## Coulomb's Law

Recall that charged objects attract some objects and repel others at a distance, without making any contact with those objects. Electric force, $F_{\mathrm{E}}$, or the force acting between two charged objects, is somewhat similar to gravity. Both are non-contact forces. Similar to the force of gravity, the electric force becomes weaker as the distance, $r$, between the charged objects increases (Figure 1). Electric force becomes stronger as the amount of charge on either object increases, in the same way that the force of gravity becomes stronger with an increase in mass of either object. In this section, you will learn more about how the electric force depends on charge and distance. You will also learn how to solve problems related to the electric force.


Figure 1 The electric force of repulsion between two identical charges decreases as the separation increases.

## The Electric Force

If you drop a tennis ball, the force of gravity is responsible for its fall. The tennis ball will take approximately 1 s to fall from a height of 5 m . How long do you think the tennis ball will take to stop as it hits the ground? It will take a lot less time to stop than it took to fall. What force is responsible for making the tennis ball stop? The electric force of repulsion between the protons in the tennis ball and the protons in the ground stop it. This electric force must be significantly stronger than gravity.

Consider two charged objects so tiny that you can model them as point particles. These objects have charges $q_{1}$ and $q_{2}$ and are separated by a distance $r$ (Figure 2). The magnitude of the electric force between $q_{1}$ and $q_{2}$ is expressed by the equation

$$
F_{\mathrm{E}}=k \frac{q_{1} q_{2}}{r^{2}}
$$

This equation represents Coulomb's law.

## Coulomb's Law

The force between two point charges is inversely proportional to the square of the distance between the charges and directly proportional to the product of the charges.

The constant $k$, which is sometimes called Coulomb's constant, has the value $k=8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}$. Do not confuse this constant $k$ with the spring constant in Hooke's law. Most physicists use the same symbol for both.

The direction of the electric force on each of the charges is along the line that connects the two charges, as illustrated in Figure 2 for the case of two like charges. As mentioned earlier, this force is repulsive for two charges with the same type of charge. Strictly speaking, the value of $F_{\mathrm{E}}$ in the equation for Coulomb's law applies only to two point charges. However, it is a good approximation whenever the sizes of the particles are much smaller than their distance of separation.
electric force $\left(F_{\mathrm{E}}\right)$ a force with magnitude and direction that acts between two charged particles


Figure 2 The electric force between two point charges $q_{1}$ and $q_{2}$ is given by Coulomb's law.

Coulomb's constant ( $k$ ) the proportionality constant in Coulomb's law; $k=8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}$

## Investigation 7.2.1

Coulomb's Law (page 366)
Now that you have learned how to calculate the electric force between two charges with a given separation, perform Investigation 7.2.1 to determine how electric force varies with distance $r$ and $\frac{1}{r^{2}}$. You will use this information with two different charge values to determine Coulomb's law.

Coulomb's law has several important properties:

- You have already seen that the electric force is repulsive for like charges and attractive for unlike charges. Mathematically, this property results from the product $q_{1} q_{2}$ in the numerator of the Coulomb's law equation. The magnitude of the electric force, $F_{\mathrm{E}}$, is always positive. In physics, it is convenient to use a negative sign to show direction. Including the negative sign of a negatively charged object would imply a direction. You never include the sign of the charge when solving problems related to Coulomb's law.
- The electric force is inversely proportional to the square of the distance between the two particles.
- The magnitude of the electric force is the magnitude of the force exerted on each of the particles. That is, a force of magnitude $F_{\mathrm{E}}$ is exerted on charge $q_{1}$ by charge $q_{2}$, and a force of equal magnitude and opposite direction is exerted on $q_{2}$ by $q_{1}$. This pairing of equal but opposite forces is described by Newton's third law, the action-reaction principle.

