6.1

Newtonian Gravitation

Early in the formation of our galaxy, tiny gravitational effects between particles began to draw matter together into slightly denser configurations. Those, in turn, exerted even greater gravitational forces, resulting in more mass joining the newly forming structures. Eventually, those repetitive and continuous gravitational effects formed and shaped our Milky Way galaxy, as depicted in **Figure 1**. The same process of gravitational attraction—on different scales—accounts for the overall structure of the entire universe, despite being the weakest of the four fundamental forces.

Gravity accounts for how the planets in our solar system move and orbit around the Sun. By the late 1700s, scientists had identified all the inner terrestrial planets as well as the gas giants, Jupiter and Saturn. Then, British astronomer William Herschel (1738–1822) used observations of the relative movements of the stars to determine that a presumed "star" was actually an additional planet. The new planet was Uranus. Scientists then observed that Uranus's path was anomalous. It seemed to respond to the pull of another distant but unknown body. Using mathematical analysis, scientists predicted where the unknown body would have to be and began searching for it. In 1846, scientists discovered the planet Neptune (**Figure 2**). CAREER LINK

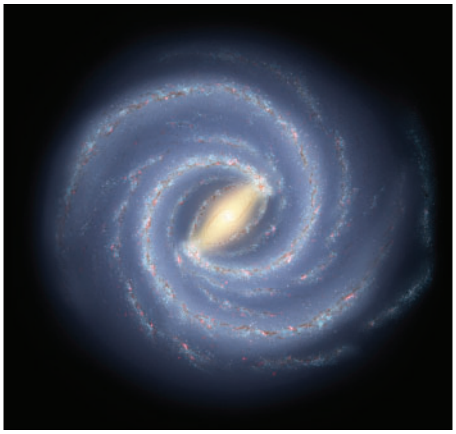

Figure 1 Our galaxy, the Milky Way, was shaped by gravitational forces, depicted here in an artist's conception.

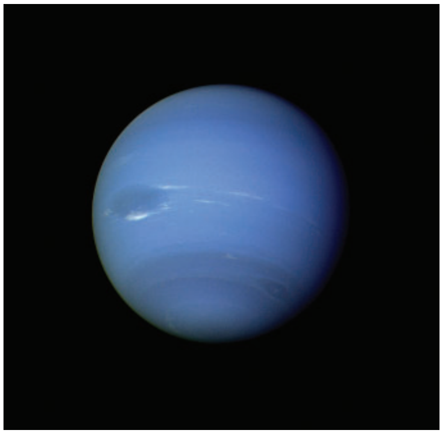

Figure 2 Scientists discovered Neptune by observing its gravitational effects on Uranus.

Universal Gravitation

The force that causes Uranus to wobble slightly in its orbit is gravity—the same force that causes Earth and the other planets to revolve around the Sun. Sir Isaac Newton, whose laws of motion provide the foundation of our study of mechanics, used known data about the solar system to describe the system of physical laws that govern the movement of celestial bodies around the Sun. Through this inquiry, he formulated the **universal law of gravitation**.

Universal Law of Gravitation

There is a gravitational attraction between *any* two objects. If the objects have masses m_1 and m_2 and their centres are separated by a distance r, the magnitude of the gravitational force on either object is directly proportional to the product of m_1 and m_2 and inversely proportional to the square of r:

$$F_g = \frac{Gm_1m_2}{r^2}$$

G is a constant of nature called the **gravitational constant**, which is equal to $6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

gravitational constant a constant that appears in the universal law of gravitation; the constant is written as G and has a value of $6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$

Newton's law of gravitation plays a key role in physics for two reasons. First, his work showed for the first time that the laws of physics apply to all objects. The same force that causes a leaf to fall from a tree also keeps planets in orbit around the Sun. This fact had a profound effect on how people viewed the universe. Second, the law provided us with an equation to calculate and understand the motions of a wide variety of celestial objects, including planets, moons, and comets.

The gravitational force is always attractive (**Figure 3**). Every mass attracts every other mass. Therefore, the direction of the force of gravity on one mass (mass 1) due to a second mass (mass 2) points from the centre of mass 1 toward the centre of mass 2.

