

Single-Slit Diffraction

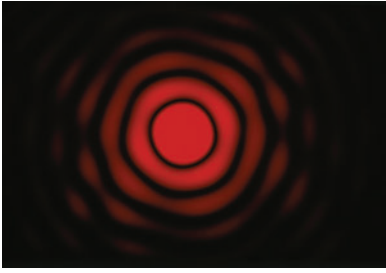


Figure 1 The interference of light passing through a single slit from a circular opening produces a pattern of concentric rings.

Fraunhofer diffraction an interference pattern that shows distinctive differences between the bright central fringe and darker flanking fringes

central maximum the bright central region in the interference pattern of light and dark lines produced in diffraction

secondary maxima the progressively less-intense bright areas, outside the central region, in an interference pattern

If you shine a beam of light through a wide-enough opening, you might expect the beam to pass through with very little diffraction. However, when light passes through a progressively narrower opening, the wave properties of light often produce unusual-looking patterns, such as the diffraction pattern shown in **Figure 1**. In this section, you will learn about the formation of such diffraction patterns.

Single-Slit Diffraction

To understand how a narrow opening (for example, $100\ \mu\text{m}$ wide) can affect light passing through it, consider the behaviour of light passing through a single slit as shown in **Figure 2(a)**. While investigating this configuration, assume that the following conditions exist:

1. The width, w , of the slit is narrow enough that a substantial amount of diffraction can occur, but not so narrow that it acts as a single point source of waves.
2. The light source is monochromatic with wavelength λ .
3. The light source is far from the slit, so that light rays that reach the slit are all travelling approximately parallel to each other.

Such configurations produce **Fraunhofer diffraction**, which shows a bright central fringe called the **central maximum**. The central maximum is flanked by dark fringes, called minima, and less-intense bright fringes, called **secondary maxima**.

Why do these fringes occur, and how can you predict where bright fringes and dark fringes will appear on the screen? The key to answering these questions is Huygens' principle. Recall from Chapter 9 that Huygens' principle states that each point on a wave front acts as a new source of waves. **Figure 2(b)** is a magnified view of the slit looking down on it from above, showing its width. According to Huygens' principle, all points across the slit width in Figure 2(b) act as wave sources, and these different waves create an interference pattern at the screen.

When the slit is divided in half (Figure 2(b)), coherent waves that interfere with each other appear as the light passes through the slit. The points in one half are labelled A, and the points in the other half are labelled B. Each of these points is a source of Huygens waves (spherical waves), including point A_1 at one edge of the slit and the corresponding source point B_1 at the centre of the slit.