Using the equation for Coulomb's law, we can show that electric forces can be extremely large. Suppose you have two boxes of electrons, each with a total charge of $q_{\mathrm{T}}=-1.8 \times 10^{8} \mathrm{C}$ separated by a distance $r$ of 1.0 m . For simplicity, assume that each box is so small that it can be modelled as a point particle. The magnitude of the total electric force is

$$
\begin{aligned}
F_{\mathrm{E}} & =k \frac{q_{1} q_{2}}{r^{2}} \\
& =\left(8.99 \times 10^{9} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\ell^{2}}\right) \frac{\left(1.8 \times 10^{8} \ell\right)\left(1.8 \times 10^{8} \ell\right)}{(1.0 \mathrm{Kr})^{2}} \\
F_{\mathrm{E}} & =2.9 \times 10^{26} \mathrm{~N}
\end{aligned}
$$

This is an extremely large force, all from just two small containers of electrons. Note that the negative sign for charge was not included.

Why, then, does the electric force not dominate everyday life? The fact is that it is essentially impossible to obtain a box containing only electrons. Recall from earlier science studies that a neutral atom contains equal numbers of electrons and protons. If our two point-like boxes had contained equal numbers of electrons and protons, their total charges, $q_{T}$, would have been zero, and so would the force calculated using Coulomb's law.

Ordinary matter consists of equal, or nearly equal, numbers of electrons and protons. The total charge is therefore either zero or very close to zero. At the atomic and molecular scales, however, it is common to have the positive and negative charges (nuclei and electrons) separated by a small distance. In this case, the electric force is not zero, and these electric forces hold matter together.

## Comparing Coulomb's Law and Universal Gravitation

The equation for Coulomb's law may seem familiar to you, even though you have only just learned it. This is because it is similar to the universal law of gravitation, which you learned in Chapter 6. Both of these laws describe the force between two objects. In addition, both depend on certain properties of the objects involved. For gravitation, this property is mass. For the electric force, this property is electric charge. Both mass and charge can be considered to be concentrated at a central point. If you think of the mass or charge as a solid sphere, you can treat this same mass or charge as if it were a point at the centre of that sphere.

Another similarity between the two laws is that both the gravitational force and the electric force decrease as the distance between the two interacting objects increases. If the distance between the objects is great (compared to the size of either object), the actual size and shape of the objects involved become mathematically irrelevant. The theoretical mass and charge at the centre point of each object are then used in calculations.

Although there are similarities between electric and gravitational forces, there are also important differences. Gravitational forces are always attractive. The direction of electric forces depends on the types of charge: unlike charges attract, and like charges repel. Also, the magnitude of the electric force is much greater than the magnitude of the gravitational force over the same distance.

## The Superposition Principle

So far, you have learned about the electric force between two point particles. Now consider how to use Coulomb's law with more than two charges. Suppose you have three particles. One particle has a charge of $q_{1}$. The second particle has a charge of $2 q_{1}$ and is a distance $2 r$ from the first particle, as shown in Figure 3. What is the electric force on a third charge, $q_{2}$, placed midway between these two charges?

Solve this problem by first using Coulomb's law to calculate the force exerted by charge $q_{1}$ on $q_{2}$. Then use Coulomb's law a second time to calculate the force exerted by the charge $2 q_{1}$ on $q_{2}$. The total force on $q_{2}$ equals the vector sum of these two separate contributions. This combining of two forces is an example of the superposition principle. The superposition principle says that the total force acting on $q_{2}$ is the sum, or superposition, of the individual forces exerted on $q_{2}$ by all the other charges in the problem. Remember that force is a vector, so be careful to add these forces as vectors. This means that, for charges not on a straight line, the solution requires some trigonometry involving triangles or the use of vector components.

When all charges lie in a straight line, the superposition principle simplifies so that you can add or subtract the various individual forces to or from one another to obtain the resultant, or net, force. As shown in Figure 3, the separation between $q_{2}$ and $q_{1}$ is $r$. Coulomb's law for this pair of charges is therefore

$$
F_{\mathrm{E}_{1}}=\frac{k q_{1} q_{2}}{r^{2}}
$$

Using the coordinate system in Figure 3, this corresponds to a force on $q_{2}$ by $q_{1}$ in the positive $x$-direction because like charges repel. So, $F_{\mathrm{E}_{1}}$ is the component of the force along the positive $x$-axis. The charges line up along the $x$-axis, so the component of the force along the $y$-axis is zero.

