9.2

# Refraction and Total Internal Reflection

When a light wave strikes a transparent material such as glass or water, some of the light is reflected from the surface (as described in Section 9.1). The rest of the light passes through (transmits) the material. **Figure 1** shows a ray that has entered a glass block that has two parallel sides. The part of the original ray that travels into the glass is called the refracted ray, and the part of the original ray that is reflected is called the reflected ray.

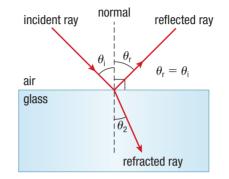


Figure 1 A light ray that strikes a glass surface is both reflected and refracted.

Refracted and reflected rays of light account for many things that we encounter in our everyday lives. For example, the water in a pool can look shallower than it really is. A stick can look as if it bends at the point where it enters the water. On a hot day, the road ahead can appear to have a puddle of water, which turns out to be a mirage. These effects are all caused by the refraction and reflection of light.

# Refraction

The direction of the refracted ray is different from the direction of the incident ray, an effect called **refraction**. As with reflection, you can measure the direction of the refracted ray using the angle that it makes with the normal. In Figure 1, this angle is labelled  $\theta_2$ . The size of this angle depends on the incident angle (which is shown as  $\theta_i$  in Figure 1) as well as the optical densities of the two media. The **optical density** of a medium is a measure of its tendency to absorb the energy of an electromagnetic wave, which is different from the material's mass density. The more optically dense a material is, the slower a wave will move through it. When light travels from a less optically dense medium such as air to a more optically dense medium such as glass, light is refracted toward the normal, as shown in Figure 1. If we extend the refracted ray in Figure 1 to show the light leaving the glass and entering air again, you would see that the light is refracted away from the normal and emerges parallel to the incident ray. This is shown in **Figure 2** on the next page. It is an example of the **principle of reversibility**, which states that the path a light ray follows remains the same if its direction of travel is reversed.

Consider the speed of light in various media. The speed of light in a vacuum is  $3.0 \times 10^8$  m/s. In most cases, this value is a good approximation of the speed of light in air. However, when light enters a different medium, it interacts with the atoms and its average speed through the medium is changed. The ratio of the speed of light in a

vacuum to the speed of light in another medium,  $\frac{c}{v}$ , is called the **index of refraction**, *n*.

 Table 1, on the next page, shows the speed of light and indices of refraction in different media.

**refraction** the bending of light as it travels at an angle from one medium to another

**optical density** the property of a material that determines how light behaves when it travels through the material

**principle of reversibility** a light ray will follow exactly the same path if its direction of travel is reversed

**index of refraction** the ratio of the speed of light in a vacuum to the speed of light in another medium

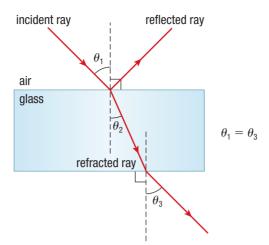
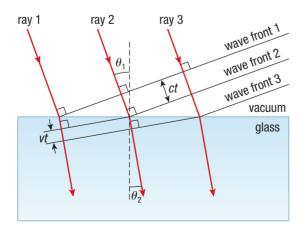


Figure 2 The refracted ray that emerges from the bottom of the glass is parallel to the incident ray.

Medium	Index of refraction	Speed of light (m/s)	Medium	Index of refraction	Speed of light (m/s)
vacuum	1.00	$2.9979 imes10^8$	lens of human eye	1.41	$2.1262 imes10^8$
air	1.0003	$2.9970 imes10^8$	quartz crystal	1.46	$2.0534 imes10^8$
ice	1.30	$2.3061  imes 10^8$	Pyrex glass	1.47	$2.0394  imes 10^8$
liquid water	1.33	$2.2541  imes 10^8$	Plexiglas (plastic)	1.51	$1.9854  imes 10^8$
aqueous humour (liquid between the lens and cornea)	1.33	2.2541 × 10 <sup>8</sup>	benzene	1.50	1.9986 × 10 <sup>8</sup>
cornea of human eye	1.38	$2.1724 imes10^8$	zircon	1.92	$1.5601  imes 10^{8}$
vitreous humour (liquid between the lens and retina)	1.38	2.1724 × 10 <sup>8</sup>	diamond	2.42	1.2388 × 10 <sup>8</sup>

**Table 1** The Speed of Light and Indices of Refraction in Different Media

Notice in Table 1 that as the index of refraction increases, the speed of light in various media decreases. **Figure 3** shows how the change in the speed of light from a vacuum to glass affects the orientation of the wave fronts and the direction of the rays inside the glass, causing refraction. Each incident ray is at a right angle to its respective wave front. The incident rays arrive at an angle of  $\theta_1$ , and the refracted rays are at an angle of  $\theta_2$ .



**Figure 3** Refraction is caused by the difference in wave speed in two media. Here the distance between the wave fronts inside the glass is less than outside the glass. The speed of light has decreased; however, the frequency of the light is the same.