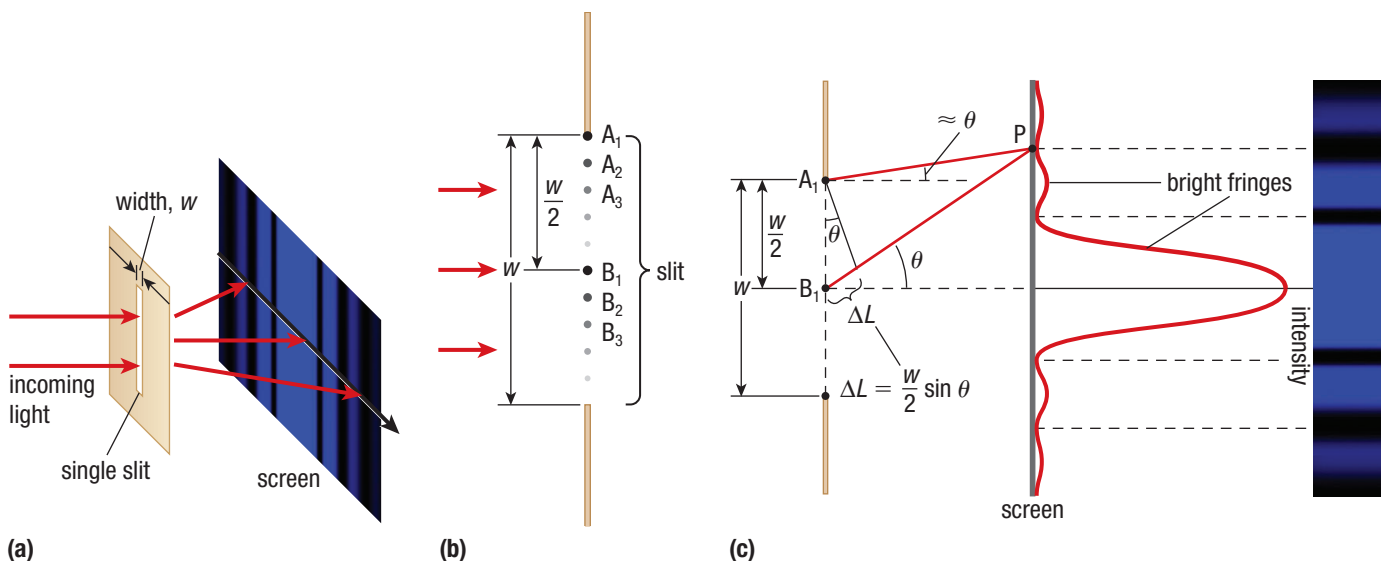


Figure 2 (a) Light passes through a single slit and hits a screen. (b) A view from above the slit looking down at its width. Each point in the slit acts as a source of spherical waves. (c) Looking down at the slit and screen, the single-slit pattern is apparent. Assume that point P is far away.

Figure 2(c) shows the distances from these points to a point P on the screen—the path lengths—and the path length difference ΔL . Although Figure 2 shows the screen close to the slit, assume that the screen is quite far away, so all the angles denoted by θ are approximately equal.

Suppose the waves from A_1 and B_1 interfere destructively when they reach a particular point P on the screen. The path length difference is $\Delta L = \frac{\lambda}{2}$. When this condition for destructive interference is met for points A_1 and B_1 , the waves from A_2 and B_2 will also interfere destructively, as will the waves from points A_3 and B_3 , and so on, for all similar pairs of points within the slit. Destructive interference produces a dark fringe on the screen. From Figure 2(c), you can see that

$$\Delta L = \frac{w}{2} \sin \theta_n$$

where w is the width of the slit.

Substituting $\frac{\lambda}{2}$ for ΔL leads to $\frac{w}{2} \sin \theta_n = \frac{\lambda}{2}$, which gives the angle of the first dark fringe on one side of the centre of the screen. On the other side, the corresponding value gives the angle of the first dark fringe to that side of the centre. The first dark fringes are thus at the angles

$$w \sin \theta_n = \lambda \quad \text{single-slit diffraction: first dark fringes (destructive interference)}$$

Figure 3 shows the method of determining the position of the second dark fringe. In Figure 3(a), the slit is divided into four regions. The points in these four regions are denoted by A, B, C, and D. Figure 3(b) shows the path lengths from the farthest point in each region (A_1 , B_1 , C_1 , and D_1) to a point P on the screen. Suppose the path length difference for the Huygens waves from A_1 and B_1 is $\Delta L = \frac{\lambda}{2}$, so these waves interfere destructively. The path length difference for C_1 and D_1 has the same value, so these waves also interfere destructively. The path length difference is the same for waves from other pairs of points A_2 and B_2 , C_2 and D_2 , and so on, so these waves also interfere destructively, resulting in no light, or a total intensity of zero, on the screen. Thus, for the second dark fringe, $\Delta L = \frac{w}{4} \sin \theta$. Setting this expression equal to $\frac{\lambda}{2}$, the interference condition in this case is the following, where the angle again corresponds to fringes left and right of the centre of the screen:

$$w \sin \theta_2 = 2\lambda \quad \text{single-slit diffraction: second dark fringes (destructive interference)}$$