Now consider the force on $q_{2}$ by the charge $2 q_{1}$ in a similar way. The separation of the charges is again $r$, so, using the equation for Coulomb's law, you get

$$
\begin{aligned}
& F_{\mathrm{E}_{2}}=k \frac{\left(2 q_{1}\right) q_{2}}{r^{2}} \\
& F_{\mathrm{E}_{2}}=\frac{2 k q_{1} q_{2}}{r^{2}}
\end{aligned}
$$

From Figure 3 you can see that this corresponds to the force on $q_{2}$ by $2 q_{1}$ in the negative $x$-direction, again because like charges repel. This is because the electric force acts along the line connecting the two charges, which in this case is the $x$-axis. For this reason, the component of the force along the $y$-axis is again zero. So, the force $F_{\mathrm{E}_{2}}$ exerted by $2 q_{1}$ on $q_{2}$ is in the negative $x$-direction, or to the left. The total force on $q_{2}$ is the sum of the two electric forces, $F_{\mathrm{E}_{1}}$ and $F_{\mathrm{E}_{2}}$. Be careful to take direction into account for the calculation. In this case, the sign of each force indicates its direction.

$$
\begin{aligned}
\vec{F}_{\mathrm{E}_{\text {net }}} & =\vec{F}_{\mathrm{E}_{1}}+\vec{F}_{\mathrm{E}_{2}} \\
& =+\frac{k q_{1} q_{2}}{r^{2}}-\frac{2 k q_{1} q_{2}}{r^{2}} \\
\vec{F}_{\mathrm{E}_{\text {net }}} & =-\frac{k q_{1} q_{2}}{r^{2}}
\end{aligned}
$$

The negative sign means that, for the positive product $q_{1} q_{2}$, the force exerted on $q_{2}$ is along the negative $x$-direction.


Figure 3 The total force on $q_{2}$ equals the sum of the forces from charges $q_{1}$ and $2 q_{1}$.
superposition principle the resultant, or net, vector acting at a given point equals the sum of the individual vectors from all sources, each calculated at the given point

The equation for Coulomb's law is more accurately represented by $\left|\vec{F}_{\mathrm{E}}\right|=\frac{k\left|q_{1}\right|\left|q_{2}\right|}{r^{2}}$. Here, the vertical bars surrounding the electric force vector, $\left|\vec{F}_{E}\right|$, represent the magnitude of the electric force, and the vertical bars surrounding the charges $q_{1}$ and $q_{2}$, $\left|q_{1}\right|$ and $\left|q_{2}\right|$, represent the absolute values of the electric charges. This representation is more accurate because the magnitude of the electric force is a strictly positive quantity. If you just use the equation $F_{\mathrm{E}}=\frac{k q_{1} q_{2}}{r^{2}}$, you will get a negative value for the magnitude of the electric force when one of the charges is negative. The absolute value bars keep the values of the charges positive. However, many people find the more accurate version of the equation cumbersome and confusing and prefer to use $F_{\mathrm{E}}=\frac{k q_{1} q_{2}}{r^{2}}$. This is what we will do when solving problems. Always remember to drop the negative signs on charges when using this equation.

The following Tutorial will give you some practice in solving problems using Coulomb's law.

## Tutorial 1 Using Coulomb's Law

This Tutorial shows how to use Coulomb's law to solve charge distribution problems
in both one and two dimensions.

## Sample Problem 1: Applying Coulomb's Law in One Dimension

An early model of the hydrogen atom depicted the electron revolving around the proton, much like Earth revolving around the Sun. The proton and electron both have mass, so they exert a gravitational force upon each other. They also have charge, so they exert an electric force on each other.
(a) The distance between the electron and the proton in a hydrogen atom is $5.3 \times 10^{-11} \mathrm{~m}$, the charge of each is $1.6 \times 10^{-19} \mathrm{C}$, the mass of the electron is $9.11 \times 10^{-31} \mathrm{~kg}$, and the mass of the proton is $1.67 \times 10^{-27} \mathrm{~kg}$. Calculate the ratio of the electric force $F_{\mathrm{E}}$ to the gravitational force $F_{g}$.
(b) Determine the accelerations of the electron caused by both the electric force and the gravitational force.