Figure 3 also shows the wave fronts at equally spaced moments in time. The speed of light in the vacuum is *c*, so the wave travels a distance of *ct* through the vacuum in time *t*. In the glass, the speed of light is reduced to *v*, so the wave travels a distance of *vt* in time *t*. Since the speed of light in a vacuum (*c*) is greater than the speed of light in the glass (*v*), the distance travelled in time *t* in the glass (*vt*) is less than the distance travelled in the vacuum (*ct*). This effect causes the wave fronts to bend at the point where they enter the glass, which means that the light rays bend toward the normal as they enter the glass. The result is that  $\theta_2$  is less than  $\theta_1$ . The angle  $\theta_2$  between the refracted light ray and the normal is called the **angle of refraction** and is often written as  $\theta_R$ . **Figure 4** shows a more detailed view of the relationship between the angles  $\theta_1$  and  $\theta_2$  as the wave fronts enter the glass.

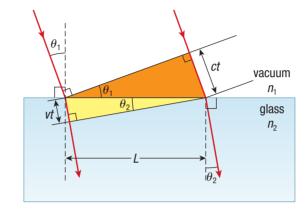


Figure 4 You can use the geometry of the wave fronts to examine the relationship between the incident and refracted angles.

The orange triangle on the glass edge in Figure 4 has sides of length *ct* and *L*, with the angle adjacent to *L* being  $\theta_1$ . The following trigonometric relationship is true for this triangle:

$$\sin\theta_1 = \frac{ct}{L}$$

The corresponding yellow triangle has sides of length vt and L, with the angle adjacent to L being  $\theta_2$ . For this triangle, the same relationship holds:

$$\sin\theta_2 = \frac{vt}{L}$$

Rearrange these equations to solve for *L*:

$$L = \frac{ct}{\sin \theta_1}$$
 and  $L = \frac{vt}{\sin \theta_2}$ 

Therefore,

$$\frac{ct}{\sin\theta_1} = \frac{vt}{\sin\theta_2}$$

Since *t* cannot equal zero, we can divide each side of the equation by *t*:

$$\frac{c}{\sin\theta_1} = \frac{v}{\sin\theta_2}$$

or

$$\sin\theta_1 = \frac{c\sin\theta_2}{v}$$

Recall that the ratio  $\frac{L}{\nu}$  is a dimensionless number called the index of refraction and is given the symbol *n*:

$$n=\frac{c}{v}$$

We can rewrite the above equation as

$$\sin\theta_1 = n_2 \sin\theta_2$$

where  $n_2$  is the index of refraction of the glass.

angle of refraction the angle that a light

ray makes with respect to the normal to the surface when it has entered a

different medium

The preceding derivation describes the refraction of light from a vacuum to glass. If you consider the common situation where light passes between two substances, such as air and glass or air and water, then the following relationship, called **Snell's law**, is true:

**Snell's Law**  
$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

where  $n_1$  and  $\theta_1$  are, respectively, the index of refraction in the incident medium and the angle of incidence, and  $n_2$  and  $\theta_2$  are, respectively, the index of refraction for the medium being studied and the angle of refraction.

Using the relationship between speed, frequency, and wavelength, as well as the fact that the frequency of light does not change when passing from one substance to another, the following equation can be derived:

$$n = \frac{c}{v}$$
$$= \frac{f\lambda_1}{f\lambda_2}$$
$$n = \frac{\lambda_1}{\lambda_2}$$

where  $\lambda_1$  is the wavelength of light in a vacuum, and  $\lambda_2$  is the wavelength of light in the substance. When the first medium is not a vacuum and has index of refraction  $n_1$ , this equation becomes

$$\frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2}$$

In **Figure 5**, incident light hits the block at angle  $\theta_1$  on side 1. It is then refracted at an angle  $\theta_2$ , which can be calculated using Snell's law if we have enough information, such as the indices of refraction of the media. When the incident light hits the boundary on the opposite side, at an angle  $\theta_2$ , the light is refracted at an angle equal to  $\theta_1$  because the surfaces are parallel.

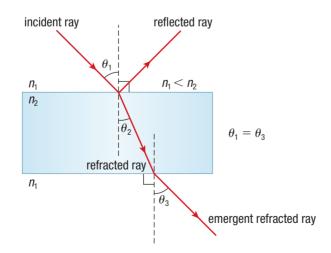


Figure 5 Snell's law can be used to calculate the angles of refraction.

In Tutorial 1, you will use Snell's law to calculate the angle of refraction as well as the speed, wavelength, and frequency for refracted light.

# Tutorial **1** Solving Problems by Applying Snell's Law

You can calculate angles of incidence and refraction using Snell's law, given information about the index of refraction. In the following Sample Problem, we will use Snell's law to solve for the angle of refraction and the speed, wavelength, and frequency of light.

### Sample Problem 1: Angle of Refraction for Light from a Vacuum into Glass

- (a) Calculate the angle of refraction for light moving from a vacuum into a plate of glass with index of refraction 1.47. The angle of incidence is 40.0°.
- (b) The light continues through the glass and emerges back into a vacuum. Calculate the angle of refraction when the light exits the glass.
- (c) Suppose the light exits into water instead of a vacuum. Calculate the angle of refraction for the light moving from glass into water. The index of refraction for water is 1.33.