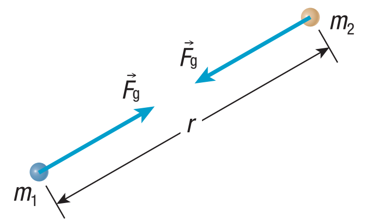

Figure 3 The gravitational force between two point masses m_1 and m_2 that are separated by a distance r is given in the universal law of gravitation.

The magnitude of the gravitational force exerted by mass 1 on mass 2 is equal to the magnitude of the gravitational force exerted by mass 2 on mass 1. Since the forces are both attractive, this result is precisely what we would expect from Newton's third law ($\vec{F}_{1\text{ on }2} = -\vec{F}_{2\text{ on }1}$). The two gravitational forces are an action–reaction pair because they are equal in magnitude and opposite in direction and they act on different members of the pair of objects.

Another important feature of the universal law of gravitation is that the force follows the inverse-square law. The **inverse-square law** is a mathematical relationship between variables in which one variable is proportional to the inverse of the square of the other variable. When applied to gravitational forces, this relationship means that the force is inversely proportional to the square of the distance between the mass centres, or $F \propto \frac{1}{r^2}$. In other words, the force of attraction drops quickly as the two objects move farther apart. No matter how large the distance between the mass centres, however, they will still experience a gravitational force. Every massive object in the universe exerts a force of attraction on every other massive object.

inverse-square law a mathematical relationship in which one variable is proportional to the inverse of the square of another variable; the law applies to gravitational forces and other phenomena, such as electric field strength and sound intensity

If both objects have a small mass compared to the distance between their centres, they will experience a small gravitational force. For the force to be noticeable, at least one of the objects must have a large mass relative to the distance between the object centres.

The Value of *g*

On Earth, we can calculate the acceleration due to the force of gravity, g, from the universal law of gravitation. Near Earth's surface, g has an approximate value of 9.8 m/s^2. The precise value of g, however, decreases with increasing height above Earth's surface based on the inverse-square law (**Table 1**). The value of g also varies on the surface of Earth because the surface varies in distance from the centre of Earth.

Table 1 Gravity versus Distance from Earth's Surface

Altitude (km)	g (m/s^2)
1 000	7.33
2 000	5.68
3 000	4.53
4 000	3.70
5 000	3.08
6 000	2.60
7 000	2.23
8 000	1.93
9 000	1.69
10 000	1.49
50 000	0.13

Calculating the Force of Gravity

The first measurement of the gravitational constant G was carried out in 1798 in a famous experiment by Henry Cavendish (1731–1810). Cavendish wanted to measure the gravitational force between two objects on Earth using two large lead spheres. He needed to use spheres with a very small distance between them or he would have found the gravitational force nearly impossible to observe. Cavendish arranged two large spheres of mass m_1 in a dumbbell configuration and suspended them from their centre point by a thin wire fibre, as shown in **Figure 4** on the next page. He placed another pair of large spheres with mass m_2 close to the suspended masses.

The gravitational forces between the pairs of masses, m_1 and m_2, caused the dumbbell to rotate. As the fibre twisted, tension in the fibre caused a force resisting the twist that increased as the rotation increased. At a certain angle, this twisting force balanced the gravitational force. By carefully measuring the angle θ, Cavendish could determine the force on the dumbbell, as well as the separation of the spheres. By also measuring the masses of the spheres and inserting the values into the universal law of gravitation, Cavendish could measure G.

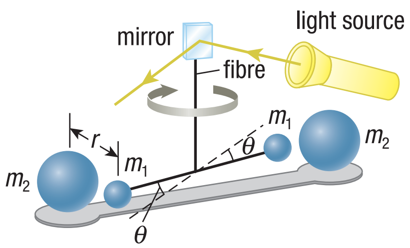

Figure 4 Cavendish used an apparatus like this one to measure the force of gravity between terrestrial objects. The amount that the light is deflected from its original path gives an indication of the angle of rotation θ.

Through this method, Cavendish measured the value of G: $6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2$. The constant G has this combination of units because it gives a gravitational force in newtons. When you multiply the units of G by two masses in kilograms and divide by the square of a distance in metres, you will be left with newtons. Following his calculation of G, Cavendish was able to calculate the mass of Earth. In fact, all masses of planetary bodies can be determined by using the universal law of gravitation. Although Cavendish conducted his experiment more than 200 years ago, his design still forms the basis for experimental studies of gravitation today. Tutorial 1 shows how to calculate the force of gravity between two spherical masses, as well as how to calculate the mass of a celestial body.

