Collisions in Two Dimensions: Glancing Collisions

5.5

So far, you have read about collisions in one dimension. In this section, you will examine collisions in two dimensions. In **Figure 1**, the player is lining up the shot so that the cue ball (the white ball) will hit another billiard ball at an angle, directing it toward the corner pocket. What component of the cue ball's momentum will be transferred to the target ball if the shot is successful?

The laws of conservation of momentum and conservation of kinetic energy will apply just as they did for one-dimensional interactions. However, to calculate momentum for two-dimensional problems, consider the *x*-components and *y*-components of force and motion independently.

Figure 1 Billiards requires players to master the use of glancing collisions.

Mini **Investigation**

Glancing Collisions

Skills: Performing, Observing, Analyzing, Communicating

SKILLS HANDBOOK A2.1

In this investigation, you will model glancing collisions to observe and analyze how they work.

Equipment and Materials: eye protection; air table; 2 pucks; marbles; billiard balls

1. Put on your eye protection. Set up an air table with two pucks.

When you unplug the air table, pull the plug and not the cord. Wear closed-toe shoes to protect your feet in case the puck flies off the table. Push the objects lightly and cautiously.

2. Push the first puck toward the second to cause a gentle head-on collision. Observe the changes in the speeds of the pucks after the collision.
3. Repeat this process, but vary the angles of collision and the initial speed of the first puck. Observe the changes in the speeds of the pucks and the directions of their final velocities.
4. Try the same investigation using different objects, such as marbles or billiard balls.

A. How does the speed of the second puck compare to the initial speed of the first puck as you vary the angle of collision? K/U T/I A

B. How did using marbles or billiard balls affect the changes in direction and velocity? How was this different from using the pucks? Briefly summarize your observations. T/I A

Components of Momentum

Dealing with collisions in two dimensions involves the same basic ideas as dealing with collisions in one dimension. Now, however, the final velocity of each object involves two unknowns: the two components of the velocity vector. For objects in motion in two dimensions, the change in momentum for each component can be considered independently:

$$\Sigma\vec{F}_x\Delta t = \Delta\vec{p}_x$$

$$\Sigma\vec{F}_y\Delta t = \Delta\vec{p}_y$$

Similarly, the conservation of momentum equation can be expressed in terms of horizontal and vertical components:

$$\vec{p}_{\mathrm{i}_{1x}} + \vec{p}_{\mathrm{i}_{2x}} = \vec{p}_{f_{1x}} + \vec{p}_{\mathrm{f}_{2x}}$$

$$\vec{p}_{\mathrm{i}_{1y}} + \vec{p}_{\mathrm{i}_{2y}} = \vec{p}_{\mathrm{f}_{1y}} + \vec{p}_{\mathrm{f}_{2y}}$$

Consider the collision of two billiard balls shown in **Figure 2**. In this shot, the cue ball (1) will collide with the target ball (2), initially at rest, sending it at an angle ϕ toward the corner pocket, and the cue ball will continue travelling at an angle θ from its original direction of travel. Both objects are travelling at an angle to the directions of their original courses. This type of collision is called a **glancing collision**. In the following Tutorial, we use components to analyze the physics of a glancing collision.

glancing collision a collision in which the first object, after an impact with the second object, travels at an angle to the direction it was originally travelling