#### Solution

(a) **Given:**  $n_2 = 1.47$ ;  $\theta_1 = 40.0^\circ$ ;  $n_1 = 1.0$ 

#### **Required:** $\theta_2$

Analysis:

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{n_1}{n_2}$$
$$\sin \theta_2 = \frac{n_1 \sin \theta_1}{n_2}$$

#### Solution:

$$\sin \theta_2 = \frac{n_1 \sin \theta_1}{n_2}$$
$$= \frac{(1.0) \sin 40.0^{\circ}}{1.47}$$
$$\theta_2 = 25.9^{\circ} \text{ (one extra digit carried)}$$

**Statement:** The angle of refraction for light moving from the vacuum into glass is  $26^{\circ}$ .

(b) **Given:** 
$$n_3 = 1.0$$
;  $\theta_2 = 25.9^{\circ}$ 

**Required:**  $\theta_3$ 

#### Analysis:

$$\frac{\sin \theta_3}{\sin \theta_2} = \frac{n_2}{n_3}$$
$$\sin \theta_3 = \frac{n_2 \sin \theta_2}{n_3}$$

#### Solution:

$$\sin \theta_3 = \frac{n_2 \sin \theta_2}{n_3}$$
$$\sin \theta_3 = \left(\frac{1.47}{1}\right) (\sin 25.9^\circ)$$
$$= 0.64$$
$$\theta_3 = 40^\circ$$

**Statement:** The angle of refraction for light moving from the glass back into the vacuum is  $40^{\circ}$ . This is the same as the angle of incidence of the ray entering the glass.

(c) **Given:**  $n_3 = 1.33$ ;  $\theta_2 = 25.9^\circ$ ;  $n_2 = 1.47$ 

**Required:**  $\theta_3$ 

#### Analysis:

So

$$\frac{\sin \theta_3}{\sin \theta_2} = \frac{n_2}{n_3}$$
$$\sin \theta_3 = \frac{n_2 \sin \theta_2}{n_3}$$
lution:
$$\sin \theta = \frac{n_2 \sin \theta_2}{n_3}$$

$$= \frac{(1.47) \sin 25.9^\circ}{1.33}$$
  
 $\theta_3 = 29^\circ$ 

**Statement:** The angle of refraction for light moving from glass into water is 29°.

# Sample Problem 2: Refraction and Lasers

Light travels at  $3.0 \times 10^8$  m/s. Laser light with a wavelength of 520 nm enters a sheet of plastic. The index of refraction for the plastic is 1.49.

- (a) Calculate the speed of the laser light in the plastic.
- (b) Calculate the wavelength of the laser light in the plastic.
- (c) Calculate the frequency of the laser light in the plastic.

### Solution

(a) **Given:** n = 1.49;  $c = 3.0 \times 10^8$  m/s

#### Required: v

#### Analysis:

$$n = \frac{c}{v}$$
$$v = \frac{c}{n}$$

Solution:

$$r = \frac{c}{n}$$
$$= \frac{3.0 \times 10^8}{1.49}$$

 $v = 2.01 \times 10^8$  m/s (one extra digit carried)

m/s

**Statement:** The speed of the laser light in the plastic is  $2.0 \times 10^8$  m/s.

(b) **Given:** 
$$n = 1.49$$
;  $\lambda_1 = 520$  nm  $= 5.2 \times 10^{-7}$  m

**Required:**  $\lambda_2$ 

# Analysis:

$$\eta = \frac{\lambda_1}{\lambda_2}$$
$$_2 = \frac{\lambda_1}{n}$$

λ.

Solution:

λ

$$\lambda_{2} = \frac{\lambda_{1}}{n}$$

$$= \frac{5.2 \times 10^{-7} \text{ m}}{1.49}$$

$$\lambda_{2} = 3.49 \times 10^{-7} \text{ m} \text{ (one extra digit carried)}$$

Statement: The wavelength of the laser light in the plastic is 3.5  $\times$  10  $^{-7}$  m.

(c) Given:  $\lambda_2 = 3.49 \times 10^{-7}$  m;  $\nu = 2.01 \times 10^8$  m/s

Required: f

Analysis:

$$f = \frac{\nu}{\lambda}$$

Solution:

$$f = \frac{v}{\lambda_2}$$
$$= \frac{2.01 \times 10^8 \text{ m/s}}{3.49 \times 10^{-7} \text{m}}$$
$$f = 5.77 \times 10^{14} \text{ Hz}$$

Statement: The frequency of the laser light in the plastic is  $5.8\times10^{14}\,\text{Hz}.$ 

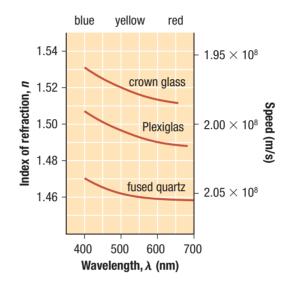
## **Practice**

- 1. In an experiment, light shines on a flat mirror with an angle of incidence of 65°. The experiment takes place underwater. What is the angle of incidence? The index of refraction for water is 1.33. [XIII] [ans: 65°]
- 2. A light ray enters an unknown medium from air. Some reflects at an angle of 47.5°, and the rest of the light is refracted at an angle of 34.0°. According to Table 1 on page 445, what is the unknown medium? **Transet** [ans: water]
- 3. Calculate the index of refraction of water when a light ray that hits the top of a glass of water at an angle of incidence of 35° is refracted at an angle of 25°. III [ans: 1.36]
- 4. Determine the speed of light in a diamond. Use  $3.0 \times 10^8$  m/s for the speed of light in air. Refer to Table 1 on page 445 for the index of refraction for diamond. [72] [ans:  $1.2 \times 10^8$  m/s]
- 5. Calculate the wavelength of light in quartz if the wavelength in a vacuum is  $5.6 \times 10^{-7}$  m and the index of refraction is 1.46. [7] [ans:  $3.8 \times 10^{-7}$  m]
- Light with a wavelength of 450 nm in a vacuum enters a sample of glass. The index of refraction of the glass is 1.45. Determine the frequency of the light inside the glass.
   [10] [ans: 6.7 × 10<sup>14</sup> Hz]

# **Dispersion and Prisms**

Visible light contains waves with a spectrum of different wavelengths. In a vacuum, all these waves travel at the same speed,  $3.0 \times 10^8$  m/s. When visible light travels from one medium to another, the speeds of the various waves change.

