Electric Potential and Electric Potential Energy Due to Point Charges

7.5

In the first section of this chapter, you saw how a Van de Graaff generator in a science museum causes the hair of anyone in contact with the device to stand on end. At that point, the discussion dealt simply with the properties of electric charge, and how the like charges (electrons) on individual hairs caused the hairs to repel each other and spread out in all directions.

We can look at this phenomenon in another way: the charge on the Van de Graaff generator creates a high electric potential on the conducting sphere of the generator. This potential provides an electric field that exerts a force on charged particles near the generator. This force, in turn, accelerates these particles. In fact, when the Van de Graaff generator was invented, it was an early type of particle accelerator. As a particle accelerator, however, the Van de Graaff generator has limitations. For one thing, it can only accelerate charges like those on the sphere. Also, if the potential on the sphere becomes too high with respect to the generator's surroundings, the air around the generator will ionize. This causes large sparks to jump between the sphere and objects at lower electric potentials, resulting in artificial lightning (**Figure 1**).

Figure 1 The charge on the Van de Graaff generator has a large electric potential difference with its surroundings. This can cause large sparks to appear between the generator and its surroundings.

At a distance, the conducting sphere of a Van de Graaff generator resembles a point charge. In this section, you will learn about electric potentials, electric potential energies, and electric potential differences for point charges and their surroundings.

Electric Potential Due to a Point Charge

Earlier you learned about the electric properties of a point charge. This simple example is useful in many situations. In Section 7.4, we defined electric potential V in terms of a test charge q and the electric potential energy E_E. The electric potential energy, in turn, depends on the position d of the test charge in an electric field $\vec{\varepsilon}$. If the charge producing the electric field has the same sign as the test charge q, the work done by the electric field on q is negative. This means that $\Delta E_E > 0$ for $\Delta d > 0$, and the sign of potential energy (and thus potential) is reversed from that of the uniform electric field.

For a point charge q, the magnitude of the electric field is

$$\varepsilon = \frac{kq}{r^2}$$

Setting the location in the field equal to the radius of the field ($\Delta d = r$), the **electric potential due to a point charge** is inversely proportional to the distance from the charge and directly proportional to the amount of charge. So, for a point charge q producing an electric field, at a distance r from the charge, the electric potential is

electric potential due to a point charge the electric potential is inversely proportional to the distance from the charge and proportional to the amount of charge producing the field

$$V = \frac{kq\Delta d}{r^2}$$

$$V = \frac{kq}{r}$$

These results reinforce what you learned about potential energy and the behaviour of like and unlike charges.

Suppose you place a source charge ($q > 0$) at the origin of the coordinate system in **Figure 2**. The charge produces a potential given by $\frac{kq}{r}$. Then you place a positive test charge nearby. Both the source and the test charge are positive, so they repel one another, and the test charge experiences a force that carries it "downhill" and away from the origin along the blue potential curve in Figure 2. When the source charge is negative, the potential is described by the lower, red curve in Figure 2. The source charge is now negative, so the attractive force carries a positive test charge along the red potential curve toward the origin.

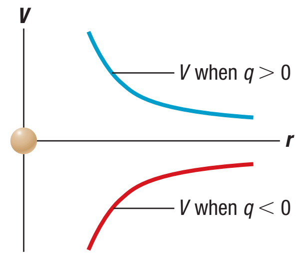

Figure 2 Electric potential near a point charge. The blue curve is for a positive point charge, and the red curve is for a negative point charge.

Only changes in potential energy are important, so you must always be clear about choosing the reference point for potential energy. Electric potential is proportional to electric potential energy, so you must also pay attention to the reference point for electric potential. For example, when dealing with a point charge, follow the standard convention by choosing $V = 0$ at an infinite distance from the source charge. Infinite distance represents a distance so far away that any potential produced by a charge is negligible and for all intents and purposes equal to zero. In many other problems, choose Earth as $V = 0$ because Earth conducts charge well. Put another way, the electric ground is the point where $V = 0$.