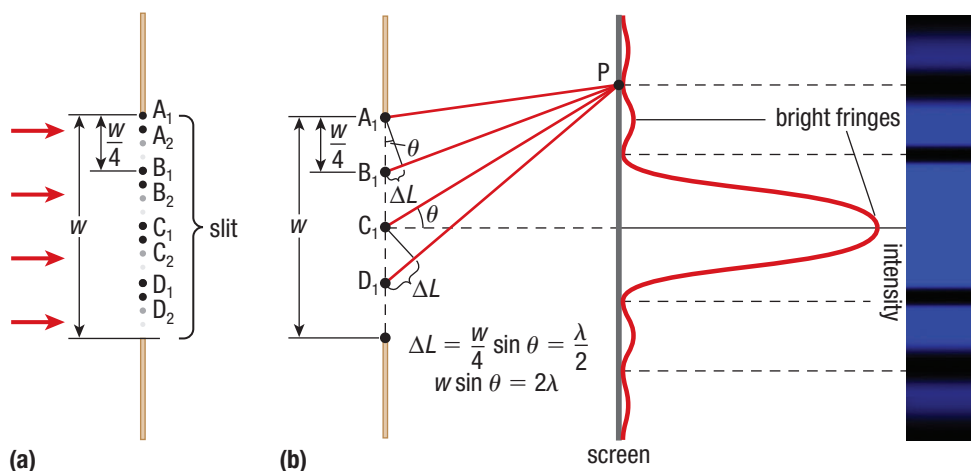


Figure 3 (a) To determine the locations of the second dark fringes, we divide the width into four regions, A, B, C, and D. (b) The locations of the second dark fringes can then be determined. In a scale drawing, the slit width would be much smaller than the width of the central diffraction peak.

The same method can predict the third and other dark fringes by dividing the slit into 6, 8, 10, or any even number of regions. The result for the n th dark fringe is

$$w \sin \theta_n = n\lambda \quad \text{single-slit diffraction: } n\text{th dark fringes (destructive interference)}$$

where $n = 1, 2, 3, \dots$. The values of θ that satisfy this equation give the angles of all dark fringes.

To determine the angles for the maxima, we use a similar method of dividing up the slit. For maxima, however, we divide the slit into an odd number of regions. Light from all but one region will interfere destructively, as in the discussion for minima, and the light from the remaining region provides the maxima.

To calculate the first maximum after the central peak, divide the slit into three regions, for example. In the equation for destructive interference of two of the regions, the effective slit width for each region is $\frac{w}{3}$, and the path length difference is a half-wavelength as always. The location of the fringe is then

$$\frac{w}{3} \sin \theta = \frac{\lambda}{2}, \text{ or}$$

$$w \sin \theta = \frac{3}{2}\lambda \quad \text{single-slit diffraction: first bright fringes (constructive interference)}$$

You can calculate additional bright fringes by breaking the slit into 5, 7, 9, or any odd number of regions. The result is that maxima occur in general when

$$w \sin \theta_m = \left(m + \frac{1}{2}\right)\lambda \quad \text{single-slit diffraction: } m\text{th bright fringes (constructive interference)}$$

where $m = 1, 2, 3, \dots$, and the values of θ that satisfy this equation give the angles of all bright fringes.

Figure 4 shows the intensity curve for single-slit diffraction. For each successive bright fringe, more of the slit acts as point sources of light that interferes destructively. The intensity of bright fringes therefore decreases away from the central peak.