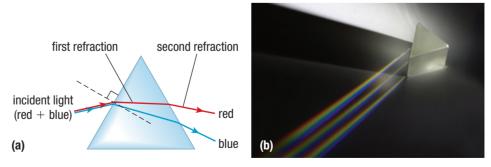
The speed of the wave depends on its frequency and wavelength. When the wave enters the new medium, its frequency does not change but its wavelength does. This means that the speed of the refracted light depends on its wavelength. This dependence of the speed of light on wavelength causes light waves to separate in a phenomenon called **dispersion**. Figure 6 shows the variation of the speed of visible light with wavelength for three materials.

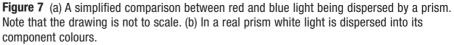


**dispersion** the separation of a wave into its component parts according to a given characteristic, such as frequency or wavelength

Figure 6 The speed of visible light varies in different materials.

The index of refraction for a material depends on the wavelength of light. The index of refraction for red light (at one end of the visible spectrum) and the index of refraction for blue light (at the opposite end of the spectrum) are slightly different for the same material. For example, Figure 6 shows that, in quartz, the index of refraction for red light is approximately 1.46 and the index of refraction for blue light 1.47. The difference between these values is 0.01, which is much smaller than the difference between the (average) indices of refraction for quartz and water (1.46 - 1.33 = 0.13). However, the difference is large enough to lead to a difference in the angle of refraction for different colours in quartz. This is true in other materials as well. **Figure 7** shows how this effect is used in a prism to disperse (separate) an incident beam of white light into its component colours.





#### UNIT TASK BOOKMARK

You can apply what you have learned about dispersion and prisms to the Unit Task on page 556.

Prisms are often composed of glass, and are triangular in shape, as shown in Figure 7(b). In Figure 7(a), the beam of light that is incident on the prism is taken, for simplicity, to be composed of two colours (red and blue), represented as two rays. Each ray is refracted once when it enters the left-hand face of the prism, and again when it leaves the prism at the right-hand face. Note that the light does not curve within the material but only changes direction at the boundaries.

The incident blue light has a slightly larger index of refraction than red light, so its angle of refraction is less than the angle of refraction of the incident red light. Inside the glass, the blue and red light components travel along slightly different paths. This path difference is increased by the second refraction when the light exits the prism. The outgoing light appears slightly bluish at one side and slightly reddish at the other side of the beam.

Another effect of the triangular shape of a prism is that the light ray leaving the prism is not parallel to the light ray that enters the prism. The angle between the incident ray and the final outgoing ray is called the **angle of deviation**. The following Tutorial examines how you can use Snell's law to solve dispersion problems involving prisms.

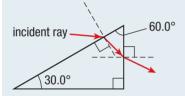
**angle of deviation** the angle between the incident ray and the final outgoing ray after reflection or refraction

## Tutorial **2** Solving Dispersion Problems Using Snell's Law

In this Tutorial, you will use Snell's law to calculate the angles at which a prism disperses different wavelengths of light.

#### Sample Problem 1: Angle of Refraction through a Prism

**Figure 8** shows a ray of light incident on a glass triangular prism. It is refracted as it passes through the first face and is then refracted again through the second face.



#### Figure 8

- (a) Calculate the angles of refraction for blue light and for red light travelling from air into the left boundary of the prism. The angle of incidence at the left boundary of the prism is 40.0°. The index of refraction for blue light in the prism is 1.47, and the index of refraction for red light is 1.46. The index of refraction for red light is 1.46. The index of refraction for air is 1.0003.
- (b) Calculate the angle of incidence for blue and red light at the right boundary of the prism.
- (c) Calculate the angle of refraction for blue and red light as the rays exit the prism.

#### Solution

(a) Given:  $n_{2 \text{ blue}} = 1.47$ ;  $n_{2 \text{ red}} = 1.46$ ;  $n_{air} = 1.0003$ ;  $\theta_1 = 40.0^{\circ}$ Required:  $\theta_{2 \text{ blue}}$ ;  $\theta_{2 \text{ red}}$ 

Analysis:  $\frac{\sin \theta_2}{\sin \theta_1} = \frac{n_{\text{air}}}{n_{\text{prism}}}$ 

Solution: For blue light:

$$\frac{\sin \theta_{2 \text{ blue}}}{\sin \theta_1} = \frac{n_1}{n_{2 \text{ blue}}}$$
$$\sin \theta_{2 \text{ blue}} = \frac{n_1 \sin \theta_1}{n_{2 \text{ blue}}}$$
$$= \frac{(1.0003) \sin 40.0^\circ}{1.47}$$

$$\theta_{2 \text{ blue}} = 25.94^{\circ}$$
 (one extra digit carried)

For red light:

$$\theta_{2 \, \text{red}} = \sin^{-1} \left( \frac{(1.0003) \sin 40.0^{\circ}}{1.46} \right)$$

 $\theta_{2 \text{ red}} = 26.13^{\circ}$  (one extra digit carried)

**Statement:** The angle of refraction at the left boundary of the prism of blue light is  $25.9^{\circ}$ . The angle of refraction of red light is  $26.1^{\circ}$ .