For two or more charges, use the superposition principle. Remember that electric potentials of negative charges are negative, and electric potentials of positive charges are positive. However, because electric potential is not a vector, the total electric potential of several point charges equals the algebraic sum of the electric potentials resulting from each of the individual charges and their specific distances from the chosen location.

Electric Potential Energy of Two Point Charges

Now consider the electric potential energy of a pair of charges. The electric potential energy of a pair of charges is the potential energy possessed by each charge in the pair because of its position. For a pair of charges, this potential energy is inversely proportional to the distance between the charges and directly proportional to the product of the two charges. For a pair of charges q_1 and q_2 separated by a distance r, use the equation for the electric field for the point charge q_2 and the definition of electric potential energy for the point charge q_1, in terms of V, to determine the potential energy for the two charges:

$$\begin{aligned} E_E &= q_1 V \\ &= q_1 \frac{kq_2}{r} \\ E_E &= \frac{kq_1q_2}{r} \end{aligned}$$

The electric potential energy of a pair of charges is then given by

$$\boxed{E_E = \frac{kq_1q_2}{r}}$$

This is qualitatively similar to the equation for Coulomb's law, except that E_E varies as $\frac{1}{r}$, not $\frac{1}{r^2}$. If the charges are initially separated by a distance r_i and then brought together to a final separation r_f, the change in the potential energy is

$$\Delta E_E = E_{Ef} - E_{Ei}$$
$$\Delta E_E = \frac{kq_1q_2}{r_f} - \frac{kq_1q_2}{r_i}$$

Note that only changes in potential energy matter, and that E_E approaches zero when the two charges are very far apart. The blue curve in **Figure 3** shows the general behaviour of the electric potential energy for two like charges. In this case, E_E is positive and increases as the charges are brought together. If the charges have opposite signs, the electric force is attractive (negative), and the potential energy is then also negative, as shown by the red curve in Figure 3. The following Tutorial will demonstrate how to solve problems involving electric potential and electric potential energy.

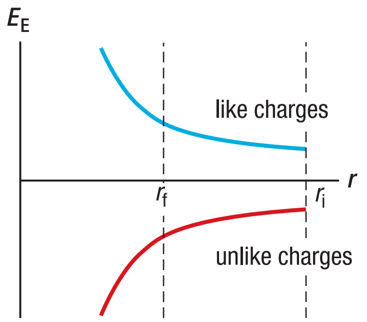

Figure 3 Electric potential energy as a function of the separation r between two charges q_1 and q_2. The blue curve is for a repulsive force, and the red curve is for an attractive force.

Tutorial 1 Electric Potential and Electric Potential Energy

Sample Problem 1 models how to calculate the total electric potential of two point charges and the work required to move a third charge into this potential. In Sample Problem 2, we will use the electric potential energy and conservation of energy to calculate the speed of a charge at a given location. Finally, in Sample Problem 3, we show how to calculate the minimum particle separation by using electric potential energy, conservation of energy, and conservation of momentum.

Sample Problem 1: Calculating the Electric Potential

A point charge with a charge of 4.00×10^{-8} C is 4.00 m due west from a second point charge with a charge of -1.00×10^{-7} C.

(a) Calculate the total electric potential due to these charges at a point P, 4.00 m due north of the first charge.

(b) Calculate the work required to bring a third point charge with a charge of 2.0×10^{-9} C from infinity to point P.

Solution

(a) **Given:** $q_1 = 4.00 \times 10^{-8}$ C; $q_2 = -1.00 \times 10^{-7}$ C; $r_{12} = 4.00$ m; $r_{1P} = 4.00$ m; $k = 8.99 \times 10^9$ N·m²/C²

Required: V_T

Analysis: The total potential at P is the sum of the potentials due to the two individual charges q_1 and q_2. We can use the equation $V = \frac{kq}{r}$ to calculate the potential due to each charge and the Pythagorean theorem to calculate the distance of point P from q_2: $r_{2P} = \sqrt{r_{12}^2 + r_{1P}^2}$.