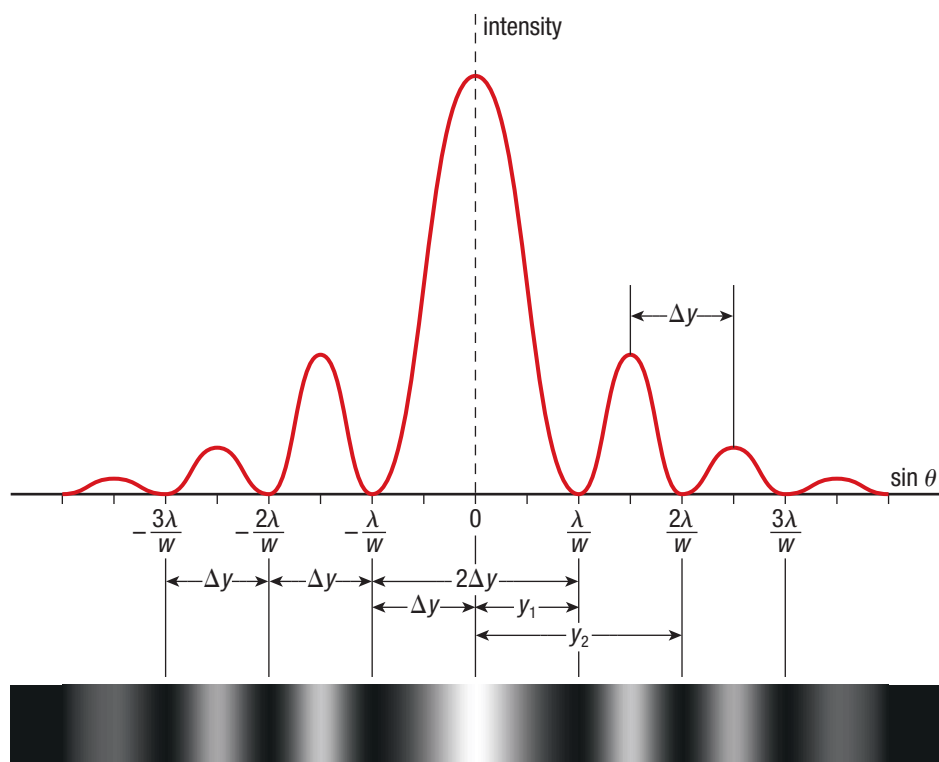


Figure 4 A full calculation of the intensity curve shows that the central bright fringe is about 20 times as intense as the bright fringes on either side.

Investigation 10.2.1

Diffraction of Light Using a Single Slit (page 545)

Now that you have learned about single-slit diffraction, perform Investigation 10.2.1 to study how changes in slit width, wavelength, and source distance affect the interference patterns.

You can approximate the width of the central bright fringe by using the angular separation of the first dark fringes on either side. You can calculate the angles for these first dark fringes by setting $n = 1$ to obtain

$$\sin \theta_1 = \frac{\lambda}{w}$$

Figure 5 shows the projection of the fringes on the screen. The distance y_1 of the first minimum from the central maximum (using $\sin \theta \approx \tan \theta$ for small angles) is then

$$y_1 = \frac{L\lambda}{w}$$

The width of the central maximum is the distance between first minima, $2y_1$, or

$$2\Delta y = 2\frac{L\lambda}{w}$$

The separation of the central maximum from the first minimum, $2\Delta y$, is half of this, or just y_1 , which gives $\lambda = \frac{w\Delta y}{L}$. From the equation for the location of the minima, the distance from each minimum to the next one is also y_1 . This is useful for predicting the positions of the fringes and the wavelength of light from the diffraction pattern. In particular, a longer wavelength or a smaller width produces a larger value of Δy and a wider diffraction pattern.

Figure 5 also shows the angles for the single-slit diffraction pattern. By determining the angle and distance from the screen, you can calculate the distances between fringes. The following Tutorial shows how to calculate the width of the central maximum of a single-slit diffraction pattern.

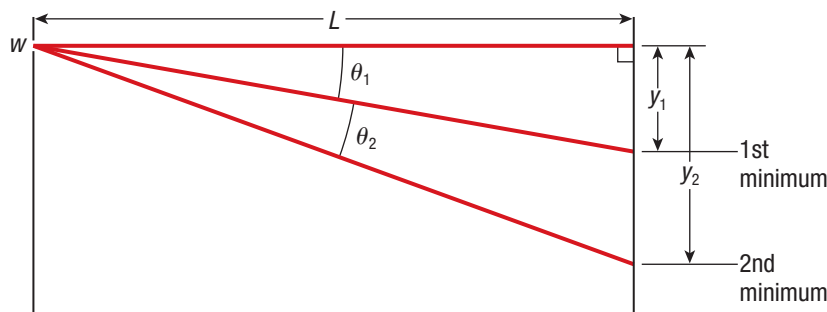


Figure 5 You can determine the value of $\sin \theta_1$ using the approximation $\sin \theta_1 \approx \tan \theta_1 = \frac{y_1}{L}$.

Tutorial 1 Analyzing Single-Slit Interference

In the following Sample Problems, you will analyze interference patterns produced during single-slit diffraction.

Sample Problem 1: Determining the Distance between Dark Fringes and the Central Maximum

Light with a wavelength of 5.40×10^2 nm is incident on a slit of width $11 \mu\text{m}$ and produces a diffraction pattern on a screen located 80.0 cm behind the slit. Calculate the distance of the first dark fringe from the central maximum on the screen.

