Diffraction and Interference of Water Waves

9.3

Have you ever noticed how people relaxing at the seashore spend so much of their time watching the ocean waves moving over the water, as they break repeatedly and roll onto the shore? Water waves behave similarly to other kinds of waves in many ways. **Figure 1** illustrates one characteristic behaviour of waves when a part of the wave enters through a narrow opening. The section of the wave that gets through acts as a source of new waves that spread out on the other side of the opening, and the waves from the two sources combine in specific ways. You will learn about these characteristic behaviours of waves in this section.

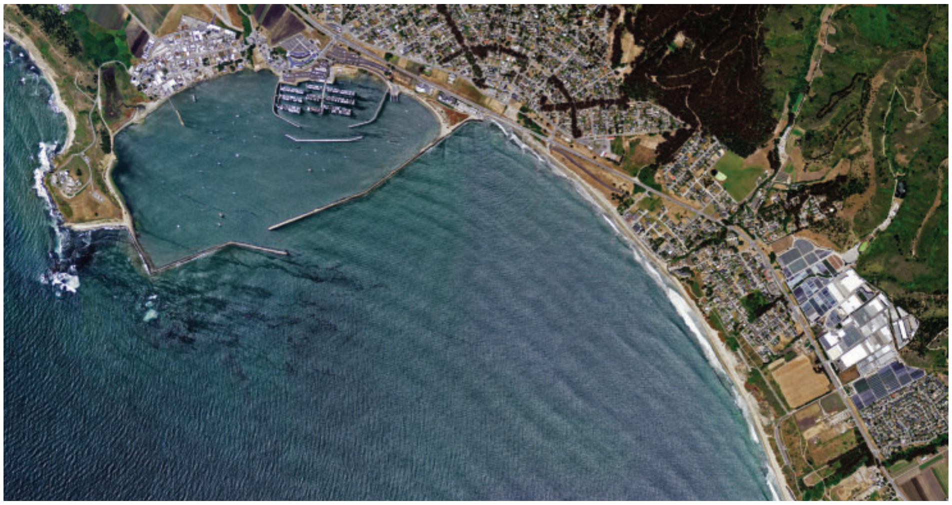


Figure 1 When part of a wave enters a narrow opening, such as this bay, new waves are created and the two waves combine in predictable ways.

Diffraction

If you observe straight wave fronts in a ripple tank, you can see that they travel in a straight line if the water depth is constant and no obstacles are in the way. If, however, the waves pass by an edge of an obstacle or through a small opening, the waves spread out. **Diffraction** is the bending of a wave as the wave passes through an opening or by an obstacle. The amount of diffraction depends on the wavelength of the waves and the size of the opening. In **Figure 2(a)**, an obstacle diffracts shorter wavelengths slightly. In **Figure 2(b)**, the same obstacle diffracts longer wavelengths more.

diffraction the bending and spreading of a wave when it passes through an opening; dependent on the size of the opening and the wavelength of the wave

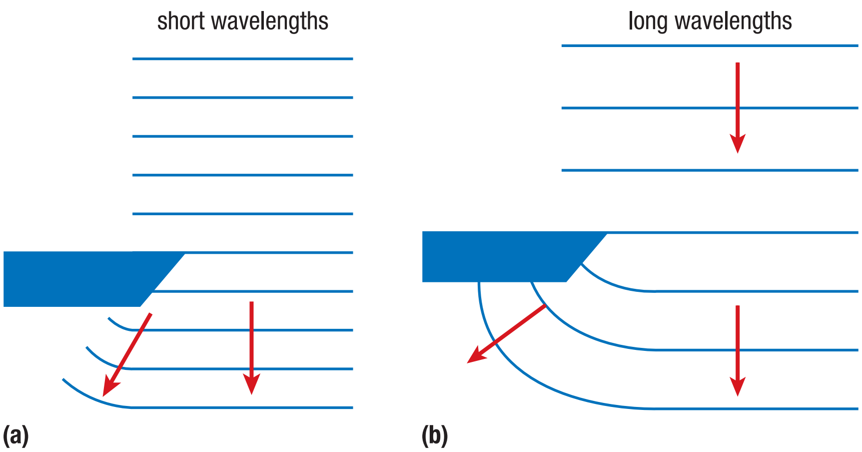


Figure 2 When waves travel by an edge, (a) shorter wavelengths diffract less than (b) longer wavelengths.

Although diffraction also occurs for sound and light waves, we will study the phenomenon first for water waves because their long wavelength allows for easier observation of the effects of diffraction.

In **Figure 3(a)**, you can see a wave encountering a slit with a certain width, *w*. In **Figure 3(b)** and **Figure 3(c)**, the width of the opening is the same but the wavelength of the incident waves has changed. Notice how the amount of diffraction changes. Therefore, as the wavelength increases but the width of the slit does not, the diffraction also increases. In Figure 3(a), the wavelength is approximately one-third of *w*. Notice that only the part of the wave front that passes through the slit creates the series of circular wave fronts on the other side. In Figure 3(b), the wavelength is about half of *w*, which means that significantly more diffraction occurs, but areas to the edge exist where no waves are diffracted. In Figure 3(c), the wavelength is approximately two-thirds of *w*, and the sections of the wave front that pass through the slit are almost all converted to circular wave fronts.

