7.4

Potential Difference and Electric Potential

In the previous section, you learned how two parallel charged surfaces produce a uniform electric field. From the definition of an electric field as a force acting on a charge, it follows that, for a given uniform electric field, charge, and particle mass, the particle undergoes a uniform acceleration. Another way of thinking about the physics of this situation is that the electric field does work on the charged particle. This view works well if the charge stays constant, but in reality the work done by the field varies with charge. What if, instead of describing the electric field in terms of force per charge, you expressed it in terms of energy per charge? As it turns out, we can describe the field in this way, as you will learn in this section.

No matter how you choose to describe the uniform electric field, its ability to accelerate charged particles with known conditions has proven useful to physicists and engineers. Devices such as particle accelerators, which can accelerate particles to speeds near the speed of light, can only work if the particles are moving in the first place. Electric fields cause the initial motion of these particles by accelerating them. Particle acceleration is important in some everyday devices as well. Inkjet printers accelerate charged ink particles toward specific parts of the paper (**Figure 1**). Old television sets and computer monitors have cathode-ray tubes. These tubes accelerate electrons toward a phosphor screen. Variations in the deflection of the accelerated electrons determine the brightness and colour of the screen.

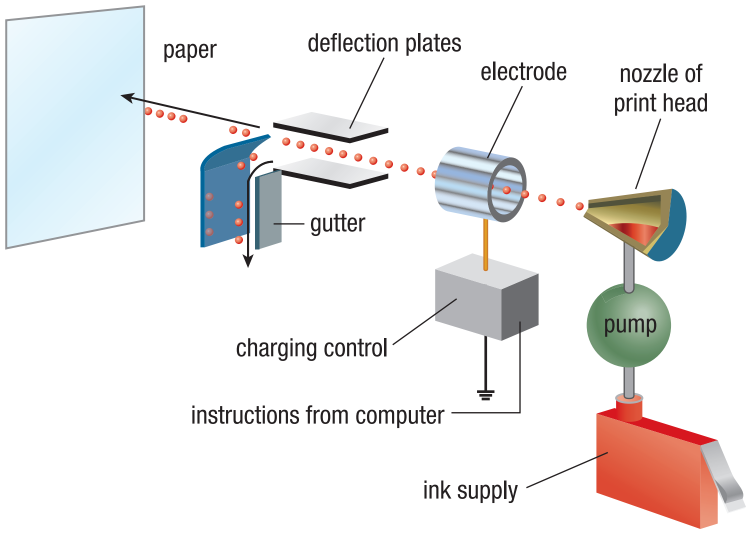


Figure 1 Ink droplets from the print head are either charged or uncharged. The uncharged droplets move to the paper undeflected, forming the letters. Charged droplets are deflected into the gutter, leaving those parts of the paper blank.

Work and Electric Potential Difference

Recall that in a region of space where the electric field $\vec{\varepsilon}$ is constant, $\vec{\varepsilon}$ has the same magnitude and direction at all points. A point charge q in this region experiences an electric force

$$\vec{F}_E = q\vec{\varepsilon}$$

The force is parallel to $\vec{\varepsilon}$, as shown in **Figure 2**.

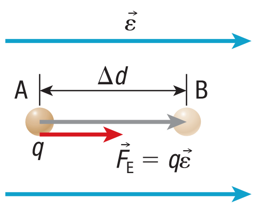


Figure 2 The electric field gives rise to an electric force, which moves in the same direction as the charge if the charge is positive.

Suppose this charge moves a certain distance Δd, starting at point A and ending at point B. For simplicity, assume this displacement is parallel to the electric force $\vec{F}_E$. According to the definition of work (W), the work done by the electric force on the charge is

$$W = F_E\Delta d$$

The electric force does work on the charge and is independent of the path it takes from A to B. We can now define the **electric potential energy**, E_E, which is the energy stored in the system that can do work W on a positively charged particle. From your studies of work and energy you know that the change in the potential energy associated with this type of force is equal to $-W$, where W is the work done by that force. A more detailed explanation of the meaning of the negative sign will follow. So, if the electric force does an amount of work W on a charged particle, the change in the electric potential energy is

electric potential energy (E_E) the energy stored in a system of two charges a distance Δd apart, or the energy stored in an electric field that can do work on a positively charged particle

