Magnetic Force on Moving Charges

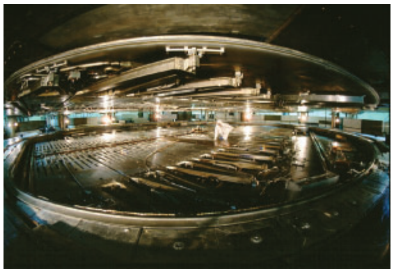

Figure 1 The 18 m cyclotron at TRIUMF is the largest in the world. The magnetic force of electromagnets guides protons along an expanding spiral path.

Each second, the cyclotron particle accelerator at TRIUMF national laboratory in Vancouver (**Figure 1**) accelerates trillions of protons to speeds of 224 000 km/s. At this speed, a proton travelling in a straight line would reach the Moon in less than 2 s. The magnetic force of a series of large electromagnets directs the protons in the cyclotron along a spiral path with an increasing radius. These powerful magnets produce a magnetic field over 10 000 times as strong as Earth's magnetic field. WEB LINK

The unit of magnetic field strength in the SI system is the **tesla** (T). A tesla is defined in terms of other SI units by this conversion:

$$1\ \text{T} = 1\ \frac{\text{kg}}{\text{C}\cdot\text{s}}$$

tesla the SI unit of measure for describing the strength of a magnetic field; $1\ \text{T} = 1\ \frac{\text{kg}}{\text{C}\cdot\text{s}}$

The magnetic field close to a refrigerator magnet has a magnitude of about 0.001 T. The magnetic field strength near Earth's surface is approximately 50 μT (5×10^{-5} T). An MRI unit, which you read about at the beginning of this chapter, can have a magnetic field strength of 7 T.

We can observe a magnetic field exerting a force on moving charges using a cathode ray tube and a magnet. In **Figure 2(a)**, a beam of electrons moves across the cathode ray tube. Bringing a magnet near the cathode ray tube (**Figure 2(b)**) shows how a magnetic force deflects the electron beam.

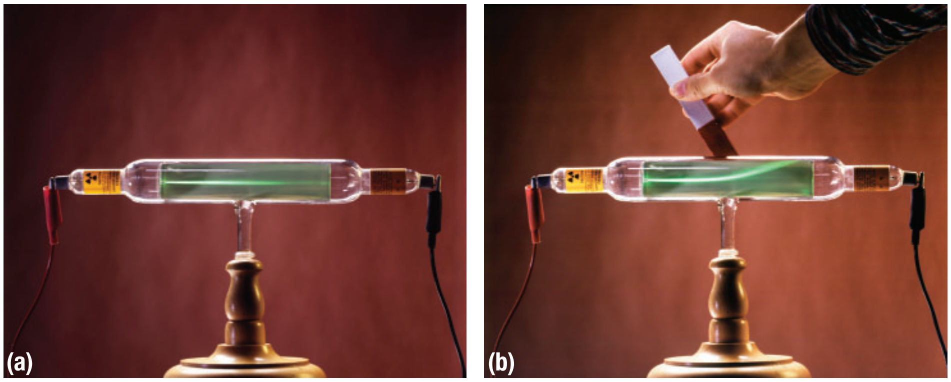

Figure 2 (a) Electrons move straight through the cathode ray tube. (b) When a bar magnet is placed near the electrons, the magnetic force deflects their path.

A Charge in a Magnetic Field

The eighteenth-century French scientist André-Marie Ampère demonstrated that two current-carrying parallel wires could be made to attract or repel one another depending on the direction of the current in each wire. He also showed that a magnetic field could be made to force a current-carrying conductor to move. Ampère's discovery applies to more than just electrons flowing in a wire. Magnetic forces act on all types of electric charges, such as electrons, protons, and ions. An important property of the magnetic force is that it depends on the velocity of the charge. The magnetic force is non-zero only if a charge is in motion. This property of the magnetic force is not shared by the gravitational force and the electric force. Those forces are both independent of velocity.