## Solution

(a) Given: $r=5.3 \times 10^{-11} \mathrm{~m} ; q=1.6 \times 10^{-19} \mathrm{C}$;
$m_{\mathrm{e}}=9.11 \times 10^{-31} \mathrm{~kg} ; m_{\mathrm{p}}=1.67 \times 10^{-27} \mathrm{~kg} ;$
$k=8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2} ; G=6.67 \times 10^{-11} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{kg}^{2}$
Required: $\frac{F_{E}}{F_{g}}$
Analysis: Use Coulomb's law to calculate $F_{\mathrm{E}}$, and use the equation for universal gravitation to calculate $F_{\mathrm{g}}$.
$F_{\mathrm{g}}=\frac{G m_{\mathrm{e}} m_{\mathrm{p}}}{r^{2}}$ and $F_{\mathrm{E}}=\frac{k q_{e} q_{\mathrm{p}}}{r^{2}}$, where $q_{\mathrm{e}}=q_{\mathrm{p}}$.
From these results, calculate $\frac{F_{\mathrm{E}}}{F_{g}}$.
Solution: $F_{\mathrm{E}}=\frac{k q_{e} q_{\mathrm{p}}}{r^{2}}$

$$
\begin{aligned}
& =\frac{\left(8.99 \times 10^{9} \mathrm{~N} \cdot \frac{\mathrm{~m}^{2}}{\mathrm{C}^{2}}\right)\left(1.6 \times 10^{-19} \mathrm{C}\right)^{2}}{\left(5.3 \times 10^{-11} \mathrm{nt}\right)^{2}} \\
F_{\mathrm{E}} & =8.193 \times 10^{-8} \mathrm{~N}(\text { two extra digits carried })
\end{aligned}
$$

$$
\begin{aligned}
F_{\mathrm{g}} & =\frac{G m_{\mathrm{e}} m_{\mathrm{p}}}{r^{2}} \\
& =\frac{\left(6.67 \times 10^{-11} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{~kg}^{2}}\right)\left(9.11 \times 10^{-31} \mathrm{~kg}\right)\left(1.67 \times 10^{-27} \mathrm{~kg}\right)}{\left(5.3 \times 10^{-11} \mathrm{kt}\right)^{2}} \\
F_{\mathrm{g}} & =3.613 \times 10^{-47} \mathrm{~N}(\text { two extra digits carried }) \\
\frac{F_{\mathrm{E}}}{F_{\mathrm{g}}} & =\frac{8.193 \times 10^{-8} \mathrm{~N}}{3.613 \times 10^{-47} \mathrm{~N}} \\
\frac{F_{\mathrm{E}}}{F_{\mathrm{g}}} & =2.3 \times 10^{39}
\end{aligned}
$$

Statement: The electric force $F_{\mathrm{E}}$ between the electron and the proton of a hydrogen atom is $2.3 \times 10^{39}$ times the gravitational force $F_{g}$ between these same particles.
(b) Given: $m_{\mathrm{e}}=9.11 \times 10^{-31} \mathrm{~kg} ; F_{\mathrm{g}}=3.613 \times 10^{-47} \mathrm{~N}$;
$F_{\mathrm{E}}=8.193 \times 10^{-8} \mathrm{~N}$

## Required: $a_{\mathrm{E}} ; a_{\mathrm{g}}$

Analysis: Use the equation for electric force to calculate $a_{\mathrm{E}}$ using $F_{\mathrm{E}}$ and $m_{\mathrm{e}}$. Likewise, use the equation for gravitational force to calculate $a_{g}$ using the values of $F_{\mathrm{g}}$ and $m_{\mathrm{e}}$.
$a_{\mathrm{E}}=\frac{F_{\mathrm{E}}}{m_{\mathrm{e}}}$ and $a_{\mathrm{g}}=\frac{F_{\mathrm{g}}}{m_{\mathrm{e}}}$. Use $1 \mathrm{~N}=1 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}^{2}$.
Solution: $a_{\mathrm{E}}=\frac{F_{\mathrm{E}}}{m_{\mathrm{e}}}$

$$
\begin{aligned}
= & \frac{8.193 \times 10^{-8} \mathrm{~kg} \cdot \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}{9.11 \times 10^{-31} \mathrm{~kg}} \\
a_{\mathrm{E}} & =9.0 \times 10^{22} \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

$$
\begin{aligned}
a_{g} & =\frac{F_{g}}{m_{e}} \\
& =\frac{3.613 \times 10^{-47} \mathrm{~kg} \cdot \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}{9.11 \times 10^{-31} \mathrm{~kg}} \\
a_{g} & =4.0 \times 10^{-17} \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