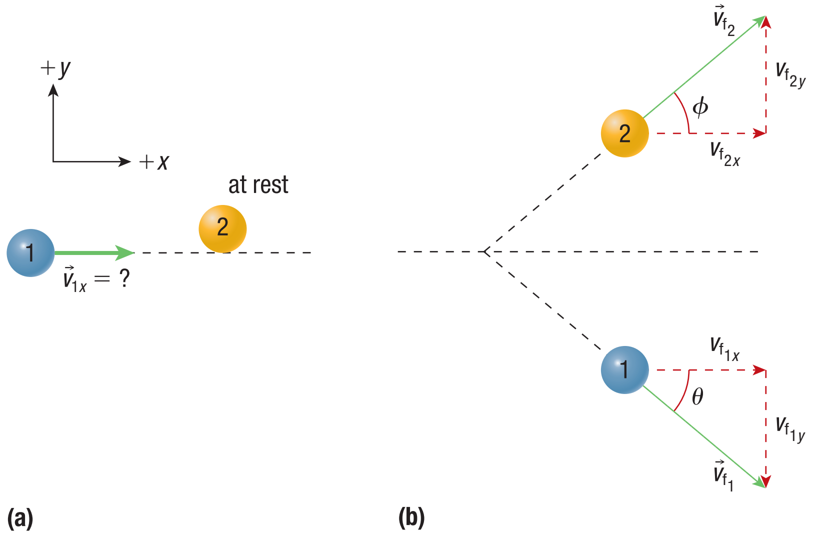

Figure 2 A cue ball striking another ball at an angle causes both balls to change direction.

Investigation 5.5.1

Conservation of Momentum in Two Directions (page 260)
After learning about the physics of glancing collisions, perform Investigation 5.5.1 to explore how momentum is conserved in collisions that occur in two dimensions.

Tutorial 1 Analysis of Glancing Collisions

In these Sample Problems, you will apply conservation of momentum in two dimensions.

Sample Problem 1: Analysis of a Glancing Collision

In a game of curling, a collision occurs between two stones of equal mass. The object stone is initially at rest. After the collision, the stone that is thrown has a speed of 0.56 m/s in some direction, represented by θ in **Figure 3.**

The object stone acquires a velocity $\vec{v}_{f_2} = 0.42$ m/s at an angle of $\phi = 30.0°$ from the original direction of motion of the thrown stone. Determine the initial velocity of the thrown stone.

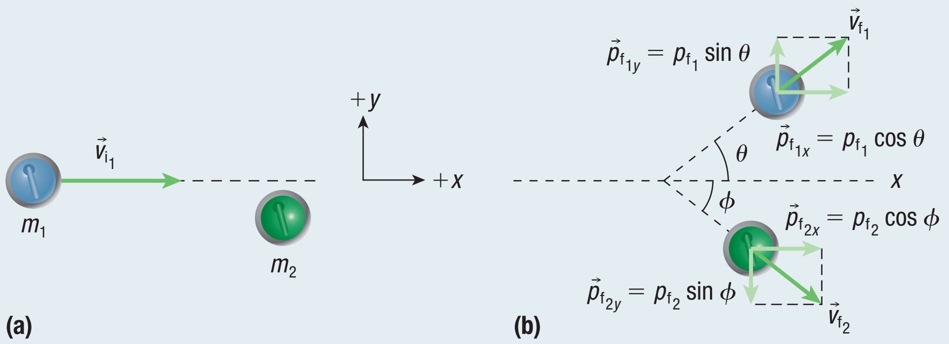

Figure 3 (a) The curling stone collides with the object stone in a glancing collision. (b) Both curling stones move off in different directions. We can analyze their final velocities to determine the initial velocity of the thrown curling stone.

Given: $m_1 = m_2$; $\vec{v}_{i_2} = 0$ m/s; $\vec{v}_{f_1} = 0.56$ m/s; $\vec{v}_{f_2} = 0.42$ m/s; $\phi = 30.0°$

Required: $\vec{v}_{i_1}$

Analysis: Choose a coordinate system to identify directions: let positive x be to the right and negative x be to the left. Let positive y be up and negative y be down.

Apply conservation of momentum independently in the x-direction and the y-direction. Begin by applying conservation of momentum in the y-direction to determine the direction of the final velocity of the thrown stone. Then apply conservation of momentum in the x-direction to calculate the initial velocity of the thrown stone.