Investigation 6.1.1

Universal Gravitation (page 308)
You have learned the basic information about the universal law of gravitation. This investigation will give you an opportunity to verify this law through an observational study.

Tutorial 1 Calculating the Force of Gravity

The following Sample Problems involve the force of gravity.

Sample Problem 1: Calculating the Force of Gravity between Two Ordinary Masses

The centres of two uniformly dense spheres are separated by 50.0 cm. Each sphere has a mass of 2.00 kg.

(a) Calculate the magnitude of the gravitational force of attraction between the two spheres.

(b) How much of an effect will this force have on the two spheres?

Solution

(a) **Given:** $G = 6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2$; $r = 50.0\ \text{cm} = 0.500\ \text{m}$; $m_1 = m_2 = 2.00\ \text{kg}$

Required: F_g

Analysis: $F_g = \dfrac{Gm_1m_2}{r^2}$

Solution:
$$F_g = \frac{Gm_1m_2}{r^2}$$
$$= \frac{\left(6.67 \times 10^{-11}\ \frac{\text{N·}\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(2.00\ \cancel{\text{kg}})(2.00\ \cancel{\text{kg}})}{(0.500\ \cancel{\text{m}})^2}$$
$$F_g = 1.07 \times 10^{-9}\ \text{N}$$

Statement: The magnitude of the gravitational force of attraction between the two spheres is 1.07×10^{-9} N.

(b) The gravitational force of attraction between the two spheres (1.07×10^{-9} N) is too small to have any noticeable effect on the motion of these two spheres under normal circumstances.

Sample Problem 2: Calculating the Force of Gravity and Solving for Mass

Eris, a dwarf planet, is the ninth most massive body orbiting the Sun. It is more massive than Pluto and three times farther away from the Sun. Eris is estimated to have a radius of approximately 1200 km. Acceleration due to gravity on Eris differs from the value of g on Earth. In this three-part problem, you will explore the force of gravity on the surface of Eris.

(a) Suppose that an astronaut stands on Eris and drops a rock from a height of 0.30 m. The rock takes 0.87 s to reach the surface. Calculate the value of g on Eris.

(b) Calculate the mass of Eris.

(c) Suppose that an astronaut stands on Eris and drops a rock from a height of 2.50 m. Calculate how long it would take the rock to reach the surface.

Solution

(a) **Given:** $\Delta d = 0.30$ m; $v_i = 0$ m/s; $\Delta t = 0.87$ s

Required: g_{Eris}

Analysis: Near the surface of Eris, the gravitational acceleration will be approximately constant, like it is near the surface of Earth. We can determine the value of g_{Eris} by using the equation for the motion of an object falling under constant acceleration: $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2$. Since $v_i = 0$, the equation simplifies to $\Delta\vec{d} = \frac{1}{2}\vec{a}\Delta t^2$.

For this problem, $a = g_{Eris}$. Choose up as positive, so down is negative.

Solution:
$$\Delta\vec{d} = \frac{1}{2}\vec{a}\Delta t^2$$
$$\vec{a} = \frac{2\Delta\vec{d}}{\Delta t^2}$$
$$\vec{g}_{Eris} = \frac{2(-0.30\text{ m})}{(0.87\text{ s})^2}$$
$$\vec{g}_{Eris} = -0.7927\text{ m/s}^2 \text{ (two extra digits carried)}$$

Statement: The value of g on Eris is 0.79 m/s^2.

(b) **Given:** $g_{Eris} = 0.7927$ m/s^2; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2; $r = 1200$ km $= 1.2 \times 10^6$ m

Required: m_{Eris}

Analysis: Use the equations for the force of gravity, $F_g = mg$, and the universal law of gravitation, $F_g = \frac{Gm_1m_2}{r^2}$.