Statement: The acceleration of the electron caused by the electric force of the proton is $9.0 \times 10^{22} \mathrm{~m} / \mathrm{s}^{2}$, and the acceleration of the electron caused by the gravitational force is $4.0 \times 10^{-17} \mathrm{~m} / \mathrm{s}^{2}$.

## Sample Problem 2: Determining Electrostatic Equilibrium

Two charges, $q_{1}=-2.00 \times 10^{-6} \mathrm{C}$ and $q_{2}=-1.80 \times 10^{-5} \mathrm{C}$, are separated by a distance, $L$, of 4.00 m . A third charge, $q_{3}=+1.50 \times 10^{-6} \mathrm{C}$, is placed somewhere between $q_{1}$ and $q_{2}$, as shown in Figure 4, where the net force exerted on $q_{3}$ by the other two charges is zero. Determine the location of $q_{3}$.


Figure 4
Given: $L=4.00 \mathrm{~m} ; q_{1}=-2.00 \times 10^{-6} \mathrm{C} ; q_{2}=-1.80 \times 10^{-5} \mathrm{C}$; $q_{3}=+1.50 \times 10^{-6} \mathrm{C} ; k=8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}$
Required: $x$, the location of $q_{3}$
Analysis: Use the superposition principle, and use the equation for Coulomb's law to calculate the forces $F_{\mathrm{E}_{1}}$ exerted on $q_{3}$ by $q_{1}$ and $F_{E_{2}}$ exerted on $q_{3}$ by $q_{2}$.
The force equations are $F_{\mathrm{E}_{1}}=\frac{k q_{1} q_{3}}{x^{2}}$ and $F_{\mathrm{E}_{2}}=\frac{k q_{2} q_{3}}{(L-x)^{2}}$.
Since the net force on $q_{3}$ is zero, $F_{\mathrm{E}_{1}}-F_{\mathrm{E}_{2}}=0$, or $F_{\mathrm{E}_{1}}=F_{\mathrm{E}_{2}}$.
Solution: $\quad F_{\mathrm{E}_{1}}=F_{\mathrm{E}_{2}}$

$$
\frac{k q_{1} q_{3}}{x^{2}}=\frac{k q_{2} q_{3}}{(L-x)^{2}}
$$

Divide both sides of the equation by the common terms $k$ and $q_{3}$, and then simplify.

$$
\begin{aligned}
\frac{q_{1}}{x^{2}} & =\frac{q_{2}}{(L-x)^{2}} \\
q_{1}(L-x)^{2} & =q_{2} x^{2} \\
q_{1}\left(L^{2}-2 L x+x^{2}\right) & =q_{2} x^{2} \\
q_{1} L^{2}-2 q_{1} L x+q_{1} x^{2} & =q_{2} x^{2} \\
\left(q_{2}-q_{1}\right) x^{2}+2 q_{1} L x-q_{1} L^{2} & =0 \\
\frac{\left(q_{2}-q_{1}\right)}{q_{1}} x^{2}+2 L x-L^{2} & =0
\end{aligned}
$$

Substitute the values for $q_{1}, q_{2}$, and $L$ into the equation, noting that $x$ is in metres with three significant digits. Solve for $x$.

$$
\begin{aligned}
& \frac{\left(1.80 \times 10^{-5} \mathrm{C}-2.00 \times 10^{-6} \mathrm{C}\right)}{2.00 \times 10^{-6} \mathrm{C}} x^{2}+ \\
& 2(4.00) x-(4.00)^{2}=0 \\
&\left(\frac{1.60 \times 10^{-5} \mathrm{C}}{2.00 \times 10^{-6} \mathrm{C}}\right) x^{2}+(8.00) x-16.0=0 \\
& 8.00 x^{2}+(8.00) x-16.0=0 \\
& x^{2}+x-2=0 \\
&(x-1)(x+2)=0 \\
& x=1 \text { or } x=-2
\end{aligned}
$$

Distance is always a positive quantity, so $x=1$.
Statement: The location of $q_{3}$, such that the electric forces from $q_{1}$ and $q_{2}$ cancel, is $x=1.00 \mathrm{~m}$, or 1.00 m to the right of $q_{1}$.

