32.0^\circ \\ \lambda &= 5.89 \times 10^{-7} \text{ m} \end{aligned}$$

Statement: The light has a wavelength of 589 nm.

Practice

1. Consider two diffraction gratings, one with 10 000 lines/cm and one with 8500 lines/cm. Compare the separations between adjacent maxima for these two gratings. K/U T/I C
2. Calculate the angular separation of successive maxima of the same colour when light with a wavelength of 660 nm is incident on a diffraction grating with 8500 lines/cm. T/I A [ans: 34°]
3. A diffraction grating produces a third-order bright fringe at an angle of 22.0° for red light with a wavelength of 694.3 nm. Calculate the number of lines per centimetre on the grating. T/I [ans: 1798 lines/cm]

CDs and DVDs as Diffraction Gratings

UNIT TASK BOOKMARK

You can apply what you have learned about diffraction gratings to the Unit Task on page 556.

When you move a CD or a DVD under white light, you can see that the data side produces a spectrum of colours that change as you move it. The disc has a reflective surface and a long, microscopically thin track that spirals thousands of times around the disc from the centre to the edge. Light reflected from adjacent edges of the spiral track interferes in the same way as light interferes from slits in a reflection diffraction grating. Interference between waves from the track edges leads to a diffraction pattern. Different wavelengths contained in the white light have interference peaks in different directions, giving the colour pattern that you see.

CDs and DVDs take advantage of destructive interference to store data. The disc-manufacturing process uses a sharply focused laser beam that burns microscopic, quarter-wavelength-deep pits at precise intervals along the length of the spiral track. Then, a reflective coating is applied to the entire disc, including the track and its pits. Upon playback, the player rotates the disc and shines a sharply focused, low-power laser beam on the track. When the beam hits the leading or trailing edge of a pit, the light briefly reflects from both the pit and the undisturbed surface of the track (called the "land"), as shown in **Figure 6**.

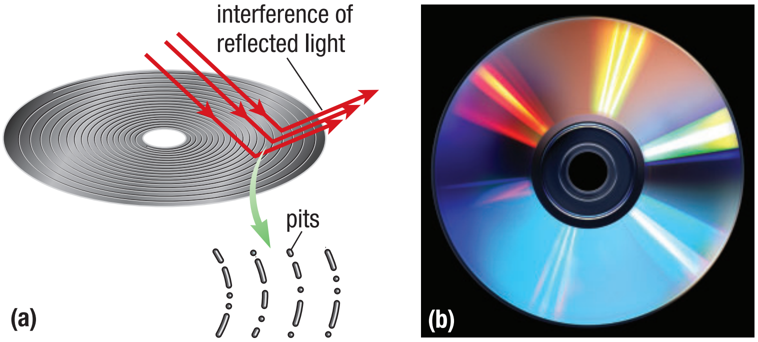

Figure 6 Interference of light reflected from nearby tracks in (a) produces the pattern of colours seen from the CD in (b).

Since the pit is a quarter-wavelength deeper than the land, the light wave that the pit reflects is half a wavelength out of phase with the wave that the adjacent land reflects. The two reflected waves interfere destructively, producing a momentary decrease in the intensity of the light reflecting from the track. A photodetector monitoring the reflected light detects these intensity changes, and the player converts them into usable data.

CDs use a near-infrared 780 nm laser, whereas DVDs use shorter wavelengths of 635 nm. The DVD's shorter wavelength allows for a smaller track separation, a smaller pit depth, and a smaller pit length. As a result, a DVD can store much more data than a CD on the same size disc. WEB LINK

Research This

Blu-ray Technology

Skills: Researching, Analyzing, Communicating — SKILLS HANDBOOK A4.1

DVD and CD players use the interference between two reflected beams to read CDs and DVDs. The interference depends on the relation between the wavelength and the difference in height between a pit and the land in the reflecting surfaces. A newer technology is called Blu-ray.