(b) **Given:**  $\theta_{2 \text{ blue}} = 25.94^{\circ}; \theta_{2 \text{ red}} = 26.13^{\circ}$ 

**Required:**  $\theta_{3 \text{ blue}}$ ;  $\theta_{3 \text{ red}}$ 

**Analysis:** If  $\theta_2$  is the angle of refraction at the left boundary and  $\theta_3$  is the angle of incidence at the right boundary, then from the geometry of the triangular prism we have  $\theta_2 + \theta_3 = 60.0^{\circ}$ (which is the angle in the top right of the triangle).

**Solution:**  $\theta_3 = 60.0^\circ - \theta_2$ For blue light:  $60.0^\circ - 25.94^\circ = 34.06^\circ$ For red light:  $60.0^\circ - 26.13^\circ = 33.87^\circ$ 

**Statement:** The angle of incidence at the right boundary of the prism of blue light is 34.1°. The angle of incidence of red light is 33.9°.

(c) **Given:**  $\theta_{3 \text{ blue}} = 34.06$ ;  $\theta_{3 \text{ red}} = 33.87$ ;  $n_{2 \text{ blue}} = 1.47$ ;  $n_{4 \text{ blue}} = 1.46; n_{air} = 1.0003$ **Required:**  $\theta_{4 \text{ blue}}; \theta_{4 \text{ red}}$ Analysis:  $\frac{\sin \theta_4}{\sin \theta_3} = \frac{n_{\text{prism}}}{n_{\text{air}}}$ Solution: For blue light:  $\sin \theta_{4 \text{ blue}} - n_{2 \text{ blue}}$  $\sin \theta_{3 \text{ blue}}$   $n_1$  $\sin\theta_{4\,\rm blue}=\frac{n_{2\,\rm blue}\sin\theta_{3\,\rm blue}}{}$ *n*<sub>1</sub>  $=\frac{(1.47)(\sin 34.06^\circ)}{1.0003}$ 1.0003  $\theta_{4 \text{ blue}} = 55.4^{\circ}$ 

For red light:

$$\theta_{4 \text{ red}} = \sin^{-1} \frac{(1.46)(\sin 33.87^{\circ})}{1.0003}$$
$$\theta_{4 \text{ red}} = 54.4^{\circ}$$

Statement: The angle of refraction for the blue light as it exits the prism is 55.4°. The angle of refraction for the red light as it exits the prism is 54.4°.

### **Practice**

- 1. Consider Figure 8 on page 451, but this time only one colour of light strikes the prism. The index of refraction for this colour in the prism is 1.465, and its angle of incidence is 40.0°.
  - (a) Determine the angle of refraction for this light at the left boundary of the prism. [ans: 26.0°]
  - (b) Determine the angle of refraction for this light as it exits the prism. [ans: 54.9°]
- 2. In Sample Problem 1(c), why is the final angle of refraction not equal to the angle of incidence? K/U C
- 3. Monochromatic light is incident on the prism in Figure 9. The index of refraction of the prism is 1.60. The angle of incidence is 55°. What angle does the outgoing ray make with the horizontal? **KUU TIL A** [ans: 21° [below the horizontal]]

# **Research** This

Figure 9

60°

#### Using Spectroscopy to Determine Whether Extra-Solar Planets Can Support Life

#### Skills: Questioning, Researching, Analyzing, Communicating

60

Research spectroscopy. Find out how scientists use star spectra (Figure 10) to look for planets that contain substances necessary for life, such as carbon dioxide, methane, oxygen, and water, in their atmospheres. Your research should aim to answer the following questions.

Figure 10

- A. How are dispersion and spectroscopy used to determine which gases are present on other planets?
- B. Name some of the key chemical compounds that astrophysicists and astrobiologists look for, and explain why these compounds are so important. 🎡 🎹
- C. When scientists use this technique, do they consider light to be a wave, a particle, or both? Explain.







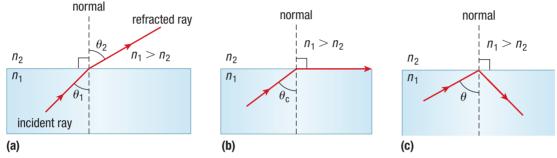
# **Total Internal Reflection**

The angle of refraction depends on the angle of incidence; however, for some values of the angle of incidence, there is no refracted ray. Consider a light ray that travels in medium 1 and meets the boundary of medium 2, as shown in **Figure 11**. If the index of refraction  $n_1$  is greater than  $n_2$ , the angle of refraction  $\theta_2$  is greater than  $\theta_1$ . Light will refract away from the normal. As the incident angle  $\theta_1$  increases, the refracted angle  $\theta_2$  also increases.