## Sample Problem 3: Applying Coulomb's Law in Two Dimensions

Two point particles have equal but opposite charges of $+q_{1}$ and $-q_{1}$. The particles are arranged as shown in Figure 5. Suppose a charge $q_{2}$ is placed on the $x$-axis as shown. $q_{1}=5.0 \times 10^{-6} \mathrm{C}$, $q_{2}=1.0 \times 10^{-6} \mathrm{C}$, and the distance between $+q_{1}$ and $-q_{1}$ is 8.0 m measured vertically along the $y$-axis. Calculate the magnitude and the direction of the net electric force on $q_{2}$.
Given: $q_{1}=5.0 \times 10^{-6} \mathrm{C} ;-q_{1}=-5.0 \times 10^{-6} \mathrm{C}$;
$q_{2}=1.0 \times 10^{-6} \mathrm{C} ; k=8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}$
Required: net electric force on $q_{2}$
Analysis: As shown in Figure 5, there are symmetrical right triangles above and below the $x$-axis, so we can use the Pythagorean theorem to calculate $r$ (the distance separating $+q_{1}$ and $q_{2}$. Then use $r$ to calculate the electric force between $+q_{1}$ and $q_{2}$ and that between $-q_{1}$ and $q_{2}$. Use trigonometry to calculate
the horizontal and vertical components of these two forces to determine the magnitude and direction of the net electric force. Use Coulomb's law to calculate the force between the charges.

Figure 5

Solution: First, calculate the magnitudes of the electric forces along the horizontal and vertical directions. Both of the forces along the $x$-axis act on the left side of the charge $q_{2}$, so the net horizontal force is the sum of the two horizontal force components.

Use the Pythagorean theorem to calculate $r$, the distance between $+q_{1}$ and $q_{2}$, and that between $-q_{1}$ and $q_{2}$.
$r=\sqrt{(4.0 \mathrm{~m})^{2}+(3.0 \mathrm{~m})^{2}}$
$r=5.0 \mathrm{~m}$
Calculate the force of $+q_{1}$ on $q_{2}, F_{\mathrm{E}_{1}}$.
$F_{\mathrm{E}_{1}}=\frac{k q_{1} q_{2}}{r^{2}}$

$$
=\frac{\left(8.99 \times 10^{9} \frac{\mathrm{~N} \cdot \mathrm{~m}^{2}}{\mathrm{C}^{2}}\right)\left(5.0 \times 10^{-6} \mathrm{C}\right)\left(1.0 \times 10^{-6} \mathrm{C}\right)}{(5.0 \mathrm{~m})^{2}}
$$