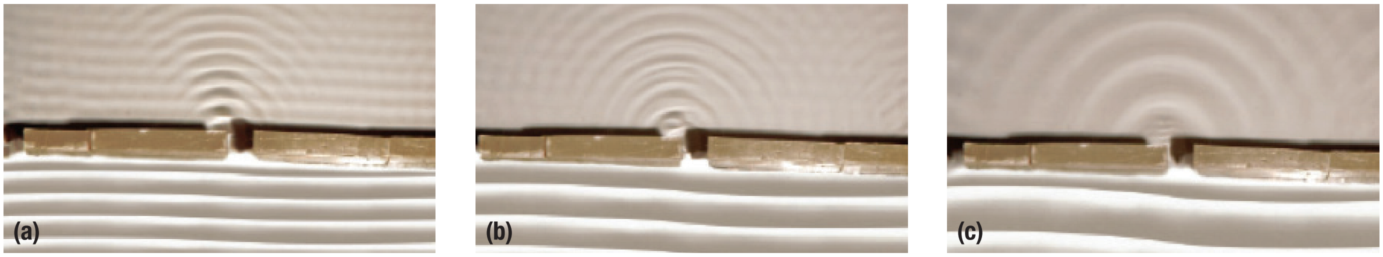


Figure 3 As the wavelength increases, the amount of diffraction increases.

What happens if you change the width of the slit but keep the wavelength fixed? As **Figure 4(a)** and **Figure 4(b)** show, as the size of the slit decreases, the amount of diffraction increases. If waves are to undergo more noticeable diffraction, the wavelength must be comparable to or greater than the slit width ($\lambda \geq w$). For small wavelengths (such as those of visible light), you need to have narrow slits to observe diffraction. **Figure 4(c)** shows what happens to the diffraction pattern when the wavelength decreases but the size of the slit does not change.

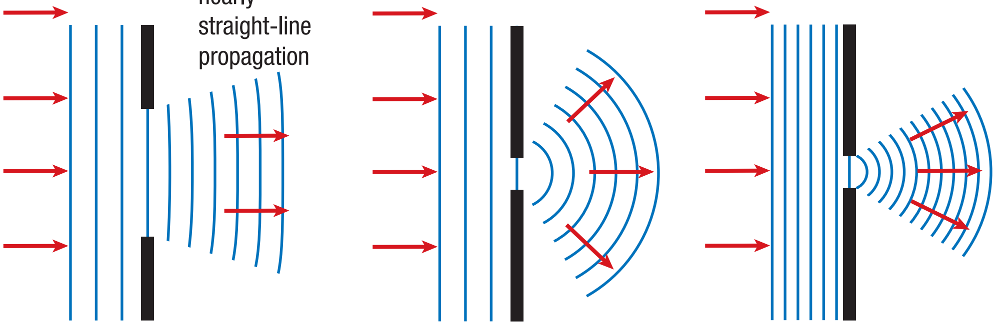


Figure 4 (a) and (b) As the size of the slit (aperture) decreases, diffraction increases. (c) With a shorter wavelength and no change in the size of the opening, there is less diffraction.

Investigation 9.3.1

Properties of Water Waves (page 487)

You have learned how observing water waves helps in understanding the properties of waves in general. This investigation will give you an opportunity to test conditions for diffraction to occur.

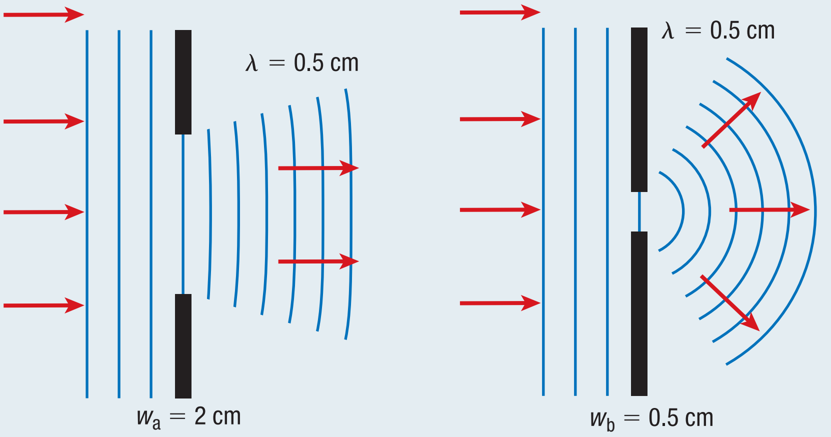
The relationship between wavelength, width of the slit, and extent of diffraction is perhaps familiar for sound waves. You can hear sound through an open door, even if you cannot see what is making the sound. The primary reason that sound waves diffract around the corner of the door is that they have long wavelengths compared to the width of the doorway. Low frequencies (the longer wavelengths of the sound) diffract more than high frequencies (the shorter wavelengths of the sound). So if a sound system is in the room next door, you are more likely to hear the lower frequencies (the bass). In the following Tutorial, you will learn how to solve problems relating to diffraction.

Tutorial 1 Diffraction

The following Sample Problem provides an example and sample calculations for the concepts of diffraction.