Given: $\lambda = 5.40 \times 10^2$ nm = 5.40×10^{-7} m;
 $w = 11 \mu\text{m} = 1.1 \times 10^{-5}$ m; $L = 80.0$ cm = 0.800 m

Required: y_1

Analysis: The distance from the central maximum to the first minimum is $\Delta y = y_1$.

$$\begin{aligned} \lambda &= \frac{w\Delta y}{L} \\ \Delta y &= \frac{L\lambda}{w} \\ \text{Solution: } \Delta y &= \frac{L\lambda}{w} \\ &= \frac{(0.800 \text{ m})(5.40 \times 10^{-7} \text{ m})}{1.1 \times 10^{-5} \text{ m}} \\ \Delta y &= 3.9 \times 10^{-2} \text{ m} \end{aligned}$$

Statement: The first dark fringe is 3.9 cm from the central maximum.

Sample Problem 2: Determining the Width of the Central Maximum

Light with a wavelength of 670 nm is incident on a slit of width $12\ \mu\text{m}$ and produces a diffraction pattern on a screen that is 30.0 cm behind the slit.

- Calculate the angular width and the absolute width, in centimetres, of the central maximum.
- Calculate the distance between the first and second minima.
- Determine the distance between the second and third maxima. Compare this answer to your answer in (b). Does this answer make sense? Explain why or why not.

Solution

- (a) **Given:** $n = 1$; $\lambda = 670\ \text{nm} = 6.7 \times 10^{-7}\ \text{m}$;
 $w = 12\ \mu\text{m} = 1.2 \times 10^{-5}\ \text{m}$; $L = 30.0\ \text{cm} = 0.300\ \text{m}$

Required: θ_1 ; $2\Delta y$, the width of the central maximum

Analysis: The distance from the central maximum to the first minimum on each side is $\Delta y = y_1$. The angle for the first

minimum is $\sin \theta_1 = \frac{\lambda}{w}$, where $n = 1$ and the angular width is $2\theta_1$. To calculate $2y_1$, use $\lambda = \frac{w\Delta y}{L}$.

Solution: $\sin \theta_1 = \frac{\lambda}{w}$

$$= \frac{6.7 \times 10^{-7}\ \text{m}}{1.2 \times 10^{-5}\ \text{m}}$$
$$= 0.05583$$
$$\theta_1 = 3.201^\circ \text{ (two extra digits carried)}$$
$$\lambda = \frac{w\Delta y}{L}$$
$$2\Delta y = 2 \frac{L\lambda}{w}$$
$$= \frac{2(0.300\ \text{m})(6.7 \times 10^{-7}\ \text{m})}{1.2 \times 10^{-5}\ \text{m}}$$
$$2\Delta y = 0.0335\ \text{m} \text{ (one extra digit carried)}$$

Statement: The angular width of the central maximum is 6.4° , corresponding to 3.4 cm on the screen.

- (b) **Given:** $\lambda = 670\ \text{nm} = 6.7 \times 10^{-7}\ \text{m}$;
 $w = 12\ \mu\text{m} = 1.2 \times 10^{-5}\ \text{m}$; $L = 30.0\ \text{cm} = 0.300\ \text{m}$

Required: distance between the first and second minima

Analysis: This is half the width of the central maximum in (a), since $\Delta y = y_1$

$$2\Delta y = 2y_1$$
$$y_1 = \frac{2\Delta y}{2}$$

Solution: $y_1 = \frac{2\Delta y}{2}$

$$= \frac{0.0335\ \text{m}}{2}$$
$$y_1 = 1.7\ \text{cm}$$

Statement: The distance between the first and second minima is 1.7 cm.

- (c) **Given:** $\lambda = 670\ \text{nm} = 6.7 \times 10^{-7}\ \text{m}$;
 $w = 12\ \mu\text{m} = 1.2 \times 10^{-5}\ \text{m}$; $L = 30.0\ \text{cm} = 0.300\ \text{m}$

Required: distance between the second and third maxima

Analysis: This is half the width of the central maximum in (a);

$$y_1 = \frac{2\Delta y}{2}$$

Solution: $y_1 = \frac{2\Delta y}{2}$

$$= \frac{0.0335\ \text{m}}{2}$$
$$y_1 = 1.7\ \text{cm}$$

Statement: The distance between the second and third maxima is 1.7 cm. Answers (b) and (c) are the same.