$$\Delta E_E = -W$$
$$\Delta E_E = -F_E \Delta d$$

Combining this equation with the equation relating force to the electric field, the change in electric potential energy when the charged particle moves from A to B in Figure 2 is

$$\begin{aligned}\Delta E_E &= -W \\ &= -F_E \Delta d\end{aligned}$$

$$\boxed{\Delta E_E = -q\varepsilon\Delta d}$$

This equation gives the *change* in the potential energy as the charge moves through a displacement Δd, in a region where the electric field is parallel to the displacement. Note that the change in the potential energy depends on the starting and ending locations but not on the path taken. In Figure 2, the displacement is along a line, but the charge may move from A to B along many other paths without affecting ΔE_E.

Electric potential energy is stored through the potential effect of the electric field on an electric charge. This effect is illustrated in **Figure 3(a)**, where the charge q is moved from point B to point A by an external force $\vec{F}_a$ directed to the left.

Force $\vec{F}_a$ results from some external agent, which could be your hand. The electric force on q points to the right, assuming that q is positive, so the displacement is opposite to the direction of the electric force. The work done by the electric field on the particle is thus negative. Therefore, according to the electric potential energy equation, the change in the electric potential energy must be positive. In other words, a positive amount of energy has now been stored in the system composed of the charge q and the electric field. That energy came from the positive amount of work done by the external force $\vec{F}_a$ that moved the charge from B to A.

As the charge moves from A back to B, the process is reversed (**Figure 3(b)**). Now the electric field does a positive amount of work on the particle because the electric force and the particle's displacement are parallel and the change in the electric potential energy is negative. Energy stored in the electric field and particle system is now taken out of the system. This energy can appear as an increase in the kinetic energy of the particle when it reaches B. Potential energy becomes kinetic energy.

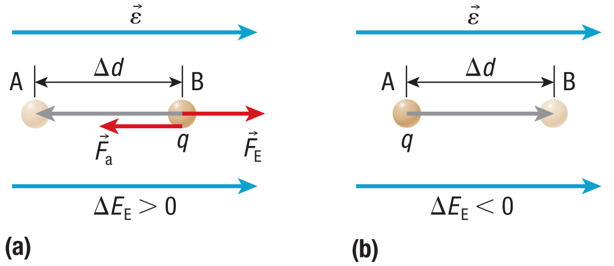


Figure 3 (a) To move q from B to A, the external force acting on a charged particle works against the electric field and produces a positive change in ΔE_E of the field and particle system. (b) The charged particle takes energy stored in the electric field and converts it to kinetic energy. This produces a negative change in ΔE_E of the field and particle system.

In the following Tutorial, you will learn more about how to solve problems that involve electric potential energy.

Tutorial 1 Solving Problems Involving Electric Potential Energy

This Tutorial explains how to determine the change in electric potential energy for a charge in a uniform electric field, given the position and magnitude of that field.

Sample Problem 1: Potential Energy Difference in an Electric Field

A charged particle moves from rest in a uniform electric field.

(a) For a proton, calculate the change in electric potential energy when the magnitude of the electric field is 250 N/C, the starting position is 2.4 m from the origin, and the final position is 3.9 m from the origin.

(b) Calculate the change in electric potential energy for an electron in the same field and with the same displacement.

(c) Calculate the change in electric potential energy for an electron accelerated in an electric field with the same magnitude but opposite direction as in (a) and (b), and with a starting position of 2.4 m from the origin and a final position of 5.0 m from the origin.

Solution

(a) **Given:** $d_i = 2.4$ m; $d_f = 3.9$ m; $q = +1.60 \times 10^{-19}$ C; $\varepsilon = 250$ N/C

Required: ΔE_E

Analysis: Use the equation for electric potential energy in terms of q, ε, and Δd, where $\Delta d = d_f - d_i$: $\Delta E_E = -q\varepsilon\Delta d = -q\varepsilon(d_f - d_i)$. Note that ΔE_E is negative for a positive charge travelling in the same direction as ε. Thus, the sign of q takes care of whether there is a gain (for negative q) or loss (for positive q) of electric potential energy. Note that we include the sign of the charge because we are dealing with energy, which is not a vector quantity and does not have a direction.