$F_{\mathrm{E}_{1}}=1.798 \times 10^{-3} \mathrm{~N}$ (two extra digits carried)
The force between a positive charge and another positive charge is repulsion, so this force is directed toward $q_{2}$ along the line connecting $+q_{1}$ and $q_{2}$.
Calculate the force of $-q_{1}$ on $q_{2}, F_{\mathrm{E}_{2}}$.
Since the magnitude of $-q_{1}$ is the same as that of $+q_{1}$, the magnitude of $F_{\mathrm{E}_{2}}$ is the same as that of $F_{\mathrm{E}_{1}}$.
$F_{\mathrm{E}_{2}}=F_{\mathrm{E}_{1}}$
$F_{\mathrm{E}_{2}}=1.798 \times 10^{-3} \mathrm{~N}$ (two extra digits carried)
The force between a negative charge and a positive charge is attraction, so this force is directed away from $q_{2}$ along the line connecting $-q_{1}$ and $q_{2}$.
Now calculate the $x$-component of $F_{\mathrm{E}_{1}}, F_{\mathrm{Ex}_{1}}$.
Let $\theta$ be the angle that $F_{\mathrm{E}_{1}}$ makes with the $x$-axis.
$\frac{F_{\mathrm{Ex}_{1}}}{F_{\mathrm{E}_{1}}}=\cos \theta$
$F_{\mathrm{EX}_{1}}=F_{\mathrm{E}_{1}} \cos \theta$
$=\left(1.798 \times 10^{-3} \mathrm{~N}\right)\left(\frac{3.0 \mathrm{mf}}{5.0 \mathrm{mI}}\right)$
$F_{\mathrm{EX}_{X_{1}}}=1.079 \times 10^{-3} \mathrm{~N}$ (two extra digits carried)
$F_{\mathrm{E}_{1}}$ is directed toward $q_{2}$, so $F_{\mathrm{Ex}_{1}}$ is directed toward $q_{2}$ along the positive $x$-direction.
By symmetry, the $x$-component of $F_{\mathrm{E}_{2}}, F_{\mathrm{Ex}_{2}}$, must have the same magnitude as the $x$-component of $F_{\mathrm{E}_{1}}$ and is directed away from $q_{2}$ along the negative $x$-direction.
$F_{E X_{2}}=F_{\mathrm{EX}_{1}}$
$F_{\mathrm{Ex}_{2}}=1.079 \times 10^{-3} \mathrm{~N}$ (two extra digits carried)
Determine the vector sum of the two horizontal force components.
$\vec{F}_{E x_{\text {net }}}=\vec{F}_{E x_{1}}+\vec{F}_{E x_{2}}$

$$
=+1.079 \times 10^{-3} \mathrm{~N}-1.079 \times 10^{-3} \mathrm{~N}
$$

$\vec{F}_{E x_{\text {net }}}=0 \mathrm{~N}$
There is no net force on $q_{2}$ along the $x$-direction.
Now calculate the $y$-component of $F_{\mathrm{E}_{1}}, F_{\mathrm{E}_{1}{ }_{1}}$.
$\frac{F_{\mathrm{E}_{y_{1}}}}{F_{\mathrm{E}_{1}}}=\sin \theta$
$F_{\mathrm{E}_{1}}=F_{\mathrm{E}_{1}} \sin \theta$
$=\left(1.798 \times 10^{-3} \mathrm{~N}\right)\left(\frac{4.0 \mathrm{mf}}{5.0 \mathrm{mf}}\right)$
$F_{\mathrm{Ey}_{1}}=1.438 \times 10^{-3} \mathrm{~N}$ (two extra digits carried)
$F_{\mathrm{E}_{1}}$ is directed toward $q_{2}$ from $+q_{1}$, so $F_{\mathrm{E}_{1}}$ is directed toward $q_{2}$ downward.
Since $F_{\mathrm{E}_{2}}=F_{\mathrm{E}_{1}}$, the $y$-component of $F_{\mathrm{E}_{2}}, F_{\mathrm{E}_{2}}$, has the same magnitude as the $y$-component of $F_{\mathrm{E}_{1}}, F_{\mathrm{E}_{1}}$.
$F_{\mathrm{Ey}_{2}}=F_{\mathrm{E}_{y_{1}}}$
$F_{\mathrm{E} y_{2}}=1.438 \times 10^{-3} \mathrm{~N}$ (two extra digits carried)
$F_{\mathrm{E}_{2}}$ is directed away from $q_{2}$ from $-q_{1}$, so $F_{\mathrm{E}_{2}}$ is also directed downward.
Determine the vector sum of the two vertical force components. Choose upward as positive, so downward is negative.
$\vec{F}_{\mathrm{E} y_{\text {net }}}=\vec{F}_{\mathrm{E} y_{1}}+\vec{F}_{\mathrm{E} y_{2}}$
$=-1.438 \times 10^{-3} \mathrm{~N}+\left(-1.438 \times 10^{-3} \mathrm{~N}\right)$
$\vec{F}_{\mathrm{Ey}_{\text {net }}}=-2.9 \times 10^{-3} \mathrm{~N}$
Statement: The net electric force acting on $q_{2}$ is $2.9 \times 10^{-3} \mathrm{~N}$ [down].