This makes sense because the distance between adjacent maxima is identical to the distance between adjacent minima because the pattern is symmetrical.

Practice

- Would the central maximum in Sample Problem 1 be wider or narrower if the slit and the screen were submerged in water? K/U
- Helium–neon laser light, with a wavelength of $7.328 \times 10^{-7}\ \text{m}$, passes through a single slit with a width of $43\ \mu\text{m}$ onto a screen 3.0 m from the slit. Calculate the separation of adjacent minima, other than those on either side of the central maximum T/I [ans: 5.1 cm]
- Monochromatic light falls onto a slit $3.00 \times 10^{-6}\ \text{m}$ wide. The angle between the first dark fringes on either side of the central maximum is 25.0° . Calculate the wavelength of the light. T/I [ans: $6.49 \times 10^{-7}\ \text{m}$]
- The first dark fringe in a single-slit diffraction pattern is located at an angle of $\theta_a = 56^\circ$. With the same light, the first dark fringe formed with another single slit is located at $\theta_b = 34^\circ$. Calculate the ratio $\frac{w_a}{w_b}$. T/I [ans: 0.67]

Resolution

The **resolution** of an optical device is the device's ability to separate closely spaced objects into distinctly different images. Light passing through a small opening is diffracted, and smaller openings produce greater diffraction. When light from two objects passes simultaneously through the same opening, the light from both sources is diffracted. It produces overlapping diffraction patterns and the images appear fuzzy. If the opening is quite small and the objects are close to each other, distinguishing between the two images can be quite challenging (**Figure 6**).

resolution the ability of an optical device to separate close objects into distinct and sharp images

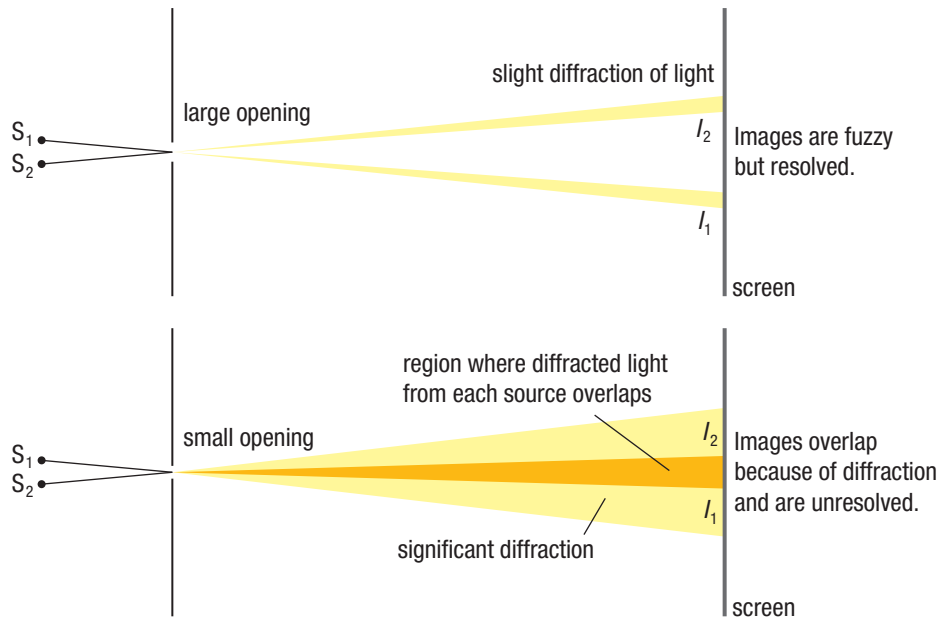


Figure 6 With a large opening, the images from two light sources are fuzzy but distinguishable. With a smaller opening, the images overlap and are difficult to distinguish.