Solution: Equate these two expressions for the gravitational force on the rock, $F_g = m_{rock}g_{Eris}$ and $F_g = \frac{Gm_{rock}m_{Eris}}{r^2}$, and use 1 N = 1 kg·m/s^2.

$$F_g = F_g$$
$$m_{rock}g_{Eris} = \frac{Gm_{rock}m_{Eris}}{r^2}$$
$$m_{Eris} = \frac{g_{Eris}r^2}{G}$$
$$= \frac{\left(0.7927\ \frac{\cancel{m}}{\cancel{s^2}}\right)(1.2 \times 10^6\ \cancel{m})^2}{6.67 \times 10^{-11}\ \frac{\cancel{kg}\cdot\cancel{m}}{\cancel{s^2}}\cdot\frac{\cancel{m^2}}{kg^{\cancel{2}}}}$$
$$m_{Eris} = 1.7 \times 10^{22}\text{ kg}$$

Statement: The mass of Eris is 1.7×10^{22} kg.

(c) **Given:** $\Delta d = 2.50$ m; $v_i = 0$ m/s; $a = g_{Eris} = 0.7927$ m/s^2

Required: Δt

Analysis: Since the value of g_{Eris} is known, we can use the equation $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2$ to determine the time it would take the rock to reach the surface. Since $v_i = 0$, the equation simplifies to $\Delta\vec{d} = \frac{1}{2}\vec{a}\Delta t^2$. Choose up as positive, so down is negative.

Solution:
$$\Delta\vec{d} = \frac{1}{2}\vec{a}\Delta t^2$$
$$\Delta t^2 = \frac{2\Delta\vec{d}}{\vec{a}}$$
$$\Delta t = \sqrt{\frac{2\Delta\vec{d}}{\vec{a}}}$$
$$= \sqrt{\frac{2(-2.50\ \cancel{m})}{-0.7927\ \frac{\cancel{m}}{s^2}}}$$
$$\Delta t = 2.5\text{ s}$$

Statement: It would take the rock 2.5 s to fall 2.50 m.

Sample Problem 3: Calculating the Force of Gravity in a Three-Body System

Figure 5 shows three large, spherical asteroids in space, which are arranged at the corners of a right triangle ABC. Asteroid A has a mass of 1.0×10^{20} kg. Asteroid B has a mass of 2.0×10^{20} kg and is 50 million kilometres (5.0×10^{10} m) from asteroid A. Asteroid C has a mass of 4.0×10^{20} kg and is 25 million kilometres (2.5×10^{10} m) away from asteroid A along the other side of the triangle.

(a) Determine the net force on asteroid A from asteroids B and C.

(b) Determine the net force on asteroid B from asteroid C.

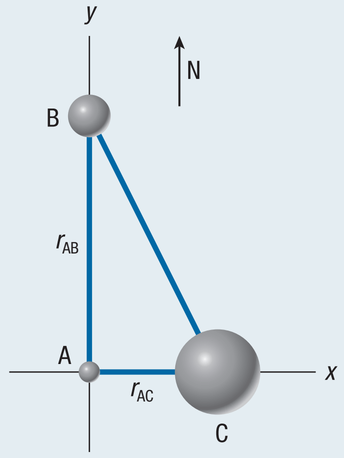

Figure 5

Solution

(a) **Given:** $m_A = 1.0 \times 10^{20}$ kg; $m_B = 2.0 \times 10^{20}$ kg; $m_C = 4.0 \times 10^{20}$ kg; $r_{AB} = 5.0 \times 10^{10}$ m; $r_{AC} = 2.5 \times 10^{10}$ m

Required: $\vec{F}_{\text{net A}}$

Analysis: The force of gravity on mass m_1 due to mass m_2 is $F_g = \frac{Gm_1m_2}{r^2}$ directed from the centre of m_1 toward the centre of m_2. The force on asteroid A from asteroid B will be along side AB, and the force on asteroid A from asteroid C will be along side AC of triangle ABC. So, use the Pythagorean theorem to determine the magnitude of the net force on asteroid A from asteroids B and C, $F_{\text{net A}} = \sqrt{F_{AB}^2 + F_{AC}^2}$, and use trigonometry to calculate the angle that the net force makes with side AC: $\theta = \tan^{-1}\left(\frac{F_{AB}}{F_{AC}}\right)$.