Figure 3 shows a particle with positive charge q moving with velocity $\vec{v}$. The direction of $\vec{v}$ is south, the magnetic field $\vec{B}$ points east, and the magnetic force on the charge is acting vertically upward. Given a magnetic field $\vec{B}$ and a charge q moving with velocity $\vec{v}$, the magnetic force exerted by $\vec{B}$ on q is

$$F_M = qvB\sin\theta$$

where θ is the angle between $\vec{v}$ and $\vec{B}$. In Figure 3, this angle is 90°, so in this case, $\sin\theta$ is equal to 1.

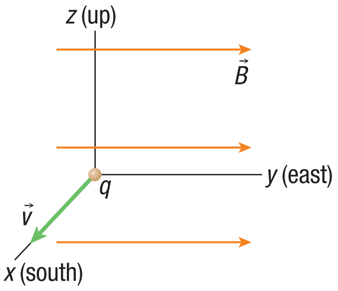

Figure 3

The magnetic force, just like any other force, is a vector quantity. Notice that the equation provides only the magnitude of the magnetic force. **Figure 4** shows how we can also determine the direction of the force using the **right-hand rule for a moving charge in a magnetic field.**

Right-Hand Rule for a Moving Charge in a Magnetic Field

If you point your right thumb in the direction of the velocity of the charge ($\vec{v}$), and your straight fingers in the direction of the magnetic field ($\vec{B}$), then your palm will point in the direction of the resulting magnetic force ($\vec{F}_M$).

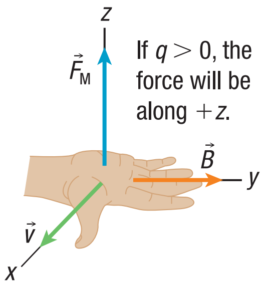

Figure 4 You can use the right-hand rule to determine the direction of the magnetic force on a moving charged particle.

Applying the right-hand rule to Figure 3, the direction of the magnetic force is vertically upward along the $+z$-axis. Using the right-hand rule gives the direction of the force when q is positive in the equation for magnetic force. The magnetic force is up for a positive charge ($q > 0$) with $\vec{v}$ and $\vec{B}$ directed as shown in Figure 4. The force for a negative charge, such as an electron, is down.

Investigation **8.2.1**

Observing the Magnetic Force on a Moving Charge (page 412)

You have learned how the magnetic force on a moving electric charge relates to the magnitude of the charge, its velocity, and the magnetic field. Now perform Investigation 8.2.1 to apply this relationship.

The magnetic force has several distinctive characteristics. One already mentioned is that $\vec{F}_M$ depends on velocity. Another characteristic different from gravitational and electric forces is that the direction of $\vec{F}_M$ is always perpendicular to both the magnetic field and the particle's velocity. The direction in which the force acts and the direction of the field are always parallel for electric and gravitational fields.

In the following Tutorial, you will calculate the magnitude of a magnetic force and use the right-hand rule to determine the direction of the force.

Tutorial 1 Calculating the Magnetic Force on Moving Charges

A charged particle moving through a magnetic field experiences a magnetic force. This Tutorial shows how to calculate the magnetic force for a negative charge and for a positive charge, and determine the direction of deflection of a charge by Earth's magnetic field.

Sample Problem 1: Magnetic Force on a Negative Charge in Motion

The electron in **Figure 5** moves at a speed of 54 m/s through a magnetic field with a strength of 1.2 T. The angle between the electron's velocity vector and the magnetic field is 90°. Assume a value for the electron's charge of $q = -e = -1.60 \times 10^{-19}$ C.

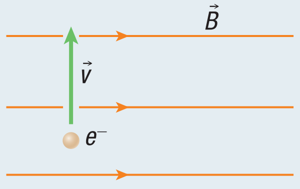

Figure 5

(a) What is the magnitude of the magnetic force on the electron? Express your answer in newtons (N), using $1\ \text{T} = 1\ \dfrac{\text{kg}}{\text{C}\cdot\text{s}}$.

(b) Use the right-hand rule to determine the direction of the magnetic force.

(c) Calculate the gravitational force on the electron. The mass of an electron is 9.11×10^{-31} kg.

(d) What is the ratio of the gravitational force on the electron to the magnetic force on the electron?

Solution

(a) **Given:** $q = 1.60 \times 10^{-19}$ C; $v = 54$ m/s; $B = 1.2$ T; $\theta = 90°$

Required: F_M

Analysis: $F_M = qvB\sin\theta$; use a positive value for q since we are calculating a magnitude.