Solution:

$$\begin{aligned}\Delta E_E &= -q\varepsilon\Delta d \\ &= -q\varepsilon(d_f - d_i) \\ &= -(+1.6 \times 10^{-19}\ \cancel{\text{C}})\left(250\ \frac{\text{N}}{\cancel{\text{C}}}\right)(3.9\ \text{m} - 2.4\ \text{m}) \\ &= -6.0 \times 10^{-17}\ \text{N}\cdot\text{m} \\ \Delta E_E &= -6.0 \times 10^{-17}\ \text{J}\end{aligned}$$

Statement: The change in electric potential energy due to the movement of the proton in the uniform electric field is -6.0×10^{-17} J. The negative sign indicates that the electric field loses potential energy by doing work on the proton.

(b) **Given:** $d_i = 2.4$ m; $d_f = 3.9$ m; $q = -1.60 \times 10^{-19}$ C; $\varepsilon = 250$ N/C

Required: ΔE_E

Analysis: Use the same equation for electric potential energy: $\Delta E_E = -q\varepsilon\Delta d = -q\varepsilon(d_f - d_i)$. Note that we include the sign of the charge to determine whether there is a gain or loss of electric potential energy.

Solution:

$$\begin{aligned}\Delta E_E &= -q\varepsilon\Delta d \\ &= -q\varepsilon(d_f - d_i) \\ &= -(-1.6 \times 10^{-19}\ \cancel{\text{C}})\left(250\ \frac{\text{N}}{\cancel{\text{C}}}\right)(3.9\ \text{m} - 2.4\ \text{m}) \\ &= 6.0 \times 10^{-17}\ \text{N}\cdot\text{m} \\ \Delta E_E &= 6.0 \times 10^{-17}\ \text{J}\end{aligned}$$

Statement: The change in electric potential energy due to the movement of the electron in the uniform electric field is 6.0×10^{-17} J. The positive value indicates that the electric field gains potential energy as the electron is accelerated in the field.

(c) **Given:** $d_i = 2.4$ m; $d_f = 5.0$ m; $q = -1.60 \times 10^{-19}$ C; $\varepsilon = -250$ N/C

Required: ΔE_E

Analysis: Note that the sign for the electric field is negative because the electron is accelerating in a direction opposite to ε. Use the same equation for electric potential energy: $\Delta E_E = -q\varepsilon\Delta d = -q\varepsilon(d_f - d_i)$.

Solution:

$$\begin{aligned}\Delta E_E &= -q\varepsilon\Delta d \\ &= -q\varepsilon(d_f - d_i) \\ &= -(-1.6 \times 10^{-19}\ \cancel{\text{C}})\left(-250\ \frac{\text{N}}{\cancel{\text{C}}}\right)(5.0\ \text{m} - 2.4\ \text{m}) \\ &= -1.0 \times 10^{-16}\ \text{N}\cdot\text{m} \\ \Delta E_E &= -1.0 \times 10^{-16}\ \text{J}\end{aligned}$$

Statement: When the direction of the electric field is reversed, the change in the electric potential energy is -1.0×10^{-16} J. The negative value indicates that the electric field loses potential energy by doing work on the electron.

Sample Problem 2: Dynamics of Charged Particles

(a) Using the law of conservation of energy, calculate the speed of the proton in part (a) of Sample Problem 1 for the given displacement. Assume that the proton starts from rest.

(b) Determine the initial speed of the electron in part (b) of Sample Problem 1, assuming its speed has decreased to half of its initial speed after the same displacement, Δd.

Solution

(a) **Given:** $d_i = 2.4$ m; $d_f = 3.9$ m; $q = 1.60 \times 10^{-19}$ C; $m = 1.67 \times 10^{-27}$ kg; $\varepsilon = 250$ N/C

Required: v

Analysis: The law of conservation of energy states that the total change in potential energy of the field and particle system and the change in kinetic energy of the particle equals zero: $\Delta E_E + \Delta E_k = 0$. The kinetic energy of the particle is related to its speed v by the equation $\Delta E_k = \frac{1}{2}mv^2$. Use this equation and the equation $\Delta E_E = -q\varepsilon\Delta d$ for the change in potential energy of the field and proton system. Note that $\Delta d = 3.9 \text{ m} - 2.4 \text{ m} = 1.5$ m and $1 \text{ N} = 1 \text{ kg}\cdot\text{m/s}^2$.