Sample Problem 1: Diffraction through a Slit

Determine and explain the difference between the diffractions observed in **Figure 5(a)** and **Figure 5(b)**.



(a) **(b)**

Figure 5

Given: $\lambda = 0.5$ cm; $w_a = 2$ cm; $w_b = 0.5$ cm

Required: diffraction analysis

Analysis: $\frac{\lambda}{w} \geq 1$

Solution:

For Figure 5(a),

$$\frac{\lambda}{w_a} = \frac{0.5 \text{ cm}}{2 \text{ cm}} = 0.25$$

$$\frac{\lambda}{w_a} < 1$$

For Figure 5(b),

$$\frac{\lambda}{w_b} = \frac{0.5 \text{ cm}}{0.5 \text{ cm}}$$

$$\frac{\lambda}{w_b} = 1$$

Statement: Since the ratio in Figure 5(a) is less than 1, little diffraction occurs. Since the ratio in Figure 5(b) is 1, more noticeable diffraction occurs.

Practice

1. Determine whether diffraction will be noticeable when water waves of wavelength 1.0 m pass through a 0.5 m opening between two rocks. T/I [ans: yes]
2. A laser shines red light with a wavelength of 630 nm onto an adjustable slit. Determine the maximum slit width that will cause significant diffraction of the light. T/I [ans: 6.3×10^{-7} m]

Interference

When two waves cross paths and become superimposed, they interact in different ways. This interaction between waves in the same medium is called **interference**. You experience interference when you listen to music and other types of sound. For example, sound waves from two speakers may reach the listener's ears at the same time, as illustrated in **Figure 6**. If the crest of one wave coincides with the crest of the other, then the waves are in phase and combine to create a resultant wave with an amplitude that is greater than the amplitude of either individual wave—resulting in a louder sound. This phenomenon is called **constructive interference**.

interference the phenomenon that occurs when two waves in the same medium interact

constructive interference the phenomenon that occurs when two interfering waves have displacement in the same direction where they superimpose

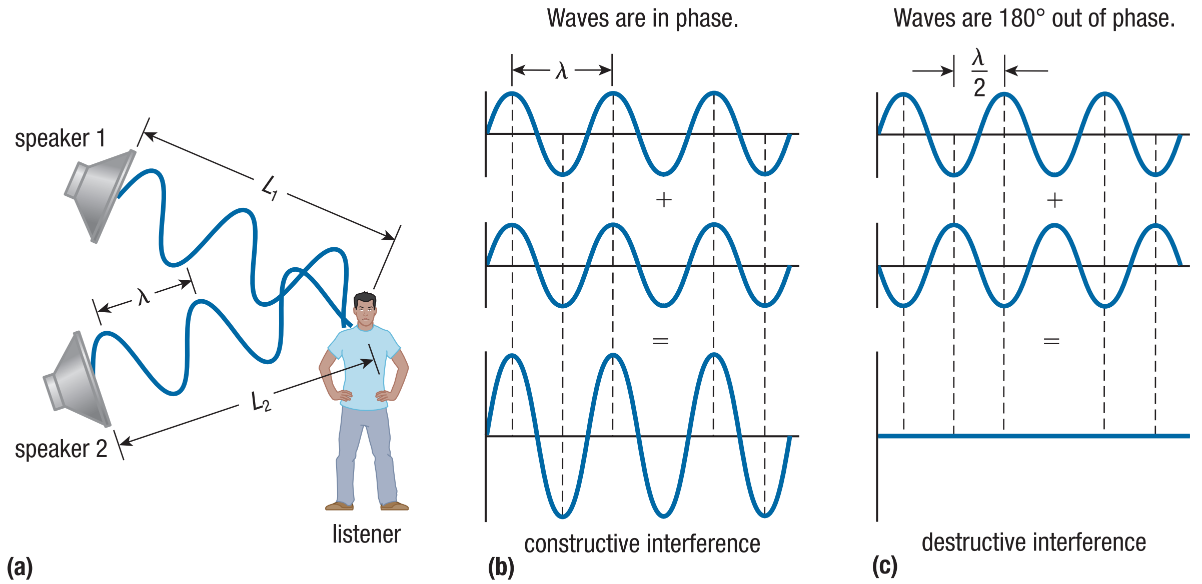


Figure 6 (a) If L_1 and L_2 are the same, the waves arrive at the listener in phase. (b) Waves interfere constructively if they arrive in phase. (c) If L_1 and L_2 differ by, say, $\frac{\lambda}{2}$, the waves arrive 180° out of phase, and they interfere destructively.

For two waves that differ in phase by $\frac{\lambda}{2}$, shown in Figure 6(c), the crest of one wave coincides with the trough of the other wave. This corresponds to a phase difference of 180°. The two waves combine and produce a resulting wave with an amplitude that is smaller than the amplitude of either of the two individual waves. This phenomenon is called **destructive interference**.

destructive interference the phenomenon that occurs when two interfering waves have displacement in opposite directions where they superimpose

The following conditions must be met for interference to occur:

1. Two or more waves are moving through different regions of space over at least some of their way from the source to the point of interest.
2. The waves come together at a common point.
3. The waves must have the same frequency and must have a fixed relationship between their phases such that over a given distance or time the phase difference between the waves is constant. Waves that meet this condition are called **coherent**.

coherent composed of waves having the same frequency and fixed phases

Figure 7 shows water waves interfering.