In many optical devices, a small lens replaces the opening. The resolution of the device is then the ability of the lens to form two sharp images. When light passes through the lens, the diffraction leads to a single bright spot surrounded by concentric circles in a diffraction pattern called an Airy disc (**Figure 7**). Two small objects will each produce an image surrounded by diffraction fringes. If the bright parts of the diffraction pattern for the first image fall on the dark areas of the other image, the two images may be inseparable. Lenses and openings that are small relative to the wavelength of light usually produce lower-resolution images by increasing the effects of diffraction, as seen in **Figure 8** on the next page. But factors other than diffraction affect the resolution of lenses such as those on cameras.

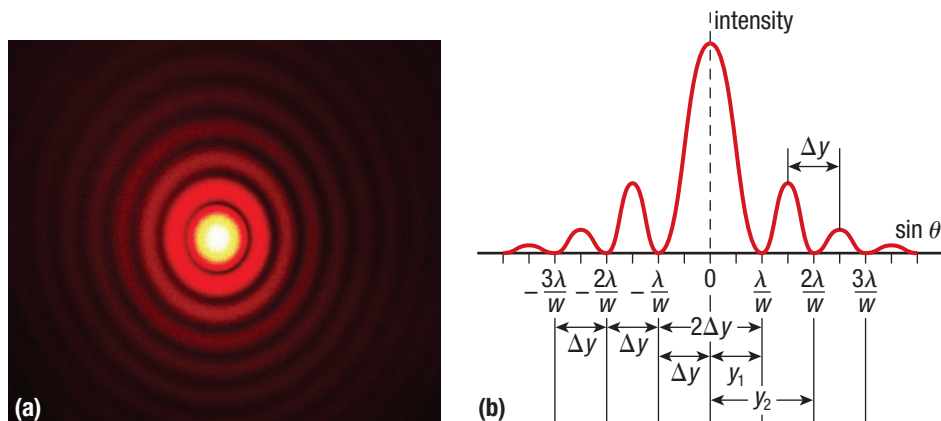


Figure 7 (a) The classical Airy disc is a pattern of concentric circles of light around a bright central spot. (b) The light intensity pattern of an Airy disc is similar to a single-slit diffraction pattern.

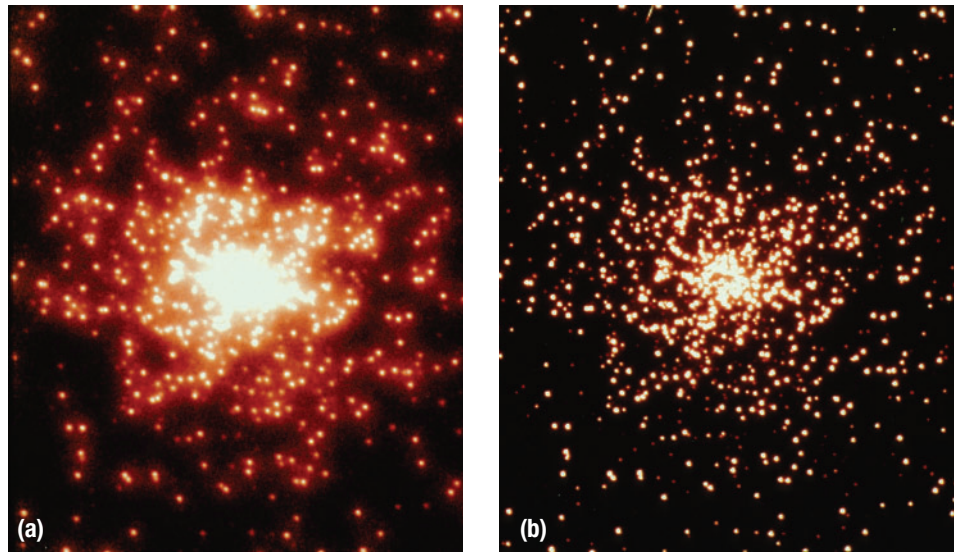


Figure 8 (a) When the resolution of an optical device is not sharp enough, stars that are in close proximity appear blurry. (b) When the resolution is sufficient, more individual stars can be clearly seen.

In an ideal set of optics, 84 % of the incident light is concentrated in the central maximum. If there are obstructions in the optical path, such as a mirror, more light will end up being spread into the rings, making the image less clear. This principle makes a refracting telescope the sharpest theoretical optical instrument for a given diameter. A refracting telescope uses a lens with no obstructions in the optical path. Reflecting telescopes use mirrors instead of lenses, but they often contain more than one mirror in the optical path.