Solution: V_T is the sum of the potentials due to the individual charges q_1 and q_2. Calculate the potential V_1 at P due to q_1.

$$V_1 = \frac{kq_1}{r_{1P}}$$
$$= \frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^{\cancel{2}}}{\text{C}^{\cancel{2}}}\right)(+4.00 \times 10^{-8}\,\cancel{\text{C}})}{4.00\,\cancel{\text{m}}}$$
$$= 8.99 \times 10^1\ \text{N}\cdot\text{m/C}$$
$$V_1 = 8.99 \times 10^1\ \text{J/C}$$

To calculate the potential V_2 at P due to q_2, first calculate the distance r_{2P} between P and q_2.

$$r_{2P} = \sqrt{r_{12}^2 + r_{1P}^2}$$
$$= \sqrt{(4.00\,\text{m})^2 + (4.00\,\text{m})^2}$$
$$r_{2P} = 5.6569\,\text{m (two extra digits carried)}$$

Then calculate V_2.

$$V_2 = \frac{kq_2}{r_{2P}}$$
$$= \frac{\left(8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^{\cancel{2}}}{\text{C}^{\cancel{2}}}\right)(-1.00 \times 10^{-7}\,\cancel{\text{C}})}{5.6569\,\cancel{\text{m}}}$$
$$= -1.5892 \times 10^2\ \text{N}\cdot\text{m/C}$$
$$V_2 = -1.5892 \times 10^2\ \text{J/C (two extra digits carried)}$$

Now calculate the total potential V_T.

$$V_T = V_1 + V_2$$
$$= 8.99 \times 10^1\,\text{J/C} + (-1.5892 \times 10^2\,\text{J/C})$$
$$V_T = -69.02\,\text{J/C (one extra digit carried)}$$

Statement: The total electric potential at point P is -69.0 J/C.

(b) **Given:** $V_T = -69.02$ J/C; $q_3 = 2.0 \times 10^{-9}$ C

Required: work required to move q_3 from $r = \infty$ to P

Analysis: Use the definition of electric potential energy. The work done on q_3 is $W = q_3(V_\infty - V_T)$. At infinity, the electric potential is zero, so $W = -q_3V_T$.

Solution: $W = -q_3V_T$

$$= -(2.0 \times 10^{-9}\,\cancel{\text{C}})\left(-69.02\,\frac{\text{J}}{\cancel{\text{C}}}\right)$$
$$W = 1.4 \times 10^{-7}\,\text{J}$$

Statement: The work done on the third point charge as it is brought from infinity to point P is 1.4×10^{-7} J. The positive value of W indicates that the electric field does work on the charge to increase the charge's kinetic energy as it approaches P.

Sample Problem 2: Electric Potential Energy and Dynamics

A point charge q_1 with charge 2.0×10^{-6} C is initially at rest at a distance of 0.25 m from a second charge q_2 with charge 8.0×10^{-6} C and mass 4.0×10^{-9} kg (**Figure 4**). Both charges are positive. Charge q_1 remains fixed at the origin, whereas q_2 travels to the right upon release. Determine the speed of charge q_2 when it reaches a distance of 0.50 m from q_1.