Solution: In the y-direction, the total momentum before and after the collision is zero.

$$p_{T_{iy}} = p_{T_{fy}} = 0$$

Therefore, after the collision:

$$mv_{f_{1y}} + mv_{f_{2y}} = 0$$

Divide both sides by m and substitute the vertical component of each velocity vector. Note that the vertical component of the first stone's velocity is directed up, so its value is positive, whereas the vertical component of the second stone's velocity is directed down, so its value is negative.

$$\frac{\cancel{m}(v_{f_{1y}} + v_{f_{2y}})}{\cancel{m}} = \frac{0}{m}$$

$$v_{f_1} \sin\theta - v_{f_2} \sin\phi = 0$$

Rearrange this equation to isolate $\sin\theta$.

$$v_{f_1} \sin\theta = v_{f_2} \sin\phi$$

$$\sin\theta = \frac{v_{f_2} \sin\phi}{v_{f_1}}$$

Substitute the given values and solve for $\sin\theta$.

$$\sin\theta = \frac{(0.42\ \cancel{\text{m/s}})(\sin 30°)}{(0.56\ \cancel{\text{m/s}})}$$

$$\sin\theta = 0.375$$

Apply the inverse sine to both sides to solve for θ.

$$\theta = \sin^{-1}0.375$$
$$\theta = 22.0° \text{ (one extra digit carried)}$$

The first stone is travelling at an angle of 22° above the horizontal after the collision.

Now use conservation of momentum in the x-direction to solve for the initial speed of the thrown stone.

$$p_{T_{ix}} = p_{T_{fx}}$$

Note that the object stone is at rest before the collision, so its initial momentum is zero.

$$mv_{i_{1x}} + \cancel{mv_{i_{2x}}} = mv_{f_{1x}} + mv_{f_{2x}}$$

Divide both sides of the equation by m.

$$\frac{\cancel{m}v_{i_{1x}}}{\cancel{m}} = \frac{\cancel{m}(v_{f_{1x}} + v_{f_{2x}})}{\cancel{m}}$$

$$v_{i_{1x}} = v_{f_{1x}} + v_{f_{2x}}$$

Substitute the known values for the final horizontal velocity components of the stones. Note that all vectors are directed to the right, so all velocities are positive.

$$\begin{aligned} v_{i_{1x}} &= v_{f_1}\cos\theta + v_{f_2}\cos\phi \\ &= (0.56\cos 22.0°)\text{ m/s} + (0.42\cos 30°)\text{ m/s} \\ &= 0.519\text{ m/s} + 0.364\text{ m/s (one extra digit carried)} \\ v_{i_{1x}} &= 0.88\text{ m/s} \end{aligned}$$

Statement: The initial velocity of the thrown stone is 0.88 m/s [right].

Sample Problem 2: Inelastic Glancing Collisions

Two cross-country skiers are skiing to a crossing of horizontal trails in the woods as shown in **Figure 4**. Skier 1 is travelling east and has a mass of 84 kg. Skier 2 is travelling north and has a mass of 72 kg. Both skiers are travelling with an initial speed of 5.1 m/s. One of the skiers forgets to look, resulting in a right-angle collision with the skis locked together after the collision. Calculate the final velocity of the two skiers.

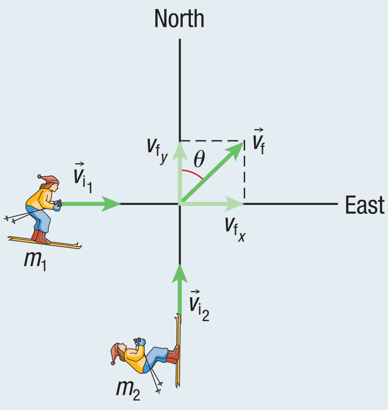

Figure 4

Given: inelastic collision; $m_1 = 84$ kg; $m_2 = 72$ kg; $\vec{v}_{i_1} = 5.1$ m/s [E]; $\vec{v}_{i_2} = 5.1$ m/s [N]