Solution: Calculate the force on asteroid A due to asteroid B, F_{AB}.

$$F_{AB} = \frac{Gm_Am_B}{r_{AB}^2} = \frac{\left(6.67 \times 10^{-11} \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(1.0 \times 10^{20}\ \cancel{\text{kg}})(2.0 \times 10^{20}\ \cancel{\text{kg}})}{(5.0 \times 10^{10}\ \cancel{\text{m}})^2}$$

$F_{AB} = 5.336 \times 10^8$ N (two extra digits carried)

Calculate the force on asteroid A due to asteroid C, F_{AC}.

$$F_{AC} = \frac{Gm_Am_C}{r_{AC}^2} = \frac{\left(6.67 \times 10^{-11} \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(1.0 \times 10^{20}\ \cancel{\text{kg}})(4.0 \times 10^{20}\ \cancel{\text{kg}})}{(2.5 \times 10^{10}\ \cancel{\text{m}})^2}$$

$F_{AC} = 4.269 \times 10^9$ N (two extra digits carried)

$\vec{F}_{\text{net A}}$ is the vector sum of $\vec{F}_{AB}$ and $\vec{F}_{AC}$. Since the force vectors lie along the sides of a right triangle, we can use the Pythagorean theorem to calculate the magnitude of the net force on asteroid A, $F_{\text{net A}}$.

$$F_{\text{net A}} = \sqrt{F_{AB}^2 + F_{AC}^2} = \sqrt{(5.336 \times 10^8\ \text{N})^2 + (4.269 \times 10^9\ \text{N})^2}$$

$F_{\text{net A}} = 4.3 \times 10^9$ N

Now calculate the angle that the net force on asteroid A makes with side AC.

$$\theta = \tan^{-1}\left(\frac{F_{AB}}{F_{AC}}\right) = \tan^{-1}\left(\frac{5.336 \times 10^8\ \cancel{\text{N}}}{4.269 \times 10^9\ \cancel{\text{N}}}\right)$$

$\theta = 7.1°$

Statement: The net force on asteroid A from asteroids B and C is 4.3×10^9 N [E 7.1° N], directed toward asteroid A.

(b) **Given:** $m_B = 2.0 \times 10^{20}$ kg; $m_C = 4.0 \times 10^{20}$ kg; $r_{AB} = 5.0 \times 10^{10}$ m; $r_{AC} = 2.5 \times 10^{10}$ m

Required: $\vec{F}_{BC}$

Analysis: Use the Pythagorean theorem to determine the distance between asteroid B and asteroid C, $r_{BC} = \sqrt{r_{AB}^2 + r_{AC}^2}$. Then use the universal law of gravitation, $F_g = \frac{Gm_1m_2}{r^2}$, directed from the centre of m_1 toward the centre of m_2. The force will act along the hypotenuse of triangle ABC, so the angle that the force makes with side AB is given by $\theta = \tan^{-1}\left(\frac{r_{AC}}{r_{AB}}\right)$.

Solution: Calculate the distance between asteroid B and asteroid C.

$$r_{BC} = \sqrt{r_{AB}^2 + r_{AC}^2} = \sqrt{(5.0 \times 10^{10}\ \text{m})^2 + (2.5 \times 10^{10}\ \text{m})^2}$$

$r_{BC} = 5.590 \times 10^{10}$ m (two extra digits carried)

Calculate the magnitude of the gravitational force between asteroid B and asteroid C.

$$F_{BC} = \frac{Gm_Bm_C}{r_{BC}^2} = \frac{\left(6.67 \times 10^{-11} \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(2.0 \times 10^{20}\ \cancel{\text{kg}})(4.0 \times 10^{20}\ \cancel{\text{kg}})}{(5.590 \times 10^{10}\ \cancel{\text{m}})^2}$$

$F_{BC} = 1.7 \times 10^9$ N

As Figure 5 indicates, the net force on asteroid B due to asteroid C acts along side BC. Calculate the angle that this force makes with side AB.

$$\theta = \tan^{-1}\left(\frac{r_{AC}}{r_{AB}}\right) = \tan^{-1}\left(\frac{2.5 \times 10^{10}\ \cancel{\text{m}}}{5.0 \times 10^{10}\ \cancel{\text{m}}}\right)$$

$\theta = 27°$

Statement: The force on asteroid B due to asteroid C is 1.7×10^9 N [S 27° E], directed toward asteroid B.