Solution: By the law of conservation of energy,

$$\Delta E_E + \Delta E_k = 0$$

$$-q\varepsilon\Delta d + \frac{1}{2}mv^2 = 0$$

$$\frac{1}{2}mv^2 = q\varepsilon\Delta d$$

$$v = \sqrt{\frac{2q\varepsilon\Delta d}{m}}$$

$$v = \sqrt{\frac{2(1.6 \times 10^{-19}\ \cancel{\text{C}})\left(250\frac{\cancel{\text{kg}}\cdot\frac{\text{m}}{\text{s}^2}}{\cancel{\text{C}}}\right)(1.5 \text{ m})}{1.67 \times 10^{-27}\ \cancel{\text{kg}}}}$$

$$v = 2.7 \times 10^5 \text{ m/s}$$

Statement: The speed of the proton accelerated for a distance of 1.5 m by an electric field of 250 N/C is 2.7×10^5 m/s.

(b) **Given:** $\Delta d = 1.5$ m; $q = -1.60 \times 10^{-19}$ C; $m = 9.11 \times 10^{-31}$ kg; $\varepsilon = 250$ N/C; $v_f = 0.5v_i$

Required: v_i

Analysis: For situations that do not involve an object at rest, the change in kinetic energy is given by the equation $\Delta E_k = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$. Use this equation and the equation $\Delta E_E = -q\varepsilon\Delta d$ for the change in potential energy of the field and electron system.

Solution: By the law of conservation of energy,

$$\Delta E_E + \Delta E_k = 0$$

$$-q\varepsilon\Delta d + \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) = 0$$

$$\frac{1}{2}m\left(\frac{1}{2}v_i\right)^2 - \frac{1}{2}mv_i^2 = q\varepsilon\Delta d$$

$$-\frac{3}{8}mv_i^2 = q\varepsilon\Delta d$$

$$v_i = \sqrt{-\frac{8q\varepsilon\Delta d}{3m}}$$

$$v_i = \sqrt{-\frac{8(-1.6 \times 10^{-19}\text{ C})\left(250\frac{\text{N}}{\text{C}}\right)(1.5 \text{ m})}{3(9.11 \times 10^{-31}\text{ kg})}}$$

$$= \sqrt{\frac{8(1.6 \times 10^{-19}\ \cancel{\text{C}})\left(250\frac{\cancel{\text{kg}}\cdot\frac{\text{m}}{\text{s}^2}}{\cancel{\text{C}}}\right)(1.5 \text{ m})}{3(9.11 \times 10^{-31}\ \cancel{\text{kg}})}}$$

$$v_i = 1.3 \times 10^7 \text{ m/s}$$

Statement: The initial speed of the electron before entering the electric field is 1.3×10^7 m/s.

Practice

1. An electron enters a uniform electric field of 145 N/C pointed toward the right. The point of entry is 1.5 m to the right of a given mark, and the point where the electron leaves the field is 4.6 m to the right of that mark. T/I A
   (a) Determine the change in the electric potential energy of the electron. [ans: 7.2×10^{-17} J]
   (b) The initial speed of the electron was 1.7×10^7 m/s when it entered the electric field. Determine its final speed. [ans: 1.1×10^7 m/s]
2. Calculate the work done in moving a proton 0.75 m in the same direction as the electric field with a strength of 23 N/C. T/I A [ans: 2.8×10^{-18} J]
3. An electron experiences a change in kinetic energy of $+4.2 \times 10^{-16}$ J. Calculate the magnitude and direction of the electric field when the electron travels 0.18 m toward the right. T/I A [ans: 1.5×10^4 N/C [toward the left]]

Electric Potential

Electric potential energy is a property of a system of charges or of a point charge in an electric field, where the field is created by other charges. In either case, this electric potential energy is not the property of a single charge alone. For this reason, the potential energy depends on the values of the charges and the electric field involved in the interaction. This leads to a new quantity called **electric potential**, V, which is a measure of how much electric potential energy is associated with a specific quantity of charge at a particular location in an electric field. Based on this definition,

electric potential (V) the value, in volts, of potential energy per unit positive charge for a given point in an electric field; 1 V = 1 J/C

$$V = \frac{E_E}{q}$$

Electric potential, or just potential, is a convenient measure because it is independent of the amount of charge at a particular location in the field. It depends only on the electric field strength at that location. For example, if you had 1 C of electrons at a particular location in a uniform electric field, you would possess a certain amount of electric potential energy. If you doubled the amount of electrons to 2 C at the same location in the electric field, you would have double the electric potential energy. In both these situations, you would have the same electric potential.