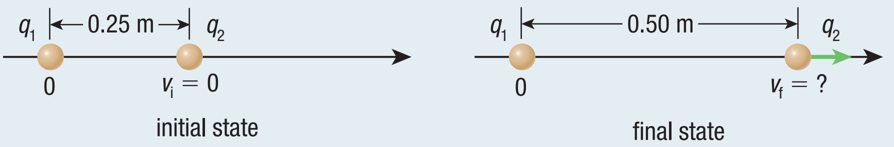

Figure 4

Given: $q_1 = 2.0 \times 10^{-6}$ C; $q_2 = 8.0 \times 10^{-6}$ C; $m = 4.0 \times 10^{-9}$ kg; $r_1 = 0.25$ m; $r_2 = 0.50$ m

Required: v_f

Analysis: No external forces are applied to the system, so energy must be conserved. We can apply the law of conservation of energy, using electric potential energy: $E_{E_i} + E_{k_i} = E_{E_f} + E_{k_f}$. Charge q_1 remains fixed at all times, so E_k equals the kinetic energy of charge q_2, and E_E equals the electric potential energy of the system of two charges. Charge q_2 is initially at rest, so the initial kinetic energy of q_2 is zero. Note that r_1 and r_2 are related by $r_2 = 2r_1$. $E_{E_i} = \dfrac{kq_1q_2}{r_1}$; $E_{E_f} = \dfrac{kq_1q_2}{2r_1}$; $E_{k_i} = \dfrac{1}{2}mv_i^2 = 0$; $E_{k_f} = \dfrac{1}{2}mv_f^2$

Solution:

$$E_{E_i} + E_{k_i} = E_{E_f} + E_{k_f}$$

$$\frac{kq_1q_2}{r_1} + 0 = \frac{kq_1q_2}{2r_1} + \frac{1}{2}mv_f^2$$

$$\frac{1}{2}mv_f^2 = \frac{kq_1q_2}{2r_1}$$

$$v_f = \sqrt{\frac{kq_1q_2}{mr_1}}$$

$$= \sqrt{\frac{\left(8.99 \times 10^9 \dfrac{\cancel{\text{kg}} \cdot \dfrac{\cancel{\text{m}}}{\text{s}^2} \cdot \text{m}^2}{\cancel{\text{C}^2}}\right)(2.0 \times 10^{-6}\ \cancel{\text{C}})(8.0 \times 10^{-6}\ \cancel{\text{C}})}{(4.0 \times 10^{-9}\ \cancel{\text{kg}})(0.25\ \cancel{\text{m}})}}$$

$$v_f = 1.2 \times 10^4 \text{ m/s}$$

Statement: The speed of charge q_2 when it reaches a distance of 0.50 m from charge q_1 is 1.2×10^4 m/s.

Sample Problem 3: A Head-on "Collision"

Two particles, a proton with charge 1.60×10^{-19} C and mass 1.67×10^{-27} kg and an alpha particle (helium-4 nucleus) with charge 3.20×10^{-19} C and mass 6.64×10^{-27} kg, are separated by an extremely large distance. They approach each other along a straight line with initial speeds of 3.00×10^6 m/s each. Calculate the separation between the particles when they are closest to each other.

Given: $q_1 = 1.60 \times 10^{-19}$ C; $q_2 = 3.20 \times 10^{-19}$ C; $m_1 = 1.67 \times 10^{-27}$ kg; $m_2 = 6.64 \times 10^{-27}$ kg; $v_{i_1} = v_{i_2} = 3.00 \times 10^6$ m/s; $r_i \rightarrow \infty$ m; $k = 8.99 \times 10^9$ N·m^2/C^2

Required: minimum separation of charges, r_f

Analysis: We can use the conservation of energy to determine the separation of the charges at the moment when both particles have converted their kinetic energy into electric potential energy: $E_{E_i} + E_{k_i} = E_{E_f} + E_{k_f}$. At an extremely large distance, $r_i \rightarrow \infty$, the electric potential energy of the pair of charges will be zero. At minimum separation, the kinetic energy of the charges will be zero, so

$$E_{E_i} = \frac{kq_1q_2}{r_i} = 0;\ E_{E_f} = \frac{kq_1q_2}{r_f};\ E_{k_i} = \frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2;\ E_{k_f} = \frac{1}{2}m_1v_{f_1}^2 + \frac{1}{2}m_2v_{f_2}^2 = 0$$