Practice

1. Two spherical asteroids have masses as follows: $m_1 = 1.0 \times 10^{20}$ kg and $m_2 = 3.0 \times 10^{20}$ kg. The magnitude of the force of attraction between the two asteroids is 2.2×10^9 N. Calculate the distance between the two asteroids. T/I [ans: 3.0×10^{10} m]
2. Jupiter has a mass of 1.9×10^{27} kg and a mean radius at the equator of 7.0×10^7 m. Calculate the magnitude of g on Jupiter, if it were a perfect sphere with that radius. T/I [ans: 26 m/s^2]
3. Uniform spheres A, B, and C have the following masses and centre-to-centre distances: $m_A = 40.0$ kg, $m_B = 60.0$ kg, and $m_C = 80.0$ kg; $r_{AB} = 0.50$ m and $r_{BC} = 0.75$ m. If the only forces acting on B are the gravitational forces due to A and C, determine the net force acting on B with the spheres arranged as in **Figures 6(a)** and **(b)**. T/I [ans: (a) 7.1×10^{-8} N [left]; (b) 8.6×10^{-7} N [W 42° S]]

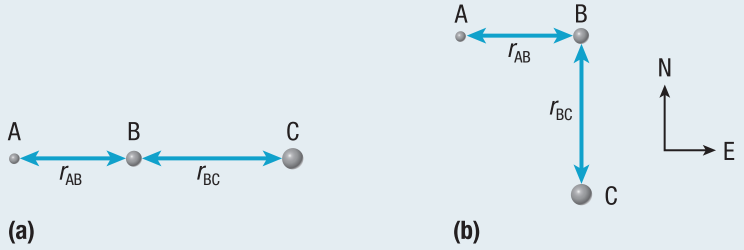

Figure 6

Gravitational Fields

The universal law of gravitation tells us that at any point in space surrounding a massive object, such as Earth, we can calculate the gravitational force on a second object sitting at that point in space. Earth has a mean radius of approximately 6380 km. So an object that is 10 km above Earth's surface, or 6390 km from Earth's centre, will have the same gravitational attraction to Earth no matter which land mass or ocean it is positioned above. A vector exists at every point in space surrounding the central object, pointing toward it and depending on the object's mass and the distance from its centre (**Figure 7**). The **gravitational field** of the central object can be represented by this collection of vectors. A gravitational field exerts forces on objects with mass. The **gravitational field strength** is the force of attraction per unit mass of an object placed in a gravitational field, and it equals the gravitational force on the object divided by the object's mass. On Earth, the gravitational field strength is approximately 9.8 N/kg. Notice that this has the same magnitude as the acceleration due to gravity on Earth's surface, and thus has the same symbol, g.

gravitational field a collection of vectors, one at each point in space, that determines the magnitude and direction of the gravitational force

gravitational field strength the magnitude of the gravitational field vector at a point in space

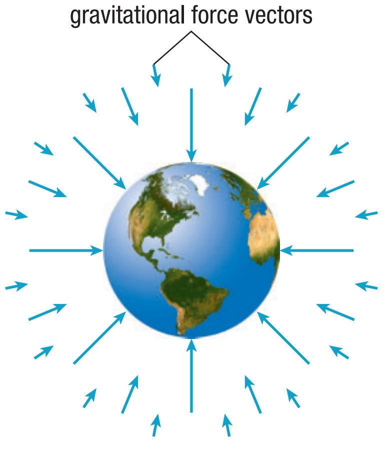

Figure 7 Earth's gravitational field strength diminishes with increasing distance from the planet's centre.

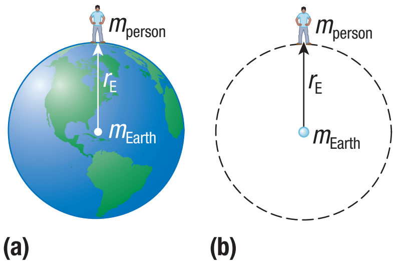

Figure 8 If we approximate Earth as a sphere, we can assume that the gravitational force that Earth exerts on a person or an object is equal to the force experienced if all the mass were located at Earth's centre.

For spherical objects, the strength of the gravitational field at a distance from the surface is the same whether the mass actually fills its volume or sits at a point in the centre. We can therefore use the gravitational force equation as though all of the object's mass were located at its centre; this is why we measure centre-to-centre distances (**Figure 8**).