The SI unit of electric potential is the volt (V), named in honour of physicist Alessandro Volta (1745–1827). The volt relates to other SI units in the following equation:

$$1\ \text{V} = 1\ \text{J/C} = 1\ \text{N·m/C}$$

The volt is used in the measurement of many electrical quantities. Note in particular that the units of volts per metre are equivalent to units of newtons per coulomb, the units for electric field strength. These equivalent units give you several different ways to express the units of the electric field. The ones most commonly used are 1 V/m = 1 N/C.

electric potential difference (ΔV) the amount of work required per unit charge to move a positive charge from one point to another in the presence of an electric field

Another convenient definition relating to electric potential energy is **electric potential difference**, ΔV. We return to the concept of the change in potential difference and the displacement of the particle: You can define the change in the potential, or potential difference, for a charge q that moves between two points:

$$\Delta V = \frac{\Delta E_E}{q}$$

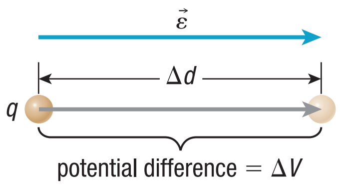
For the case of a uniform electric field, the equation for electric potential difference becomes

$$\begin{aligned}\Delta V &= \frac{\Delta E_E}{q}\\ &= \frac{-W}{q}\\ &= \frac{-q\varepsilon\Delta d}{q}\\ \Delta V &= -\varepsilon\Delta d\end{aligned}$$

This relationship shows how a non-uniform electric field varies with the change in electric potential (that is, electric potential difference) and the change in position in the field:

$$\varepsilon = -\frac{\Delta V}{\Delta d}$$

This equation states that the magnitude of the electric field is largest in regions where V is large and changes rapidly with small changes in displacement. Conversely, the electric field is zero in regions where V is constant. Notice that because of the negative sign in the equation, the electric field points from regions of high potential to regions of low potential (**Figure 4**). If we consider a circuit in which a battery is the source of electrical energy, a positive test charge will naturally move from the positive terminal where a high potential exists to the negative terminal where a low potential exists. Since the electric field points from positive to negative, the positive test charge will also move in the same direction as the field. Conversely, electrons will naturally travel from a region of low potential to a region of high potential, in a direction opposite to the direction of the electric field.



$$\varepsilon = -\frac{\Delta V}{\Delta d}$$

Figure 4 The magnitude and direction of the electric field are related to how the electric potential V changes with position.

The relation between the electric field and the changes in the potential and position involves the component of the electric field that is in the direction parallel to the displacement Δd. The electric field is a vector, so if you want to determine $\vec{\varepsilon}$ in a particular direction, you must consider how the potential V changes for a test charge moving along that direction.

With this equation and knowledge of $\vec{\varepsilon}$, you can, in principle at least, calculate how the electric potential changes as you move from place to place within the find. Strictly speaking, this relation only holds for small steps Δd in a constant field. You can, however, combine the potential changes ΔV from many such small steps to determine the change in the potential difference over a large distance. The following Tutorial will demonstrate how to solve problems involving electric potential.

UNIT TASK **BOOKMARK**

You can apply what you have learned about electric potential to the Unit Task on page 422.

Tutorial **2** Solving Problems Related to Electric Potential

This Tutorial shows how to use electric potential to solve problems related to a charge in a uniform electric field.

Sample Problem 1: TV Tubes and Particle Accelerators

The cathode-ray tubes in old television sets and computer monitors work in a way that is similar to certain parts in particle accelerators. Both devices accelerate particles in a similar way, using the uniform electric field between conducting plates. Looked at another way, the particles accelerate as they move through an electric potential difference.