Solution:

$$E_{E_i} + E_{k_i} = E_{E_f} + E_{k_f}$$

$$0 + \frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{kq_1q_2}{r_f} + 0$$

$$r_f = \frac{2kq_1q_2}{m_1v_{i_1}^2 + m_2v_{i_2}^2}$$

$$= \frac{2\left(8.99 \times 10^9 \frac{\cancel{\text{kg}} \cdot \frac{\text{m}}{\cancel{\text{s}^2}} \cdot \text{m}^2}{\cancel{\text{C}^2}}\right)(1.60 \times 10^{-19}\ \cancel{\text{C}})(3.20 \times 10^{-19}\ \cancel{\text{C}})}{(1.67 \times 10^{-27}\ \cancel{\text{kg}})\left(3.00 \times 10^6 \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)^2 + (6.64 \times 10^{-27}\ \cancel{\text{kg}})\left(3.00 \times 10^6 \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)^2}$$

$$r_f = 1.23 \times 10^{-14}\ \text{m}$$

Statement: The minimum separation of the proton and the alpha particle is 1.23×10^{-14} m.

Practice

1. Three charges, $q_1 = +6.0 \times 10^{-6}$ C, $q_2 = -3.0 \times 10^{-6}$ C, $q_3 = -3.0 \times 10^{-6}$ C, are located at the vertices of an equilateral triangle (**Figure 5**). K/U T/I
   (a) Calculate the electric potential at the midpoint of each side of the triangle. [ans: between q_1 and q_2 is $V_1 = 7.6 \times 10^3$ J/C; between q_1 and q_3 is $V_2 = 7.6 \times 10^3$ J/C; between q_2 and q_3 is $V_3 = -1.5 \times 10^4$ J/C]
   (b) Calculate the total electric potential energy of the group of charges. [ans: -8.1×10^{-2} J]
2. Four point charges, each with $q = 4.5 \times 10^{-6}$ C, are arranged at the corners of a square of side length 1.5 m. Determine the electric potential at the centre of the square. K/U T/I [ans: 1.5×10^5 J/C]
3. Two electrons start at rest with a separation of 5.0×10^{-12} m. Once released, the electrons accelerate away from each other. Calculate the speed of each electron when they are a very large distance apart. K/U T/I [ans: 7.1×10^6 m/s]
4. Two protons move toward each other. They start at infinite separation. One has an initial speed of 2.3×10^6 m/s, and the other has an initial speed of 1.2×10^6 m/s. Calculate the separation when the protons are closest to each other. K/U T/I [ans: 4.1×10^{-14} m]

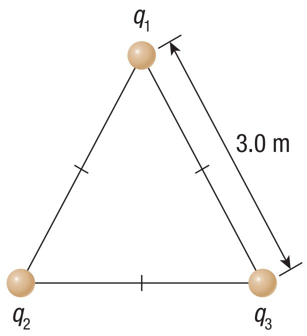

Figure 5

The Van de Graaff Generator

The Van de Graaff generator, discussed earlier, produces an electric potential on the conducting sphere, or dome, by separating charge at one place, and moving the charge to the conductor. As you can see in **Figure 6**, a moving cloth or rubber conveyor-style belt receives electrons from a lower conducting comb at point A. The negatively charged part of the belt moves upward toward the conducting sphere and gives up the negative charge to the upper comb at point B. The charge travels to the conducting sphere and gives the dome a net negative charge. As the generator continues to run, the amount of charge on the sphere builds. When the charge reaches high enough levels, the electrons jump from the sphere through the air to another conductor brought close to the sphere, producing the artificial lightning seen in Figure 1.