Solution:

$$F_M = qvB\sin\theta$$
$$= (1.60 \times 10^{-19}\ \cancel{\text{C}})\left(54\ \frac{\text{m}}{\text{s}}\right)\left(1.2\ \frac{\text{kg}}{\cancel{\text{C}}\cdot\text{s}}\right)(\sin 90°)$$
$$= 1.037 \times 10^{-17}\ \text{kg}\cdot\text{m/s}^2$$
$$F_M = 1.037 \times 10^{-17}\ \text{N (two extra digits carried)}$$

Statement: The magnitude of the magnetic force on the electron is 1.0×10^{-17} N.

(b) Using the right-hand rule for a positive charge, point the fingers of your right hand in the direction of the magnetic field. Then point your thumb in the direction of the velocity of the particle. Your palm points in the direction of the magnetic force for a positive charge. In this case, the magnetic force for a positive charge points into the page, but the magnetic force for an electron is in the opposite direction of the magnetic force for a positive charge and points out of the page.

(c) **Given:** $m_e = 9.11 \times 10^{-31}$ kg

Required: F_g

Analysis: The gravitational force on the electron is $m_e g$, the product of the mass of the electron and the gravitational acceleration.

Solution:

$$F_g = m_e g$$
$$= (9.11 \times 10^{-31}\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)$$
$$F_g = 8.928 \times 10^{-30}\ \text{N (two extra digits carried)}$$

Statement: The gravitational force on the electron is 8.9×10^{-30} N.

(d) **Given:** $F_g = 8.928 \times 10^{-30}$ N; $F_M = 1.037 \times 10^{-17}$ N

Required: $\dfrac{F_g}{F_M}$

Analysis: Divide the gravitational force by the magnetic force.

Solution:

$$\frac{F_g}{F_M} = \frac{8.928 \times 10^{-30}\ \cancel{\text{N}}}{1.037 \times 10^{-17}\ \cancel{\text{N}}}$$
$$\frac{F_g}{F_M} = 8.6 \times 10^{-13}$$

Statement: The ratio of the gravitational force to the magnetic force on the electron is 8.6×10^{-13} : 1.

Sample Problem 2: Magnetic Force on a Positive Charge in Motion

A proton is moving along the x-axis at a speed of 78 m/s. It enters a magnetic field of strength 2.7 T. The angle between the proton's velocity vector and the magnetic field is 38° (**Figure 6**). The mass of a proton is 1.67×10^{-27} kg.

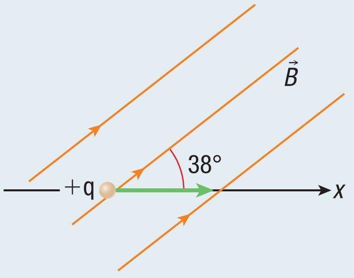

Figure 6

(a) Calculate the initial magnitude and the direction of the magnetic force on the proton.

(b) Determine the proton's initial acceleration.

Solution

(a) **Given:** $q = 1.60 \times 10^{-19}$ C; $v = 78$ m/s; $B = 2.7$ T; $\theta = 38°$

Required: magnitude and direction of magnetic force, F_M

Analysis: Use $F_M = qvB\sin\theta$ to calculate the magnitude of the force. Use the right-hand rule to determine the direction of the force.

Solution:

$$F_M = qvB\sin\theta$$
$$= (1.60 \times 10^{-19}\ \text{C})\left(78\ \frac{\text{m}}{\text{s}}\right)\left(2.7\ \frac{\text{kg}}{\text{C}\cdot\text{s}}\right)(\sin 38°)$$
$$= 2.075 \times 10^{-17}\ \text{kg}\cdot\text{m/s}^2$$
$$F_M = 2.075 \times 10^{-17}\ \text{N (two extra digits carried)}$$

Point the fingers of your right hand in the direction of the magnetic field, and direct your thumb along the velocity vector of the proton. Your palm is facing in the direction of the magnetic force—out of the page.

Statement: The magnitude of the magnetic force on the proton is 2.1×10^{-17} N. The direction of the magnetic force is out of the page.