When  $\theta_2$  reaches 90°, the refracted ray will travel parallel to the surface (Figure 11(b)). The value of the angle of incidence at which this occurs is called the **critical angle**,  $\theta_c$ . If the angle of incidence  $\theta_1$  increases beyond the critical angle, there is no refracted ray, and the incident light ray is totally reflected at the boundary (Figure 11(c)). This effect, called **total internal reflection**, can occur when light encounters a boundary between an initial medium with a higher index of refraction and a second medium with a lower index of refraction.

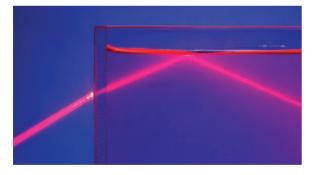
**critical angle** the smallest angle of incidence at which a light ray passing from one medium to another less refractive medium can be totally reflected from the boundary between the two

total internal reflection an effect that occurs when light encounters a boundary between a medium with a higher index of refraction and one with a lower index of refraction



**Figure 11** (a) Different angles of incidence cause different angles of refraction. (b) At the critical angle, the angle of refraction is  $90^{\circ}$ . (c) When the angle of incidence is greater than the critical angle, no light exits medium 1.

**Figure 12** shows an example of total internal reflection. Here, light from a laser enters the side of a tank of water and travels through it. At the air–water boundary the angle of incidence exceeds the critical angle, so there is no refracted ray. The ray is reflected as though it had hit a surface like a mirror. The incident and reflected rays obey the law of reflection:  $\theta_i = \theta_r$ .



**Figure 12** At  $\theta_i > \theta_c$ , the laser light totally reflects within the water.

By applying Snell's law, you can determine the critical angle since  $\theta_1 = \theta_c$  and  $\theta_2 = 90^\circ$ . From Snell's law,

 $n_1 \sin \theta_{\rm c} = n_2 \sin 90^{\circ} = n_2$ 

Therefore,

$$\sin\theta_{\rm c}=\frac{n_2}{n_1}$$

UNIT TASK BOOKMARK

You can apply what you have learned about total internal reflection and Snell's law to the Unit Task on page 556.

Since the sine of an angle can never be greater than 1, there can be no critical angle unless  $n_1$  is greater than  $n_2$ . This means that total internal reflection only occurs when light travels through a medium and encounters a boundary of another medium with a lower index of refraction,  $n_1 > n_2$ .

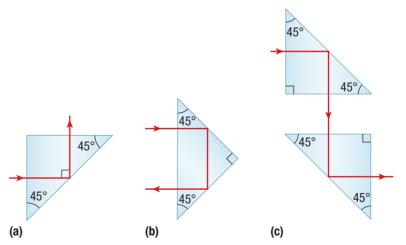
Total internal reflection can occur even when the media appear to be otherwise transparent. If you look carefully at a glass of water, you can observe that at certain angles you do not see the table on which the glass sits. Instead, you see a reflection of the inside of the glass. However, if you pick up the glass you will immediately see your fingers.

Total internal reflection has many practical applications. When combined with glass or prisms, total internal reflection can change the direction of a light ray, as shown in **Figure 13**.



Figure 13 Total internal reflection can be used to make devices that change the direction of a light beam.

Prisms that are isosceles right-angled triangles are often used to control the direction of light. As **Figure 14(a)** shows, light enters from a side, is turned 90°, and exits out the other side. **Figure 14(b)** shows how to use the two short sides of a prism to make the light entering the base take two 90° turns before exiting back out the base. **Figure 14(c)** shows an application in which two prisms are used together to alter the path of a ray of light, as in the periscope of a submarine.





**Figure 15** Retro-reflectors left on the Moon in a series of Apollo missions reflect laser light back toward Earth.

Figure 14 Various configurations of prisms can control the direction of light rays.

Retro-reflectors left on the surface of the Moon by astronauts during some of the Apollo missions are an interesting application of the reflection of light. These retro-reflectors, shown in **Figure 15**, contain hundreds of prisms and are oriented to reflect laser light back to Earth. Much like the reflector on a bicycle, these retro-reflectors have the property of always reflecting an incoming light ray back in the direction it came from. When scientists aim a laser at the retro-reflector on the Moon, the reflection enables them to measure the Earth–Moon distance with a precision of about 15 cm.

You can also make use of total internal reflection and dispersion to achieve a filterlike effect using a semicircular piece of glass, as shown in **Figure 16**. If a ray of light travels toward the centre of the semi-circle, at point P, it will strike the round edge along the normal and not refract. Imagine now rotating the piece of glass to make larger angles of incidence. Because the index of refraction depends on the wavelength of light, the critical angle for the glass–air interface also depends on the wavelength of light. In fact, the critical angle for blue light is less than the critical angle for red light. Therefore, as the angle increases, the light emerging along the flat edge of the glass becomes increasingly red in colour as other wavelengths totally reflect.

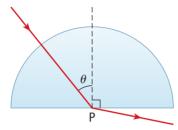


Figure 16 Total internal reflection combined with dispersion can turn a semicircular piece of glass into a red filter.