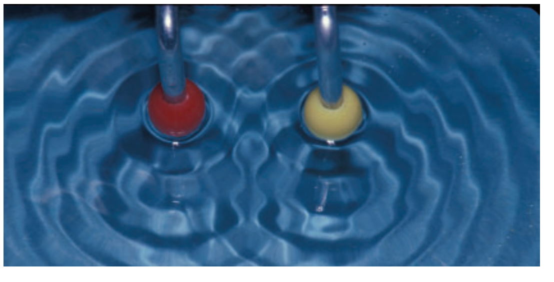


Figure 7 Interference of water waves in a ripple tank

In Figure 7, two point sources have identical frequencies. The sources produce waves that are in phase and have the same amplitudes. Successive wave fronts travel out from the two sources and interfere with each other. Constructive interference occurs when a crest meets a crest or when a trough meets a trough. Destructive interference occurs when a crest meets a trough (resulting in zero amplitude.)

Symmetrical patterns spread out from the sources, producing line locations where constructive and destructive interference occur. A **node** is a place where destructive interference occurs, resulting in zero amplitude (a net displacement of zero). As shown in **Figure 8**, the interference pattern includes lines of maximum displacement, caused by constructive interference, separated by lines of zero displacement, caused by destructive interference. The lines of zero displacement are called **nodal lines**.

node a point along a standing wave where the wave produces zero displacement

nodal line a line or curve along which destructive interference results in zero displacement

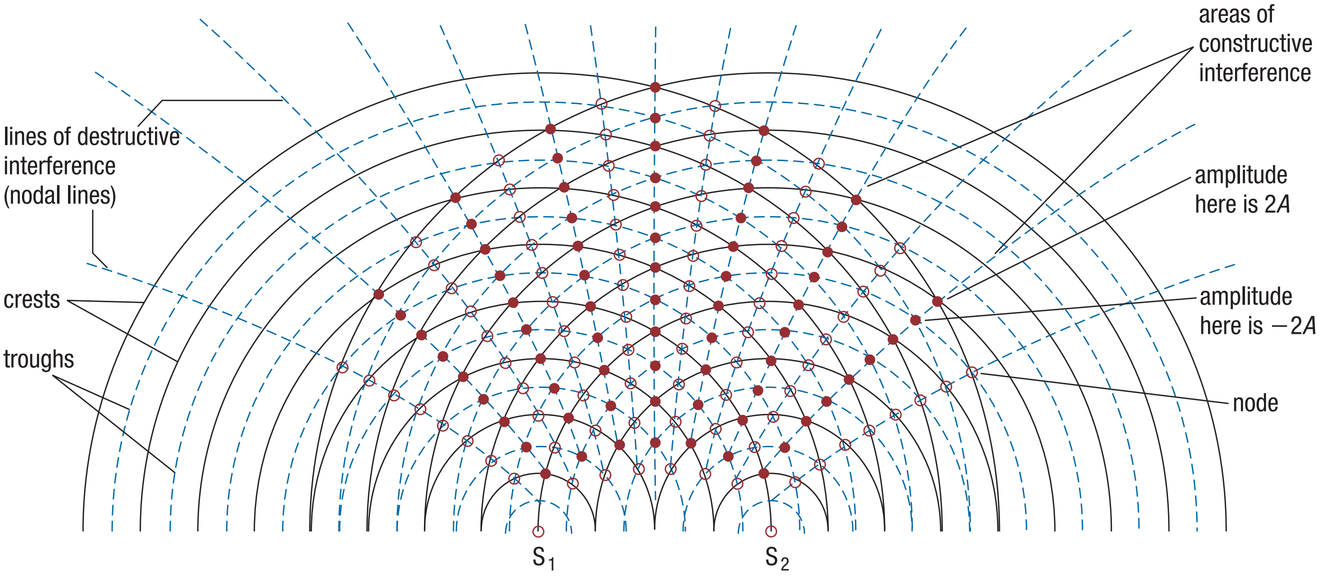


Figure 8 Interference pattern between two identical sources each with positive amplitude *A*

When the frequency of the two sources increases, the wavelength decreases, which means that the nodal lines come closer together and the number of nodal lines increases. When the distance between the two sources increases, the number of nodal lines also increases. The symmetry of the pattern does not change when these two factors are changed. However, if the relative phase of the two sources changes, then the pattern shifts, as shown in **Figure 9**, but the number of nodal lines stays the same. In Figure 9(a), the sources are in phase, but in Figure 9(b), there is a phase difference of 180°.

Investigation **9.3.2**

Interference of Waves in Two Dimensions (page 488)
So far, you have read about two-point-source interference in theory, with diagrams and photographs. This investigation gives you an opportunity to see and measure interference for yourself.