(a) An electron leaves the negative plate of a cathode-ray tube and travels toward the positive plate. The electric potential difference between the plates is 1.5×10^4 V. Using the law of conservation of energy and the definition of electric potential difference, calculate the speed of an electron as it reaches the positive plate in a cathode-ray tube. Assume that the electron is initially at rest. The mass of an electron is 9.11×10^{-31} kg.

(b) Calculate the magnitude of the electric field at a distance of 15 cm, which is at the end of the cathode-ray tube.

(a) **Given:** $q = -1.60 \times 10^{-19}$ C; $m = 9.11 \times 10^{-31}$ kg; $\Delta V = 1.5 \times 10^4$ V; $v_i = 0$ m/s

Required: v_f

Analysis: Use the equation for the law of conservation of energy, $\Delta E_E + \Delta E_k = 0$, and the equation for the electric potential difference, $\Delta E_E = q\Delta V$. The kinetic energy of the particle initially at rest is related to its final speed v_f by the equation $\Delta E_k = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$. Since $v_i = 0$, $\Delta E_k = \frac{1}{2}mv_f^2$. Note that 1 V = 1 N·m/C = 1 kg·m/s²·m/C.

Solution: By the law of conservation of energy,

$$\Delta E_E + \Delta E_k = 0$$

$$q\Delta V + \frac{1}{2}mv_f^2 = 0$$

$$\frac{1}{2}mv_f^2 = -q\Delta V$$

$$v_f^2 = \frac{-2q\Delta V}{m}$$

$$v_f = \sqrt{-\frac{2q\Delta V}{m}}$$

Calculate the final speed v_f of the electron as it reaches the positive plate.

$$v_f = \sqrt{-\frac{2(-1.6 \times 10^{-19}\ \text{C})(1.5 \times 10^4\ \text{V})}{9.11 \times 10^{-31}\ \text{kg}}}$$

$$= \sqrt{\frac{2(1.6 \times 10^{-19}\ \cancel{\text{C}})\left(1.5 \times 10^4\ \cancel{\text{kg}} \cdot \frac{\text{m}}{\text{s}^2} \cdot \frac{\text{m}}{\cancel{\text{C}}}\right)}{9.11 \times 10^{-31}\ \cancel{\text{kg}}}}$$

$$v_f = 7.3 \times 10^7\ \text{m/s}$$

Statement: The speed of the electron as it reaches the positive plate is 7.3×10^7 m/s.

(b) **Given:** $\Delta d = 15$ cm $= 0.15$ m; $\Delta V = 1.5 \times 10^4$ V

Required: ε

Analysis: Use the equation for the electric field in terms of potential difference and displacement:

$$\varepsilon = -\frac{\Delta V}{\Delta d}$$

Solution: $\varepsilon = -\dfrac{\Delta V}{\Delta d}$

$$= -\frac{1.5 \times 10^4\ \text{V}}{0.15\ \text{m}}$$

$$\varepsilon = -1.0 \times 10^5\ \text{V/m}$$

Statement: The electric field has a magnitude of 1.0×10^5 V/m. The negative sign indicates that the field extends from high to low potential. The electric field vector always points from regions of high V to regions of low V because $\vec{\varepsilon}$ is parallel to the direction that a positive test charge would move if it were placed at that location.

Sample Problem 2: Calculating the Speed of a Deflected Charged Particle

An electron moves horizontally with a speed of 1.6×10^6 m/s between two horizontal parallel plates. The plates have a length of 12.5 cm, and a plate separation that allows a charged particle to escape even after being deflected (**Figure 5**). The magnitude of the electric field within the plates is 150 N/C. Calculate the final velocity of an electron as it leaves the plates.

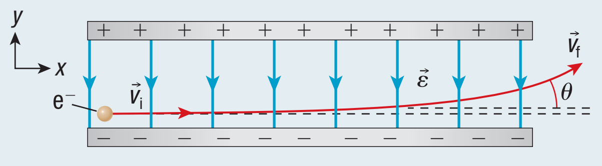


Figure 5

Given: $v_i = 1.6 \times 10^6$ m/s; $L = 12.5$ cm $= 0.125$ m; $q = -1.60 \times 10^{-19}$ C; $\varepsilon = -150$ N/C

Required: $\vec{v}_f$

Analysis: The electric field is uniform and directed downward between the plates, so the electric force acting on a proton is also constant and directed downward. A constant downward force means a constant downward acceleration on a positive charge. However, because the charge is negative, the acceleration must be upward. To calculate the magnitude of the final velocity, solve for each component of the velocity: the constant horizontal component and the accelerated upward component. Then use the equation $v_f = \sqrt{v_{x_f}^2 + v_{y_f}^2}$ for the magnitude of the final velocity.