## Practice

1. Determine the magnitude of the electric force between two charges of $1.00 \times 10^{-4} \mathrm{C}$ and $1.00 \times 10^{-5} \mathrm{C}$ that are separated by a distance of 2.00 m . TIT [ans: 2.25 N ]
2. Two particles of charge $q$ and $-2 q$ (with $q$ positive) are located as shown in Figure 6. A third charge $q_{3}$ is placed on the $x$-axis to the left of $q$. The total electric force on $q_{3}$ is zero. The charges are separated by 1.000 m . Determine where $q_{3}$ must be located. $\mathbb{K N U T I T}$ [ans: 2.414 m [left of $q$ ]]


Figure 6
3. Three point charges are placed at the following points on the $x$-axis: $+2.0 \mu \mathrm{C}$ at $x=0,-3.0 \mu \mathrm{C}$ at $x=40.0 \mathrm{~cm}$, and $-5.0 \mu \mathrm{C}$ at $x=120.0 \mathrm{~cm}$. Determine the force on the $-3.0 \mu \mathrm{C}$ charge.
TII [ans: 0.55 N toward the negative $x$-direction, or 0.55 N [left]]

### 7.2 Review

## Summary

- According to Coulomb's law, the force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between the charges, given as $F_{\mathrm{E}}=\frac{k q_{1} q_{2}}{r^{2}}$.
- Coulomb's constant, $k$, is equal to $8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}$.
- Coulomb's law applies to point charges and to charges that can be concentrated equivalently in points located at the centre, when the sizes of the charges are much smaller than their distance of separation.
- There are similarities between the electric force and the gravitational force.
- The superposition principle states that the total force acting on a charge $q$ is the vector sum, or superposition, of the individual forces exerted on $q$ by all the other charges in the problem.


## Questions

1. Two charged objects have a repulsive force of 0.080 N . The distance separating the two objects is tripled. Determine the new force. mim
2. Two charged objects have an attractive force of 0.080 N . Suppose that the charge of one of the objects is tripled and the distance separating the objects is tripled. Calculate the new force. ITI
3. Determine the magnitude of the electric force between two electrons separated by a distance of 0.10 nm (approximately the diameter of an atom).
4. Two point charges are separated by a distance $r$. Determine the factor by which the electric force between them changes when the separation is reduced by a factor of 1.50 .
5. Determine the distance of separation required for two $1.00 \mu \mathrm{C}$ charges to be positioned so that the repulsive force between them is equivalent to the weight (on Earth) of a 1.00 kg mass.
6. Consider an electron and a proton separated by a distance of 1.0 nm .
(a) Calculate the magnitude of the gravitational force between them.
(b) Calculate the magnitude of the electric force between them.
(c) Explain how the ratio of these gravitational and electric forces would change if the distance were increased to 1.0 m .
7. Particles of charge $q$ and $3 q$ are placed on the $x$-axis at $x=-40$ and $x=50$, respectively. A third particle of charge $q$ is placed on the $x$-axis, and the total electric force on this particle is zero. Determine the position of the particle.
8. Two charges of $2.0 \times 10^{-6} \mathrm{C}$ and $-1.0 \times 10^{-6} \mathrm{C}$ are placed at a separation of 10 cm . Determine where a third charge should be placed on the line connecting the two charges so that it experiences no net force due to these two charges.
9. Three charges with $q=+7.5 \times 10^{-6} \mathrm{C}$ are located as shown in Figure 7, with $L=25 \mathrm{~cm}$. Determine the magnitude and direction of the total electric force on each particle listed below.


Figure 7
(a) the charge at the bottom
(b) the charge on the right
(c) an electron placed at the origin
10. Two pith balls, each with a mass of 5.00 g , are attached to non-conducting threads and suspended from the same point on the ceiling. Each thread has a length of 1.00 m . The balls are then given an identical charge, which causes them to separate. At the point that the electric and gravitational forces balance, the threads are separated by an angle of $30.0^{\circ}$. Calculate the charge on each pith ball.