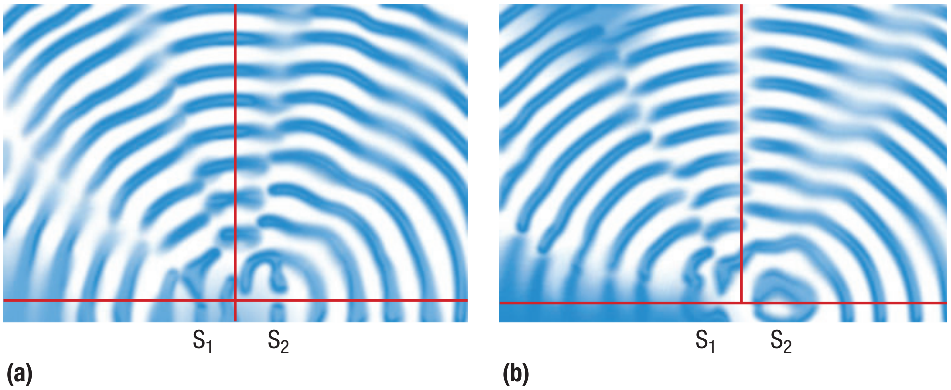


Figure 9 Effect of phase change on interference pattern for (a) zero phase difference and (b) 180° phase difference

Mini Investigation

Interference from Two Speakers

Skills: Performing, Observing, Analyzing, Communicating — SKILLS HANDBOOK A2.1

At the start of the discussion on interference, you considered sound waves coming from two speakers. In this investigation, your teacher will set up two speakers in the classroom, which will emit a single-frequency sound. You will examine how the sound intensity varies from place to place because of interference.

Equipment and Materials: speakers; plan of classroom

1. Before you begin, decide how to identify areas of constructive and destructive interference. How will these manifest themselves with sound?
2. Mark the location of the speakers on the classroom plan.
3. Your teacher will turn on the speakers.
4. Walk around the different areas of the classroom and mark on your plan where constructive and destructive interference occur.

A. How should your distances to the two speakers differ to get constructive interference? K/U

B. How should your distances to the two speakers differ to get destructive interference? K/U

C. Use your results marked on your class plan to identify two locations for either constructive or destructive interference, estimate their distances from the speakers, and use these estimates to estimate the wavelength of the sound. A

WEB LINK

Mathematics of Two-Point-Source Interference

You can measure wavelength using the interference pattern produced by two point sources and develop some mathematical relationships for studying the interference of other waves. **Figure 10** shows an interference pattern produced by two point sources in a ripple tank.

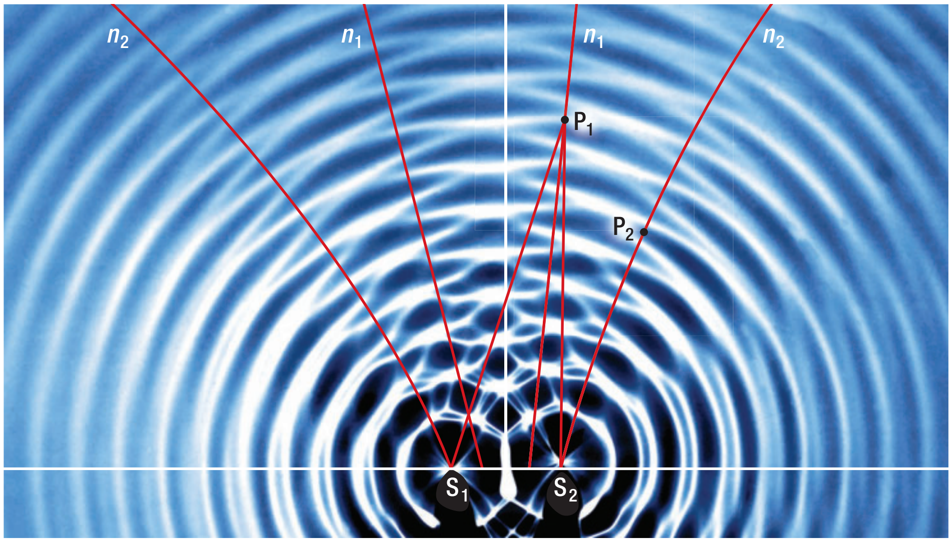


Figure 10 Ripple tank interference patterns can be used to develop relationships to study interference.

The two sources, S_1 and S_2, separated by a distance of three wavelengths, are vibrating in phase. The bisector of the pattern is shown as a white line perpendicular to the line that joins the two sources. You can see that each side of the bisector has an equal number of nodal lines, which are labelled n_1 and n_2 on each side. A point on the first nodal line (n_1) on the right side is labelled P_1, and a point on the second nodal line (n_2) on the right side is labelled P_2. The distances P_1S_1, P_1S_2, P_2S_1, and P_2S_2 are called **path lengths**.

path length the distance from point to point along a nodal line

If you measure wavelengths, you will find that $P_1S_1 = 4\lambda$ and $P_1S_2 = \frac{7}{2}\lambda$. The **path length difference**, Δs_1, on the first nodal line is equal to

path length difference the difference between path lengths, or distances

$$\Delta s_1 = |P_1S_1 - P_1S_2|$$
$$= \left|4\lambda - \frac{7}{2}\lambda\right|$$
$$\Delta s_1 = \frac{1}{2}\lambda$$

This equation applies for any point on the first nodal line. We use the absolute value because only the size of the difference in the two path lengths matters, not which one is greater than the other. A node can be found symmetrically on either side of the perpendicular bisector. The path length difference, Δs_2, on the second nodal line is equal to

$$\Delta s_2 = |P_2S_1 - P_2S_2|$$
$$\Delta s_2 = \frac{3}{2}\lambda$$

Extending this to the nth nodal line, the equation becomes

$$\Delta s_n = |P_nS_1 - P_nS_2|$$
$$\Delta s_n = \left(n - \frac{1}{2}\right)\lambda$$

where P_n is any point on the nth nodal line. Thus, by identifying a specific point on a nodal line and measuring the path lengths, you can use this relationship to determine the wavelength of the interfering waves.