# **Fibre Optics**

**Fibre optics** is a technology that uses flexible strands of glass, called optical fibres, to conduct and transmit light. Recall that light undergoes partial reflection and refraction when it encounters an interface between materials with different indices of refraction. Although an optical fibre is cylindrical, its diameter is large compared to the wavelength of light. Consequently, the reflection of light within the optical fibre is similar to reflection from a flat interface. Even if the fibre is curved, the light remains inside due to total internal reflection (**Figure 17**).

fibre optics a technology that uses glass or plastic wire (fibre) through which data are transmitted using internally reflected light impulses

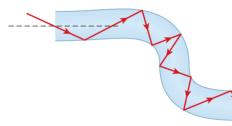


Figure 17 Light moves through the fibre optic cable by reflection.

The optical fibre in Figure 17 is a simplified illustration. All practical optical fibres consist of at least two different types of glass, as shown in **Figure 18**. The central core is surrounded by an outer layer called the cladding. The core and the cladding are both made of glass, but with different compositions and different indices of refraction. The index of refraction of the cladding is less than the core's index of refraction, enabling total internal reflection of light within the core.

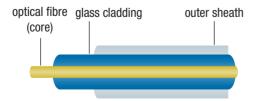


Figure 18 Fibre optic cable is surrounded by a layer of glass cladding as well as a protective outer sheath.

In an application, light from a laser enters the fibre at one end, such that the angle of incidence permits the light to always undergo total internal reflection at the boundary between the core and the cladding. Signals, such as a telephone signal, are carried from one end of the fibre to the other by pulses of laser light within the core. These fibres can be very long—many kilometres—and a single fibre can carry many more simultaneous signals than is possible with a conventional metal wire. Optical fibres can be quite flexible, and an optical transmission line can be constructed by bundling parallel fibres together.

The range of angles at which light can enter the fibre is called the acceptance cone (**Figure 19**). If incoming light is at an angle within this cone, then it will be transmitted through the fibre by total internal reflection. If incoming light is at an angle outside this cone, then it can exit out of the fibre core. The size of the acceptance cone can be adjusted. It would become larger if the critical angle for the core were made smaller. Using a material with a smaller index of refraction for the cladding material makes the difference between the indices of refraction for the core and cladding materials greater.

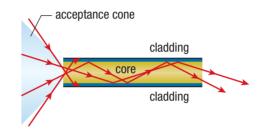


Figure 19 Light from within the acceptance cone travels inside the fibre optic core. Light from outside the acceptance cone can exit outside of the fibre core.

If a fibre optic cable bends too much at one point, an incident light ray can hit the side at an angle less than  $\theta_c$ . The possibility of total internal reflection is lost, and the light can exit the core fibre. If a light ray travelling inside the fibre meets the side wall of the fibre with angle of incidence  $\theta_1$ , there is a reflected ray with an angle of reflection equal to  $\theta_1$ . If, however, the fibre is bent to the point that  $\theta_1$  is less than the critical angle  $\theta_c$ , then total internal reflection is lost, the refracted ray in the air travels at an angle  $\theta_2$ , and there is a loss of light within the fibre.

Fibre optic cables can be made very small and flexible, so they have many uses for transporting signals and light through small or dangerous places. Doctors can study a patient's internal tissues using an instrument called an endoscope (**Figure 20**) which uses fibre optics. With an endoscope, a doctor can check a patient's digestive system for internal blockages, damaged tissues, stomach ulcers, and other medical issues without performing major surgery.

Fibre optic cables have transformed communications by allowing signals to travel quickly (at the speed of light) across long distances with relatively little loss of signal strength. Silica fibres, which are commonly used, have a loss rate of approximately 5 % to 10 % of the signal intensity per kilometre. There are some cables with losses of less than 5 %. However, over large distances technicians have to boost the signal along the way to take care of the signal intensity losses. Many communications companies in Ontario and across Canada depend on fibre optic technology for Internet and telephone services. Even transatlantic cables, which were once made of metal, now consist of fibre optic cables, including the CANTAT-3, which runs from Nova Scotia on the western end to Iceland, the Faroe Islands, England, Denmark, and Germany on the eastern end. **W** CAREER LINK



Figure 20 An endoscope uses fibre optic cables to allow doctors to explore this patient's wrist without invasive surgery.

The Sample Problem in this Tutorial models how to solve problems related to total internal reflection and the critical angle.

# **Sample Problem 1:** The Critical Angle for a Water–Air Boundary

- (a) Calculate the critical angle for light passing through water (index of refraction 1.33) into air (index of refraction 1.0003).
- (b) Describe what an underwater swimmer sees if she looks toward the surface at angles of  $40^\circ$ ,  $\theta_c$  from (a), and  $60^\circ$ relative to the normal.

# Solution

(a) **Given:**  $n_2 = 1.0003$  (index of refraction for air);  $n_1 = 1.33$ (index of refraction for water)

**Required:**  $\theta_c$ 

Analysis: 
$$\sin \theta_{\rm c} = \frac{n_2}{n_1}$$
  
 $\theta_{\rm c} = \sin^{-1} \left( \frac{n_2}{n_1} \right)$   
Solution:  $\theta_{\rm c} = \sin^{-1} \left( \frac{n_2}{n_1} \right)$ 

$$\theta_{\rm c} = 48.8^{\circ}$$

Statement: The critical angle for light passing through water into air is 48.8°.