To calculate the gravitational field strength as a function of a central spherical mass, we combine the universal law of gravitation with Newton's second law. Let us calculate the acceleration due to gravity g on a small mass m_{object} near the surface of a spherical planet of mass m_{planet} and radius r.

$$F_g = F_g$$

$$m_{object}g = \frac{Gm_{planet}m_{object}}{r^2}$$

$$g = \frac{Gm_{planet}}{r^2}$$

We can apply this formula to other planets and stars by substituting the appropriate values for m_{planet} and r. The value of g depends on the mass of the central body, the distance from that body's centre, and the gravitational constant, G. An object's acceleration due to gravity does not depend on its own mass. Tutorial 2 shows how to calculate the gravitational field strength on other planets.

Tutorial 2 Solving Problems Related to Gravitational Field Strength

In the following Sample Problem, you will learn how to calculate the gravitational field strength.

Sample Problem 1: Determining the Gravitational Field Strength

(a) Calculate the magnitude of the gravitational field strength on the surface of Saturn, assuming that it is perfectly spherical with a radius of 6.03×10^7 m. The mass of Saturn is 5.69×10^{26} kg.

(b) Determine the ratio of Saturn's gravitational field strength to Earth's gravitational field strength (9.8 N/kg).

Solution

(a) **Given:** $G = 6.67 \times 10^{-11}$ N·m^2/kg^2; $m_{Saturn} = 5.69 \times 10^{26}$ kg; $r = 6.03 \times 10^7$ m

Required: g_{Saturn}

Analysis: $g_{Saturn} = \frac{Gm_{Saturn}}{r^2}$

Solution:

$$g_{Saturn} = \frac{Gm_{Saturn}}{r^2}$$

$$= \frac{\left(6.67 \times 10^{-11} \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(5.69 \times 10^{26}\ \cancel{\text{kg}})}{(6.03 \times 10^7\ \cancel{\text{m}})^2}$$

$g_{Saturn} = 10.438$ N/kg (two extra digits carried)

Statement: The gravitational field strength on the surface of Saturn is 10.4 N/kg.

(b) **Given:** $g_{Saturn} = 10.438$ N/kg; $g_{Earth} = 9.8$ N/kg

Required: $g_{Saturn} : g_{Earth}$

Analysis:

Calculate $\frac{g_{Saturn}}{g_{Earth}}$.

Solution:

$$\frac{g_{Saturn}}{g_{Earth}} = \frac{10.438 \frac{\cancel{\text{N}}}{\cancel{\text{kg}}}}{9.8 \frac{\cancel{\text{N}}}{\cancel{\text{kg}}}}$$

$$\frac{g_{Saturn}}{g_{Earth}} = 1.1$$

Statement: The ratio of Saturn's gravitational field strength to Earth's gravitational field strength is 1.1:1.

Practice

1. The radius of a typical white dwarf star is just a little larger than the radius of Earth, but a typical white dwarf has a mass that is similar to the Sun's mass. Calculate the surface gravitational field strength of a white dwarf with a radius of 7.0×10^6 m and a mass of 1.2×10^{30} kg. Compare this to the surface gravitational field strength of Earth. T/I A [ans: 1.6×10^6 N/kg]
2. Suppose that Saturn expanded until its radius doubled, while its mass stayed the same. Determine the gravitational field strength on the new surface relative to the old surface. T/I [ans: $\frac{1}{4} g_{\text{Saturn}}$]

Tutorial 2 demonstrates that the gravitational field strength on the surface of Saturn is only slightly greater than the gravitational field strength on the surface of Earth. Each planet in our solar system has a different gravitational field strength that depends on the radius and the mass of the planet. **Table 2** lists the relative values for all the planets in our solar system.

Table 2 Surface Gravitational Field Strength of the Planets in the Solar System

Planet	Value of g_{planet} relative to Earth	Value of g (N/kg)
Mercury	0.38	3.7
Venus	0.90	8.8
Earth	1.00	9.8
Mars	0.38	3.7
Jupiter	2.53	24.8
Saturn	1.06	10.4
Uranus	0.90	8.8
Neptune	1.14	11.2

Research This

Gravitational Field Maps and Unmanned Underwater Vehicles

Skills: Researching, Analyzing, Communicating

SKILLS HANDBOOK A4.1

Unmanned underwater vehicles (UUVs) navigate underwater without a human driver to conduct searches, collect images, or recover submerged materials. Some UUVs are operated remotely by a human pilot, and others operate independently. To ensure that UUVs stay on course, gravitational field maps are used to correct errors in UUV navigation systems.