(b) **Given:** $F_M = 2.075 \times 10^{-17}$ N; $m = 1.67 \times 10^{-27}$ kg

Required: acceleration, a

Analysis: $a = \frac{F_M}{m}$

Solution:

$$a = \frac{F_M}{m}$$
$$= \frac{2.075 \times 10^{-17}\ \text{kg}\frac{\text{m}}{\text{s}^2}}{1.67 \times 10^{-27}\ \text{kg}}$$
$$a = 1.2 \times 10^{10}\ \text{m/s}^2$$

Statement: The proton's initial acceleration is 1.2×10^{10} m/s^2.

Sample Problem 3: Determining the Effect of Earth's Magnetic Field on the Direction of Lightning Strikes

During a thunderstorm, positive charge accumulates near the top of a cloud, and negative charge accumulates near the bottom of a cloud (**Figure 7**). When the charge buildup is strong enough, negative charge moves rapidly from the cloud to the ground as a lightning strike. Assume the charge velocity vector is perpendicular to the ground, and Earth's magnetic field is horizontal, directed north. Determine the direction of the deflection of this charge by Earth's magnetic field.

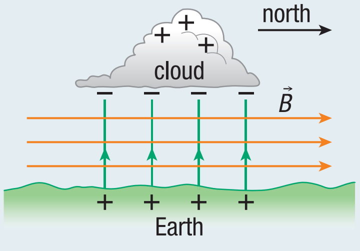

Figure 7

Given: Negative electric charge moves downward. Earth's magnetic field is directed to the north.

Required: direction of deflection of charge

Analysis: The deflection is in the direction of the magnetic force due to Earth's magnetic field. Apply the right-hand rule for a moving charge in a magnetic field to determine the direction of the magnetic force.

Solution: Point the fingers of your right hand in the direction of the velocity of the positive charge, from the ground toward the cloud. Next, point your fingers in the direction of the magnetic field, north. Your palm points in the direction of the resulting magnetic force, into the page, which is west.

Statement: When lightning strikes, Earth's magnetic field deflects the charge toward the west.

Practice

1. A proton with a mass of 1.67×10^{-27} kg is moving horizontally eastward at 9.4×10^4 m/s as it enters a magnetic field of strength 1.8 T. The field is directed vertically upward (**Figure 8**). T/I

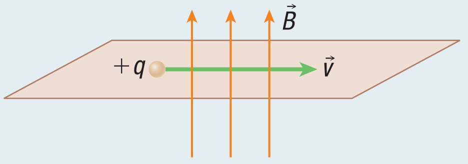

Figure 8

(a) Calculate the magnitude and direction of the magnetic force on the proton. [ans: 2.7×10^{-14} N [S]]

(b) Determine the gravitational force on the proton. [ans: 1.6×10^{-26} N]

(c) Determine the ratio of the gravitational force to the magnetic force on the proton. [ans: 6.0×10^{-13} : 1]

2. Determine the magnitude and direction of the magnetic field for an electron moving upward through a uniform magnetic field with a speed of 3.5×10^5 m/s. The particle experiences a maximum magnetic force of 7.5×10^{-14} N [right]. T/I [ans: 1.3 T [horizontal, into the page]]

3. The cyclotron at TRIUMF accelerates protons from a central injection point outward along an increasing spiral path (**Figure 9**). The protons reach a speed of 2.24×10^8 m/s. Magnets then direct the protons out of the spiral along a linear path toward a target. T/I A

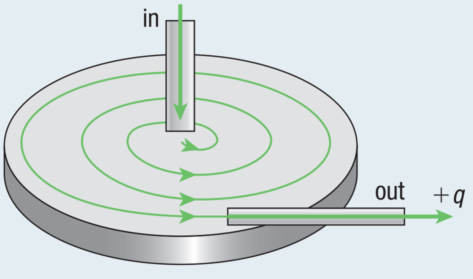

Figure 9

(a) Suppose a proton passes through a magnetic field that has strength 0.56 T directed vertically upward at the moment just *before* the proton leaves the spiral. What are the magnitude and direction of the magnetic force on the proton at this moment? [ans: 2.0×10^{-11} N [horizontally outward from the spiral]]

(b) What is the magnitude of the magnetic force acting on the proton at the moment just *after* it leaves the spiral if the only magnetic field around it is Earth's field of 5.5×10^{-5} T? [ans: 2.0×10^{-15} N]