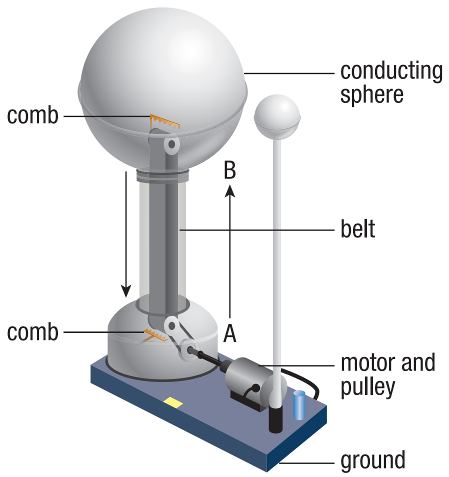

Figure 6 In a Van de Graaff generator, charge is carried to the dome by a moving belt. The charge is deposited on the belt at point A and conveyed to the dome at point B.

7.5 Review

Summary

- The electric potential at a distance r from a point charge q is $V = \frac{kq}{r}$.
- The total electric potential at a point P for a system of charges equals the algebraic sum of the potentials at P due to each individual charge at a distance that separates the charge and P.
- For an electric potential from a point source charge q_1, the work done moving a test charge q_2 from an initial separation r_i to a final separation r_f equals the change in the electric potential energy:
  $$W = E_{E_f} - E_{E_i}$$
  $$W = \frac{kq_1q_2}{r_f} - \frac{kq_1q_2}{r_i}$$
- For a system of two charges, q_1 and q_2, separated by a distance r, the electric potential energy is $E_E = \frac{kq_1q_2}{r}$.
- The potential energy difference of two point charges, q_1 and q_2, as the separation between them changes from r_i to r_f is
  $$\Delta E_E = E_{E_f} - E_{E_i}$$
  $$\Delta E_E = \frac{kq_1q_2}{r_f} - \frac{kq_1q_2}{r_i}$$
- When the amount of charge on the dome of a Van de Graaff generator is high enough, the electrons may jump from the sphere, producing artificial lightning.

Questions

1. A charged particle is released from rest and begins to move as a result of the electric force from a nearby proton. For the following particles, answer these questions. Does the particle move to a region of higher or lower potential energy? Does the particle move to a region of higher or lower electric potential? K/U T/I
   (a) The particle is a proton.
   (b) The particle is an electron.
2. Two particles are at locations where the electric potential is the same. Do these particles have the same electric potential energy? Explain. K/U T/I
3. How much work is required to move a charge from one spot to another with the same electric potential? Explain your answer. K/U T/I
4. Two point particles with charges $q_1 = 4.5 \times 10^{-5}$ C and $q_2 = 8.5 \times 10^{-5}$ C have a potential energy of 40.0 J. Calculate the distance between the charges, in centimetres. K/U T/I
5. Two point particles with charges $q_1 = 4.5 \times 10^{-5}$ C and $q_2 = 8.5 \times 10^{-5}$ C are initially separated by a distance of 2.5 m. They are then brought closer together so that the final separation is 1.5 m. Determine the change in the electric potential energy. K/U T/I
6. Two point charges with charges $q_1 = 3.5 \times 10^{-6}$ C and $q_2 = 7.5 \times 10^{-6}$ C are initially very far apart. They are then brought together, with a final separation of 2.5 m. Calculate how much work it takes to bring them together. K/U T/I
7. A simple model of a hydrogen atom shows the electron and proton as point charges separated by a distance of 5.00×10^{-11} m. Calculate how much work is required to break apart these two charges and to separate them by a very large distance when the electron is initially at rest. T/I A
8. A Van de Graaff generator used for classroom demonstrations has a conducting spherical dome with a radius of 15 cm. The electric potential produced by the charged generator is -8.5×10^4 V near its surface. The sphere has a uniform distribution of charge, so you may assume that all charge on the sphere is concentrated at the sphere's centre. Assume that the electric potential at a large distance and at Earth's surface is zero. K/U T/I A
   (a) Calculate the charge on the sphere.
   (b) Calculate the magnitude of the electric field near the surface of the sphere.
   (c) In which direction does the electric field point?