This technique will not work if the wavelengths are too small or the point P is too far away from the sources, because the path length difference is too small to measure accurately. If either or both of those conditions apply, you need to use another method to calculate the path length difference.

For any point P_n, the path length difference is AS_1 (**Figure 11(a)**):

$$|P_nS_1 - P_nS_2| = AS_1$$

When P_n is very far away compared to the separation of the two sources, d, then the lines P_nS_1 and P_nS_2 are nearly parallel (**Figure 11(b)**).

UNIT TASK **BOOKMARK**

You can apply what you have learned about interference to the Unit Task on page 556.

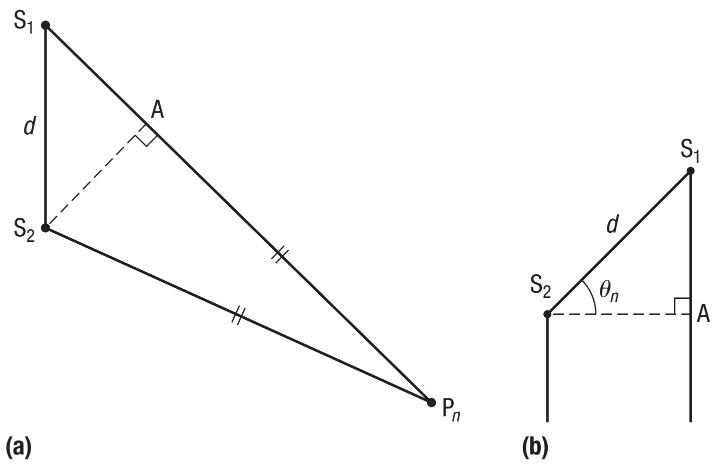


Figure 11 When developing the equations to calculate path length difference, it is important to consider the distance to P_n to be far enough away that P_nS_1 and P_nS_2 can be considered parallel.

In Figure 11(b), the lines AS_2, S_1S_2, and AS_1 form a right-angled triangle, which means that the difference in path length can be written in terms of the angle θ_n, since

$$\sin\theta_n = \frac{AS_1}{d}$$

Rearranging,

$$d\sin\theta_1 = AS_1$$

However, on the previous page we established that

$$AS_1 = |P_nS_1 - P_nS_2|$$

$$AS_1 = \left(n - \frac{1}{2}\right)\lambda$$

Combining this with the expression for AS_1 in terms of $\sin\theta_1$ leads to

$$\sin\theta_n = \left(n - \frac{1}{2}\right)\frac{\lambda}{d}$$

The angle for the nth nodal line is θ_n, the wavelength is λ, and the distance between the sources is d.

Using this equation, you can approximate the wavelength for an interference pattern. Since $\sin\theta_n$ cannot be greater than 1, it follows that $\left(n - \frac{1}{2}\right)\frac{\lambda}{d}$ cannot be greater than 1. The number of nodal lines on the right side of the pattern is the maximum number of lines that satisfies this condition. By counting this number and measuring d, you can determine an approximate value for the wavelength. For example, if d is 2.0 cm and n is 4 for a particular pattern, then

$$\left(n - \frac{1}{2}\right)\frac{\lambda}{d} \approx 1$$

$$\left(4 - \frac{1}{2}\right)\frac{\lambda}{2.0\text{ cm}} \approx 1$$

$$1.75\text{ cm} \approx \frac{1}{\lambda}$$

$$\lambda \approx 0.57\text{ cm}$$

Although it is relatively easy to measure θ_n for water waves in a ripple tank, for light waves, the measurement is not as straightforward. The wavelength of light is very small and so is the distance between the sources. Therefore, the nodal lines are close together. You need to be able to measure $\sin\theta_n$ without having to measure the angle directly. Assuming that a pair of point sources is vibrating in phase, you can use the following derivation.

For a point P_n that is on a nodal line and is distant from the two sources, the line from P_n to the midpoint of the two sources P_nC can be considered to be parallel to P_nS_1. This line is also at right angles to AS_2. The triangle P_nBC in **Figure 12** is used to determine $\sin\theta'_n$. In this derivation, d is the distance between the sources, x_n is the perpendicular distance from the right bisector to the point P_n, L is the distance from P_n to the midpoint between the two sources, and n is the number of the nodal line.