The components of the velocity are given by $v_{x_f} = v_i = \dfrac{L}{\Delta t}$ (Δt can be calculated from v_i and L) and $v_{y_f} = v_{y_i} + a_y\Delta t = a_y\Delta t$ (since $v_{y_i} = 0$).

By combining the equations for Newton's second law, $F_{net} = ma$, and the electric force on a charge in an electric field ε, $F_E = q\varepsilon$, we can determine the vertical acceleration $a_y = \dfrac{F_{net}}{m} = \dfrac{F_E}{m} = \dfrac{q\varepsilon}{m}$.

Solution: Calculate a_y.

$$a_y = \frac{q\varepsilon}{m}$$

$$= \frac{(-1.6 \times 10^{-19}\ \cancel{\text{C}})\left(-150 \dfrac{\cancel{\text{kg}} \cdot \frac{\text{m}}{\text{s}^2}}{\cancel{\text{C}}}\right)}{9.11 \times 10^{-31}\ \cancel{\text{kg}}}$$

$a_y = 2.634 \times 10^{13}$ m/s² (two extra digits carried)

Calculate Δt.

$$v_i = \frac{L}{\Delta t}$$

$$\Delta t = \frac{L}{v_i}$$

$$= \frac{0.125\ \cancel{m}}{1.6 \times 10^6\ \frac{\cancel{m}}{s}}$$

$\Delta t = 7.812 \times 10^{-8}$ s (two extra digits carried)

Calculate v_{y_f}.

$$v_{y_f} = a_y \Delta t$$

$$= \left(2.634 \times 10^{13}\ \frac{m}{s^{\cancel{2}}}\right)(7.812 \times 10^{-8}\ \cancel{s})$$

$v_{y_f} = 2.058 \times 10^6$ m/s (two extra digits carried)

Now calculate the magnitude of the net velocity.

$$v_f = \sqrt{v_{x_f}^2 + v_{y_f}^2}$$

$$= \sqrt{(1.6 \times 10^6\ \text{m/s})^2 + (2.058 \times 10^6\ \text{m/s})^2}$$

$v_f = 2.6 \times 10^6$ m/s

Calculate the angle θ to determine the direction of the electron.

$$\theta = \tan^{-1}\left(\frac{v_{y_f}}{v_{x_f}}\right)$$

$$= \tan^{-1}\left(\frac{2.058 \times 10^6\ \frac{\cancel{m}}{\cancel{s}}}{1.6 \times 10^6\ \frac{\cancel{m}}{\cancel{s}}}\right)$$

$\theta = 52°$

Statement: The final velocity of the electron is 2.6×10^6 m/s [E 52° N].

Practice

1. An old television cathode-ray tube creates a potential difference of 1.6×10^4 V across the parallel accelerating plates. These plates accelerate a beam of electrons toward the target phosphor screen. The separation between the plates is 12 cm. K/U T/I
   (a) Using the principle of energy conservation and the definition of electric potential difference, calculate the speed at which the electrons strike the screen. [ans: 7.5×10^7 m/s]
   (b) Calculate the magnitude of the electric field. [ans: 1.3×10^5 N/C]
2. Four parallel plates are connected in a vacuum as shown in **Figure 6**. An electron at rest in the hole of plate X is accelerated to the right. The electron passes through holes at W and Y with no acceleration at all. It then passes through the hole at Y and slows down as it heads to plate Z. T/I

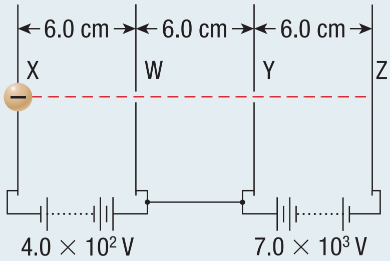


Figure 6

   (a) Calculate the speed of the electron at hole W. [ans: 1.2×10^7 m/s]
   (b) Calculate the distance, in centimetres, from plate Z to the point at which the electron changes direction. [ans: 5.7 cm [to the left of Z]]
3. An electron enters a parallel plate apparatus that is 8.0 cm long and 4.0 cm wide, as shown in **Figure 7**. The electron has a horizontal speed of 6.0×10^7 m/s. The potential difference between the plates is 6.0×10^2 V. Calculate the electron's velocity as it leaves the plates. K/U A [ans: 6.0×10^7 m/s [E 3.3° N]]