1. Research Blu-ray technology.

A. Why is the technology called Blu-ray? K/U

B. How does the technology work? K/U

C. What can a Blu-ray recording do that is superior to a CD or a DVD recording? K/U A

D. What enables Blu-ray to accomplish these improvements? K/U

E. Analyze, assess, and compare CD, DVD, and Blu-ray technologies in a visual format. Describe any disadvantages, hazards, and concerns associated with Blu-ray technologies. T/I C A

WEB LINK

10.3 Review

Summary

- A diffraction grating consists of a large number of closely spaced parallel slits.
- Diffraction gratings produce interference patterns that are similar to those from a double slit, but the maxima are far narrower and more intense.
- The angle, θ_m, for the mth-order maximum of a diffraction grating with slit spacing w and light wavelength λ is given by $m\lambda = w \sin\theta_m$.
- The colours that you can see in a CD or a DVD result from interference similar to the interference of light from a diffraction grating.

Questions

1. When a CD reflects white light, the result is a rainbow-like display of different colours. Explain what this indicates about the surface of the CD. K/U C A
2. A diffraction grating has 2800 lines/cm. Determine the distance between two lines in the grating. T/I A
3. Light incident on a diffraction grating with 10 000 lines/cm produces first-order, second-order, and third-order maxima at angles of 31.2°, 36.4°, and 47.5°, respectively. Determine the wavelength, in nanometres, of light that produces each maximum. T/I A
4. A square diffraction grating of width 2.0 cm contains 6000 slits. At what angle does blue light with a wavelength of 450 nm produce the first intensity maximum? T/I A
5. Red light with a wavelength of 600.0 nm is incident on a diffraction grating with a slit spacing of 25 μm. At what angle from $\theta = 0°$ is the first-order maximum in intensity? T/I A
6. Light with a wavelength of 780 nm from a laser pointer is incident on a diffraction grating with a screen located 10 m behind it. The maxima near $\theta = 0°$ are spaced 0.50 m apart. Determine the spacing between the lines in the diffraction grating. T/I A
7. In an experiment, light is reflected on a diffraction grating that has 300 lines/cm, and the diffraction grating is 0.84 m from a screen. The distance between the $m = 0$ and $m = 3$ bright fringe is 3.6 cm. Calculate the wavelength of the light. K/U T/I A
8. Determine the maximum order number possible in an interference pattern when light with a wavelength of 5.4×10^{-7} passes through a diffraction grating with 3000 lines/cm. T/I
9. The molecular planes in a crystal act as a diffraction grating when X-rays are incident on the crystal (**Figure 7**). The molecular planes in a crystal are 0.50 nm apart, and the X-rays have a wavelength of 0.050 nm. K/U T/I C A

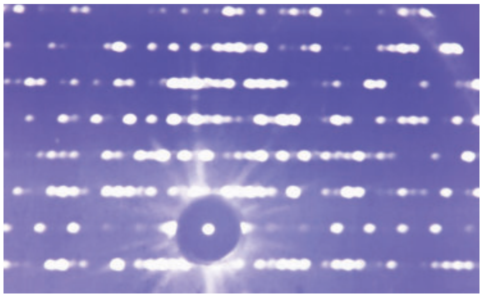

Figure 7 The white spots show the diffraction pattern as X-rays pass through a protein. From the pattern, scientists can determine the structure of the protein.

   (a) Assume that the maxima in the diffraction pattern are given by the same equation that applies for a grating with slits with the same value of w. Determine the angles for the first three maxima.
   (b) Assume that light with a wavelength of 600 nm is incident on the crystal instead of X-rays. At what angle is the first bright fringe?
   (c) What does this show about the prospects of using visible light for diffraction by crystals? Explain your answer.
10. Light with a wavelength of 5.00×10^2 nm produces a first-order maximum at an angle of 20.0° in a specific spectroscope. When the measurement is repeated with the same spectroscope on a distant star that is known to have a planet in orbit about it, the same light produces a first-order maximum at 18.0°. Determine the index of refraction of the atmosphere on the planet as it passes in front of its host star and the star's spectrum is analyzed. K/U T/I A