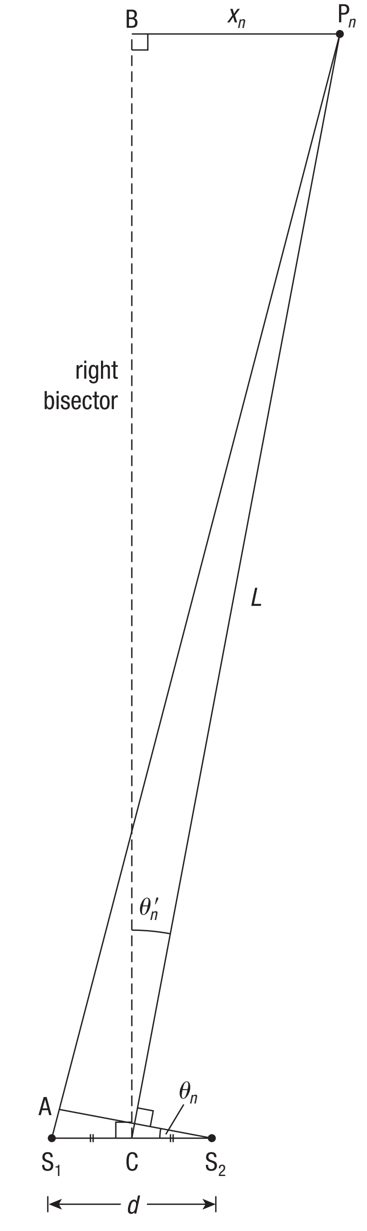


Figure 12 Use the triangle P_nBC to measure $\sin\theta'_n$.

$$\sin\theta'_n = \frac{x_n}{L}$$

Figure 13 is an enlarged version of the triangle with base CS_2 in Figure 12. Since

$$\sin\theta_n = \frac{\left(n - \frac{1}{2}\right)\lambda}{d}$$

and, since the right bisector, CB, is perpendicular to S_1S_2, you can see in Figure 13 that $\theta'_n = \theta_n$. So we can say that

$$\frac{x_n}{L} = \frac{\left(n - \frac{1}{2}\right)\lambda}{d}$$

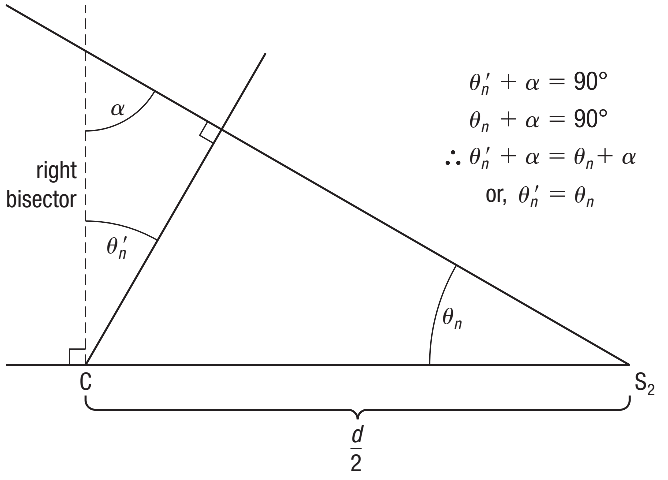


Figure 13 Enlargement of the base triangle from Figure 12

The following Tutorial will demonstrate how to solve problems involving two-dimensional interference.

Tutorial 2 Interference in Two Dimensions

This Tutorial provides examples and sample calculations for the concepts of two-point-source interference.

Sample Problem 1: Interference in Two Dimensions

The distance from the right bisector to a point P on the second nodal line in a two-point interference pattern is 4.0 cm. The distance from the midpoint between the two sources, which are 0.5 cm apart, to point P is 14 cm.

(a) Calculate the angle θ_2 for the second nodal line.

(b) Calculate the wavelength.

Solution

(a) **Given:** $n = 2$; $x_2 = 4.0$ cm; $L = 14$ cm

Required: $\sin\theta_2$

Analysis: $\sin\theta_n = \frac{x_n}{L}$

$$\theta_2 = \sin^{-1}\frac{x_2}{L}$$

Solution:

$$\theta_2 = \sin^{-1}\frac{x_2}{L}$$

$$= \sin^{-1}\left(\frac{4.0\text{ cm}}{14\text{ cm}}\right)$$

$$\theta_2 = 17°$$

Statement: The angle of the second nodal line is 17°.

(b) **Given:** $n = 2$; $x_2 = 4.0$ cm; $L = 14$; $d = 0.5$ cm

Required: λ

Analysis: $\frac{x_n}{L} = \frac{\left(n - \frac{1}{2}\right)\lambda}{d}$

$$\lambda = \frac{x_n d}{\left(n - \frac{1}{2}\right)L}$$

Solution: $\lambda = \frac{x_2 d}{\left(n - \frac{1}{2}\right)L}$

$$\lambda = \frac{(4.0 \text{ cm})(0.5 \text{ cm})}{\left(\frac{3}{2}\right)(14 \text{ cm})}$$

$$\lambda = 0.095 \text{ cm}$$

Statement: The wavelength is 9.5×10^{-2} cm.

Sample Problem 2: Determining Wave Pattern Properties Using Differences in Path Length

Two identical point sources are 5.0 cm apart, in phase, and vibrating at a frequency of 12 Hz. They produce an interference pattern. A point on the first nodal line is 5 cm from one source and 5.5 cm from the other.