1. Research UUVs (also called autonomous underwater vehicles, or AUVs), and locate several examples of both human-operated and independent UUVs.
2. Research gravitational field maps and how they work. Find one example of a visualization of gravitational field data.
3. Research the factors that affect gravitational fields and why and how the fields can vary.
4. Determine how gravitational field maps are used to correct UUV navigation systems.

A. Explain in your own words how a gravitational field map works, how it is created, and how it is used. T/I A

B. Draw a diagram highlighting the design features and functions of two UUV examples. K/U C

C. Create a one-page report or short presentation outlining how gravitational field maps are used to correct UUV navigation systems. K/U C A

WEB LINK

6.1 Review

Summary

- The universal law of gravitation states that the force of gravitational attraction between any two objects is directly proportional to the product of the masses of the objects and inversely proportional to the square of the distance between their centres: $F_g = \frac{Gm_1m_2}{r^2}$.
- The gravitational constant, $G = 6.67 \times 10^{-11}$ N·m^2/kg^2, was first determined experimentally by Henry Cavendish in 1798.
- The gravitational field can be represented by a vector at each point in space. A gravitational field exerts forces on objects with mass. The gravitational field strength at a distance r from a body of mass m equals the magnitude of gravitational acceleration at that distance: $g = \frac{Gm}{r^2}$.

Questions

1. At what altitude above Earth would your weight be one-half your weight on the surface? Use Earth's radius, r_E, as the unit. T/I A
2. In a hydrogen atom, a proton and an electron are 5.3×10^{-11} m apart. Calculate the magnitude of the gravitational attraction between the proton and the electron. The mass of a proton is 1.67×10^{-27} kg, and the mass of an electron is 9.11×10^{-31} kg. T/I A
3. Two objects are a distance r apart. The distance r increases by a factor of 4. K/U
   (a) Does the gravitational force between the objects increase or decrease? Explain your answer.
   (b) By what factor does the gravitational force between the objects change?
4. A satellite of mass 225 kg is located 8.62×10^6 m above Earth's surface. T/I A
   (a) Determine the magnitude and direction of the gravitational force. (Hint: The values for Earth's mass and radius can be found in Appendix B.)
   (b) Determine the magnitude and direction of the resulting acceleration of the satellite.
5. On the surface of Titan, a moon of Saturn, the gravitational field strength has a magnitude of 1.3 N/kg. Titan's mass is 1.3×10^{23} kg. What is Titan's radius? T/I A
6. Earth's gravitational field strength at the surface is 9.80 N/kg. Determine the distance, as a multiple of Earth's radius, r_E, above Earth's surface at which the magnitude of the acceleration due to gravity is 3.20 N/kg. T/I A
7. Calculate the gravitational field strength of the Sun at a distance of 1.5×10^{11} m from its centre (Earth's distance). T/I
8. The gravitational field strength between two objects is the sum of two vectors pointing in opposite directions. Somewhere between the objects, the vectors will cancel, and the total force will be zero. Determine the location of zero force as a fraction of the distance r between the centres of two objects of mass m_1 and m_2. T/I A
9. A 537 kg satellite orbits Earth with a speed of 4.3 km/s at a distance of 2.5×10^7 m from Earth's centre. K/U T/I
   (a) Calculate the acceleration of the satellite.
   (b) Calculate the gravitational force on the satellite.
10. Calculate the value of Mercury's surface gravitational field strength, and compare your answer to the value provided in Table 2 on page 295. K/U
11. The gravitational field strength is 5.3 N/kg at the location of a 620 kg satellite in orbit around Earth. K/U T/I
    (a) Calculate the satellite's altitude. (Hint: The values for Earth's mass and radius can be found in Appendix B.)
    (b) Determine the gravitational force on the satellite.
12. Through experimentation, Henry Cavendish was able to determine the value of the gravitational constant. Explain how to use his result together with astronomical data on the motion of the Moon to determine the mass of Earth. T/I C A
13. Determine the location between two objects with masses equal to Earth's mass and the Moon's mass where you could place a third mass so that it would experience a net gravitational force of zero. T/I A