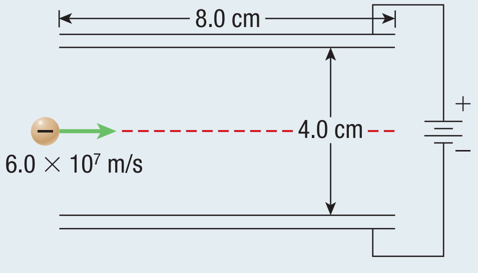


Figure 7

7.4 Review

Summary

- The change in electric potential energy depends on the electric field, the charge being moved, and the charge's displacement: $\Delta E_E = -q\varepsilon\Delta d$.
- For $\Delta E_E > 0$, work is done against the electric field ($-W$), resulting in energy stored in the field. For $\Delta E_E < 0$, work is done by the electric field ($+W$) on a particle moving in the field, which typically increases the kinetic energy of the particle.
- The electric potential is the electric potential energy per unit charge at a given point in an electric field: $V = \frac{E_E}{q}$.
- The magnitude of an electric field varies with the electric potential difference and the change in position in the field: $\varepsilon = -\frac{\Delta V}{\Delta d}$.

Questions

1. An electron moves from an initial location between parallel plates where the electric potential is $V_i = 30$ V to a final location where $V_f = 150$ V. K/U T/I
   (a) Determine the change in the electron's potential energy.
   (b) Determine the average electric field along a 10 cm–long line segment that connects the initial and final locations of the electron. Be sure to give both the magnitude and the direction of $\vec{\varepsilon}$.
2. The electric potential difference between two parallel metal plates is ΔV. The plates are separated by a distance of 3.0 mm and the electric field between the plates is $\varepsilon = 250$ V/m. Calculate ΔV. K/U T/I
3. A proton of mass 1.67×10^{-27} kg moves from a location where $V_i = 75.0$ V to a spot where $V_f = -20.0$ V. K/U T/I
   (a) Calculate the change in the proton's kinetic energy.
   (b) Replace the proton with an electron, and determine its change in kinetic energy.
4. An electron moves from a region of low potential to a region of higher potential where the potential change is +45 V. K/U T/I
   (a) Calculate the work, in joules, required to push the electron.
   (b) What is doing the work?
5. The electrons in an old TV picture tube are accelerated through a potential difference of 2.5×10^4 V. K/U T/I A
   (a) Do the electrons move from a region of high potential to a region of low potential, or vice versa?
   (b) Calculate the change in the kinetic energy of one of the electrons.
   (c) Calculate the final speed of an electron when the initial speed is zero.
6. A pair of parallel plates has an electric field of 2.26×10^5 N/C. Determine the change in the electric potential between points that are 2.55 m (initial) and 4.55 m (final) from the plates. K/U T/I A
7. An electron with a horizontal speed of 4.0×10^6 m/s and no vertical component of velocity passes through two horizontal parallel plates, as shown in **Figure 8**. The magnitude of the electric field between the plates is 150 N/C. The plates are 6.0 cm long. K/U T/I

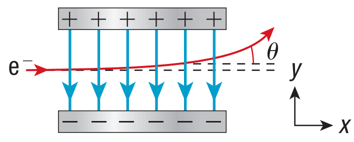


Figure 8

   (a) Calculate the vertical component of the electron's final velocity.
   (b) Calculate the final velocity of the electron.
8. An electric field of 20 N/C exists along the x-axis in space. Calculate the potential difference $\Delta V = V_B - V_A$, where the points A and B are given by
   (a) A = 0 m; B = 4 m
   (b) A = 4 m; B = 6 m T/I A
9. Points A and B are at the same potential. Determine the net work done in moving a charge from point A to point B. K/U T/I
10. The potential at a point is 20 V. Calculate the work done in bringing a charge of 0.5 C to this point. K/U A