(a) Determine the wavelength.

(b) Determine the speed of the waves.

Solution

(a) **Given:** $n = 1$; $P_1S_1 = 5.5$ cm; $P_1S_2 = 5.0$ cm; $d = 5.0$ cm

Required: λ

Analysis: $|P_nS_1 - P_nS_2| = \left(n - \frac{1}{2}\right)\lambda$

Solution:

$$|P_nS_1 - P_nS_2| = \left(n - \frac{1}{2}\right)\lambda$$

$$|P_1S_1 - P_1S_2| = \left(1 - \frac{1}{2}\right)\lambda$$

$$|5.5 \text{ cm} - 5.0 \text{ cm}| = \left(1 - \frac{1}{2}\right)\lambda$$

$$0.5 \text{ cm} = \left(\frac{1}{2}\right)\lambda$$

$$\lambda = 1.0 \text{ cm}$$

Statement: The wavelength is 1.0 cm.

(b) **Given:** $f = 12$ Hz; $\lambda = 1.0$ cm

Required: v

Analysis: $v = f\lambda$

Solution:

$$v = f\lambda$$
$$= (12 \text{ Hz})(1.0 \text{ cm})$$
$$v = 12 \text{ cm/s}$$

Statement: The speed of the waves is 12 cm/s.

Practice

1. Two point sources, S_1 and S_2, are vibrating in phase and produce waves with a wavelength of 2.5 m. The two waves overlap at a nodal point. Calculate the smallest corresponding difference in path length for this point. T/I A [ans: 1.2 m]
2. A point on the third nodal line from the centre of an interference pattern is 35 cm from one source and 42 cm from the other. The sources are 11.2 cm apart and vibrate in phase at 10.5 Hz. T/I
   (a) Calculate the wavelength of the waves [ans: 2.8 cm]
   (b) Calculate the speed of the waves [ans: 29 cm/s]
3. Two point sources vibrate in phase at the same frequency. They set up an interference pattern in which a point on the second nodal line is 29.5 cm from one source and 25.0 cm from the other. The speed of the waves is 7.5 cm/s. T/I
   (a) Calculate the wavelength of the waves. [ans: 3.0 cm]
   (b) Calculate the frequency at which the sources are vibrating. [ans: 2.5 Hz]

9.3 Review

Summary

- Diffraction is the bending and spreading of a wave whose wavelength is comparable to or greater than the slit width, or where $\lambda \geq w$.
- Interference occurs when two waves in the same medium meet.
- Constructive interference occurs when the crest of one wave meets the crest of another wave. The resulting wave has an amplitude greater than that of each individual wave.
- Destructive interference occurs when the crest of one wave meets the trough of another wave. The resulting wave has an amplitude less than that of each individual wave.
- Waves with shorter wavelengths diffract less than waves with longer wavelengths.
- A pair of identical point sources that are in phase produce a symmetrical pattern of constructive interference areas and nodal lines.
- The number of nodal lines in a given region will increase when the frequency of vibration of the sources increases or when the wavelength decreases.
- When the separation of the sources increases, the number of nodal lines also increases.
- The relationship that can be used to solve for an unknown variable in a two-point-source interference pattern is $|P_nS_1 - P_nS_2| = \left(n - \frac{1}{2}\right)\lambda$.

Questions

1. Under what condition is the diffraction of waves through a slit maximized? K/U T/I
2. Two loudspeakers are 1.5 m apart, and they vibrate in phase at the same frequency to produce sound with a wavelength of 1.3 m. Both sound waves reach your friend in phase where he is standing off to one side. You realize that one of the speakers was connected incorrectly, so you switch the wires and change its phase by 180°. How does this affect the sound volume that your friend hears? K/U
3. (a) Determine the maximum slit width that will produce noticeable diffraction for waves of wavelength 6.3×10^{-4} m.
   (b) If the slit is wider than the width you calculated in (a), will the waves diffract? Explain your answer. K/U T/I
4. Two speakers are 1.0 m apart and vibrate in phase to produce waves of wavelength 0.25 m. Determine the angle of the first nodal line. T/I
5. What conditions are necessary for the interference pattern from a two-point source to be stable? K/U T/I
6. Two identical point sources are 5.0 cm apart. A metre stick is parallel to the line joining the two sources. The first nodal line intersects the metre stick at the 35 cm and 55 cm marks. Each crossing point is 50 cm away from the middle of the line joining the two sources. K/U T/I C A
   (a) Draw a diagram illustrating this.
   (b) The sources vibrate at a frequency of 6.0 Hz. Calculate the wavelength of the waves.
   (c) Calculate the speed of the waves if the frequency of the sources is the same as in part (a).
7. A student takes the following data from a ripple tank experiment where two point sources are in phase: $n = 3$, $x_3 = 35$ cm, $L = 77$ cm, $d = 6.0$ cm, $\theta_3 = 25°$, and the distance between identical points on 5 crests is 4.2 cm. From these data, you can work out the wavelength in three different ways. T/I
   (a) Carry out the relevant calculations to determine the wavelength.
   (b) Which piece of data do you think has been incorrectly recorded?