4. An electron moving at a velocity of 6.7×10^6 m/s [E] enters a magnetic field with a magnitude of 2.3 T directed at an angle of 47° to the direction of motion and upward in the vertical plane. T/I

(a) What are the magnitude and direction of the magnetic force on the electron? [ans: 1.8×10^{-12} N [N]]

(b) What is the electron's acceleration? Its mass is 9.11×10^{-31} kg. [ans: 2.0×10^{18} m/s^2]

(c) What would the acceleration be if the charged particle were a proton of mass 1.67×10^{-27} kg? [ans: 1.1×10^{15} m/s^2]

8.2 Review

Summary

- The formula for calculating the magnitude of the magnetic force on a moving charge is $F_M = qvB \sin \theta$, where θ is the angle between $\vec{v}$ and $\vec{B}$.
- You can use the right-hand rule to determine the direction of the magnetic force on a moving particle. First, point your right thumb in the direction of $\vec{v}$. Then, point your fingers in the direction of $\vec{B}$. If the charge is positive, your palm indicates the direction of a force on the charge. The direction of force is in the opposite direction for a negative charge.

Questions

1. You have now learned three forms of the right-hand rule. Explain the use of each. K/U
2. The charged particle in **Figure 10** moves east. It experiences a force that acts out of the page. Does the particle have a positive charge or a negative charge? K/U

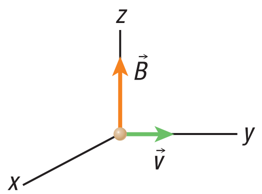

Figure 10

3. A charged particle moving along the $+y$-axis passes through a uniform magnetic field oriented in the $+z$ direction. A magnetic force acts on the particle in the $-x$ direction. Does the particle have positive charge or negative charge? How would the force change if the charge of the particle were tripled but the velocity were halved? K/U
4. A proton moves at a speed of 1.4×10^3 m/s in a direction perpendicular to a magnetic field with a magnitude of 0.85 T. T/I
   (a) Calculate the magnitude of the magnetic force on the proton.
   (b) What would the magnitude of the magnetic force be if the particle in (a) were an electron?
5. An electron with speed 235 m/s moves through a magnetic field of 2.8 T. The magnitude of the force on the electron is 5.7×10^{-17} N. What angle does the electron's velocity vector make with $\vec{B}$? T/I
6. The particle in **Figure 11** has a positive charge. Its initial velocity is to the left when it passes into a magnetic field directed into the plane of the page. In which direction is the particle deflected? K/U

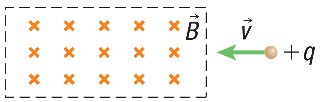

Figure 11

7. A particle of charge $q = 6.4\ \mu$C is moving at an angle of 27° with respect to the y-axis and a speed of 170 m/s. It passes through a uniform magnetic field of magnitude 0.85 T parallel to the y-axis in the x–y plane. T/I
   (a) Calculate the magnitude of the magnetic force on the particle.
   (b) Determine the direction of the magnetic force.
   (c) What would the magnetic force be on an identical particle travelling at the same speed along the y-axis? Explain.
8. The particle in **Figure 12** has a negative charge. Its velocity vector lies in the x–y plane and makes an angle of 75° with the y-axis. If the magnetic field is along the $+x$ direction, what is the direction of the magnetic force on the particle? T/I

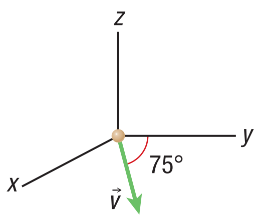

Figure 12

9. A particle of charge $q = -7.9\ \mu$C has a speed of 580 m/s. The particle lies in the x–y plane and travels at an angle of 55° with respect to the x-axis. There is a uniform magnetic field of magnitude 1.3 T parallel to the y-axis. What are the magnitude and direction of the magnetic force on the particle? T/I
10. An alpha particle is a helium nucleus that consists of two protons bound with two neutrons; its mass is 6.644×10^{-27} kg. An alpha particle moving through a perpendicular magnetic field of 1.4 T experiences an acceleration of 2.4×10^3 m/s^2. T/I A
    (a) What is the force on the alpha particle?
    (b) What is the alpha particle's speed?