# Practice

So

- 1. Replace the water in Sample Problem 1(a) with a transparent fluid that has a lower index of refraction. Describe the change to the critical angle at the liquid-air boundary.
- 2. Calculate the critical angle for light travelling through a layer of benzene floating on water at the benzene-water boundary. Use the indices of refraction in Table 1 on page 445. [ans: 62.5°]
- 3. Calculate the critical angle for light passing through a thin rod made from glass with an index of refraction of 1.40 when the rod is surrounded by air (index of refraction 1.0003). [20] [ans: 45.6°]
- 4. Diamonds, with a high index of refraction of 2.42, are known for their attractive sparkle under lights. Similar-looking materials, such as crown glass (index of refraction 1.52) and zircon (index of refraction 1.92), do not have the same sparkle (Figure 21). Calculate the critical angle between each material and air, and express how the result explains the sparkle. **T/I C A** [ans:  $\theta_{c,d} = 24.4^{\circ}$ ;  $\theta_{c,g} = 41.2^{\circ}$ ;  $\theta_{c,z} = 31.4^{\circ}$ ]

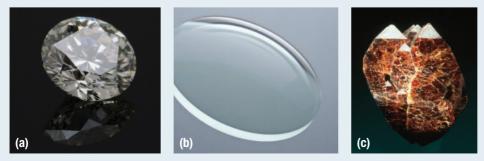


Figure 21 (a) Diamond has an index of refraction of 2.42. (b) Crown glass has an index of refraction of 1.52. (c) Zircon has an index of refraction of 1.92.

(b) The path of a light ray is reversible, so a light ray that reaches the swimmer's eve must travel the same path as a light rav that leaves the swimmer's eye. The 40° angle is less than the critical angle. A light ray directed from the swimmer's eve to the surface at an angle less than the critical angle will pass through the surface, so the swimmer sees light from outside the water when she looks at an angle of 40°.

A light ray directed from her eve to the surface at the critical angle will travel along the surface, so the swimmer sees light travelling along the surface when she looks at  $\theta_{\rm c}$ .

The 60° angle is greater than the critical angle. A light ray directed from her eye to the surface at an angle greater than the critical angle will reflect from the surface back into the water, so she sees light from underwater when she looks at 60°.

# 9.2 Review

# Summary

- Snell's law describes the relationship between the incident and refracted angles of a light ray and the indices of refraction of two media:  $n_1 \sin \theta_1 = n_2 \sin \theta_2$ .
- The index of refraction, *n*, of a medium is the ratio of the speed of light in a vacuum, *c*, to the speed of light in the medium, *v*:  $n = \frac{c}{v}$ .
- The index of refraction, n, of a medium is equal to the ratio of the wavelength of light in a vacuum, λ<sub>1</sub>, to the wavelength of light in the medium, λ<sub>2</sub>: n = λ<sub>1</sub>/λ<sub>2</sub>. For two different media, n<sub>1/2</sub> = λ<sub>2/λ1</sub>. The frequency of light is unchanged.
- Dispersion is the separation of a wave into its component parts according to a given characteristic, such as wavelength.
- When light passes from one medium to another, partial reflection and partial refraction can occur, and the wavelength changes based upon the index of refraction for the second medium.
- Total internal reflection occurs when light is completely reflected at a boundary between two media. Two conditions must be met: (1) the incident light must originate in the more optically dense medium, and (2) the angle of incidence must be greater than the critical angle.
- The critical angle can be calculated using  $\theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right)$ .

# Questions

- 1. Why is a beam of light said to be "bent" when it undergoes refraction?
- 2. Define "angle of deviation" in your own words.
- 3. Light from a laser has a wavelength of 630 nm in a vacuum. Calculate its wavelength in water (index of refraction is 1.33). 77
- The speed of light in a medium is measured to be 3.0 × 10<sup>8</sup> m/s. Calculate the index of refraction of the medium. <sup>™</sup>
- 5. A piece of glass (n = 1.47) is coated with a thin film of water, with light incident from below. Calculate the angle of refraction of the final outgoing ray when the angle of incidence is 30.0°.
- 6. Light travels through an optical fibre (n = 1.44) to air. The angle of incidence of light in the fibre is 30.0°. Calculate the angle of refraction outside the fibre. 111
- Suppose that the angle of incidence of a laser beam in water aimed at the surface is 50.0°. Use Snell's law to calculate the angle of refraction.
- 8. (a) Calculate the critical angle for light travelling through glass (n = 1.65) and water (n = 1.33).
  - (b) Does the light start in the glass or in the water?

- Light travels from air into a transparent material that has an index of refraction of 1.30. The angle of refraction is 45°.
  - (a) Calculate the angle of incidence.
  - (b) Determine the critical angle for total internal reflection to occur in the transparent material.
- 10. A ray of light passes from water (n = 1.33) into carbon disulfide (n = 1.63).
  - (a) Calculate the angle of refraction when the angle of incidence is 30.0°.
  - (b) Is it possible for total internal reflection to occur in this case? Explain your answer.
- 11. Give an example where total internal reflection is used in medicine.
- 12. Research transatlantic fibre optic cables. 🌒 🎹 🖸
  - (a) What plans for new submarine cables are currently being developed?
  - (b) Discuss some of the pros and cons of transatlantic submarine cables in a short paragraph.
- Over long distances, the intensity of the signal carried by light in a fibre optic cable is reduced. Research what causes the signal to be reduced. Provide one reason for why this happens. Image Market A.