Optical microscopes have small-diameter objective lenses. Due to diffraction, the best optical microscopes can typically achieve a magnification no greater than 1000 times. One way to improve resolution is to use shorter-wavelength light. Therefore, an ultraviolet microscope can achieve a much greater resolution than one that uses visible light.

Most observatory telescopes use curved mirrors (**Figure 9**). Astronomers use the telescopes with the largest mirrors to view the most distant objects, not only because they collect more light, but also because the large apertures reduce diffraction effects and offer greater resolution.



Figure 9 The telescope inside the Canada–France–Hawaii Observatory (left) detects visible light and infrared radiation.

10.2 Review

Summary

- Monochromatic light passing through a single slit produces a diffraction pattern that consists of a bright central region surrounded by alternating light and dark bands (maxima and minima) resulting from constructive and destructive interference.
- Fraunhofer diffraction is a special case of diffraction that shows distinctive differences between the central fringe and darker flanking fringes.
- In single-slit diffraction, the dark bands (minima) occur at angles θ_n that satisfy $\sin \theta_n = \frac{n\lambda}{w}$ ($n = 1, 2, \dots$)
- In single-slit diffraction, the bright bands (maxima) occur at angles θ_m that satisfy $\sin \theta_m = \frac{(m + \frac{1}{2})\lambda}{w}$ ($m = 1, 2, \dots$)
- The distance between successive minima, and successive maxima, is given by $\Delta y = \frac{\lambda L}{w}$, and the width of the central maximum is $2\Delta y$.
- The resolution of an optical instrument is its ability to separate closely spaced objects into distinctly different images limited by diffraction.

Questions

1. Light with a wavelength of 794 nm produces a single-slit diffraction pattern in which the ninth dark fringe lies at 6.48 cm from the direction of the central maximum. The distance to the screen is 1.0 m. Determine the width of the slit. **T/I**
2. Red light ($\lambda = 600.0$ nm) is diffracted by a single slit, and the first dark fringe occurs at $\theta_1 = 6.9^\circ$. Determine the width of the slit. **T/I**
3. A slit of width 0.15 mm is located 10.0 m from a screen. Light with a wavelength of 450 nm passes through this slit. Determine the distance between the first and the third dark fringes on the screen. **T/I**
4. Light with a wavelength of 550 nm passes through a single slit and illuminates a screen 2.0 m away. The distance from the first dark fringe to the centre of the interference pattern is 5.5 mm. Determine the width of the slit. **T/I**
5. Light from a helium–neon laser ($\lambda = 630$ nm) passes through a single slit to a screen 3.0 m from the slit. The slit is 0.25 mm wide. What is the width of the central maximum on the screen? **T/I**
6. Yellow light from a vapour lamp passes through a single slit 0.0295 cm wide to a screen 60.0 cm away. A first-order dark fringe is 0.120 cm from the centre of the central maximum. Determine the wavelength of the yellow light. **K/U T/I**
7. Suppose monochromatic light incident on a single slit produces a diffraction pattern. **K/U**
 - (a) How would the pattern differ if you doubled the wavelength of the light?
 - (b) How would the pattern differ if you doubled both the wavelength and slit width at the same time?
8. Blue light passing through a single narrow slit forms an interference pattern. Determine how the spacing of the maxima would be different if you replaced blue light with green light. Explain your reasoning. **K/U T/I C**
9. A doorway acts as a single slit for light that passes through it, but a doorway is much larger than the wavelength of visible light, so the diffraction angles are very small. Calculate the angle of the tenth minimum for a typical doorway. **K/U T/I**
10. Digital images consist of pixels, or small coloured dots. If an image is 84 pixels wide by 62 pixels high, how could you achieve better resolution? (Hint: Consider aperture and distance.) **K/U T/I A**
11. The star Mizar is actually a double star that only appears as two distinct stars when resolved by a telescope. Explain why. **K/U T/I C**
12. Explain how a single-slit interference pattern differs from a double-slit interference pattern. **K/U T/I C**