Required: $\vec{v}_f$

Analysis: According to the law of conservation of momentum, $\vec{p}_{T_i} = \vec{p}_{T_f}$. Since the initial velocities are at right angles to each other, as shown in **Figure 5**, you can calculate the total velocity and momentum using the Pythagorean theorem and trigonometry:

$$p^2 = p_1^2 + p_2^2, \text{ and } \tan\theta = \left(\frac{p_1}{p_2}\right)$$

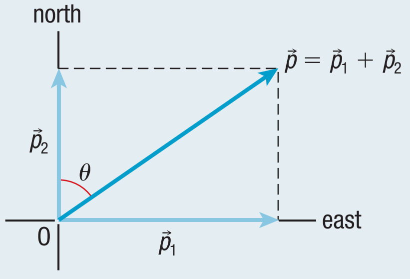

Figure 5

Solution: The first skier's momentum is

$$\vec{p}_1 = m_1\vec{v}_1$$
$$= (84 \text{ kg})\left(5.1\ \frac{\text{m}}{\text{s}}\right)[\text{E}]$$
$$\vec{p}_1 = 428 \text{ kg·m/s } [\text{E}] \text{ (one extra digit carried)}$$

The second skier's momentum is

$$\vec{p}_2 = m_2\vec{v}_2$$
$$= (72 \text{ kg})\left(5.1\ \frac{\text{m}}{\text{s}}\right)[\text{N}]$$
$$\vec{p}_2 = 367 \text{ kg·m/s } [\text{N}] \text{ (one extra digit carried)}$$

The magnitude of the total momentum can be calculated by applying the Pythagorean theorem:

$$p^2 = p_1^2 + p_2^2$$
$$p = \sqrt{p_1^2 + p_2^2}$$
$$= \sqrt{\left(428\ \frac{\text{kg·m}}{\text{s}}\right)^2 + \left(367\ \frac{\text{kg·m}}{\text{s}}\right)^2}$$
$$p = 564 \text{ kg·m/s (one extra digit carried)}$$

The direction can be determined by applying the tangent ratio:

$$\tan\theta = \left(\frac{p_1}{p_2}\right)$$
$$\theta = \tan^{-1}\left(\frac{p_1}{p_2}\right)$$
$$= \tan^{-1}\left(\frac{428\ \frac{\text{kg·m}}{\text{s}}}{367\ \frac{\text{kg·m}}{\text{s}}}\right)$$
$$\theta = 49°$$

The direction of the two skiers is [N 49° E].

Conservation of momentum tells us that the final total momentum of the skiers must equal this initial momentum. Since the collision is perfectly inelastic, both skiers have the same final velocity:

$$\vec{p}_f = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$$
$$= (m_1 + m_2)\vec{v}_f$$
$$\vec{v}_f = \frac{\vec{p}_f}{m_1 + m_2}$$
$$= \frac{564\ \frac{\cancel{\text{kg}}\text{·m}}{\text{s}}}{84\ \cancel{\text{kg}} + 72\ \cancel{\text{kg}}}[\text{N } 49° \text{ E}]$$
$$\vec{v}_f = 3.6 \text{ m/s } [\text{N } 49° \text{ E}]$$

Statement: After the collision, the skiers are travelling together with a velocity of 3.6 m/s [N 49° E].

Practice

1. Two freight trains have a completely inelastic collision at a track crossing. Engine 1 has a mass of 1.4×10^4 kg and is initially travelling at 45 km/h [N]. Engine 2 has a mass of 1.5×10^4 kg and is initially travelling at 53 km/h [W]. Calculate the final velocity. T/I A
   [ans: 9.7 m/s [N 52° W]]
2. A star of mass 2×10^{30} kg moving with a velocity of 2×10^4 m/s [E] collides with a second star of mass 5×10^{30} kg moving with a velocity of 3×10^4 m/s at a right angle to the path of the first star. If the two join together, what is their common velocity? T/I A [ans: 2×10^4 m/s [15° to the initial path of the second star]]

5.5 Review

Summary

- The laws of conservation of momentum and conservation of kinetic energy for collisions in two dimensions are the same as they are for one-dimensional collisions.
- Momentum is conserved for elastic and inelastic collisions.
- Kinetic energy is conserved only in elastic collisions.
- The fact that momentum is a vector quantity means that problems involving two-dimensional collisions can be solved by independently analyzing the x-components and y-components.

Questions

1. Two balls of equal mass undergo a collision (see **Figure 6**). Ball 1 is initially travelling horizontally with a speed of 10.0 m/s, and ball 2 is initially at rest. After the collision, ball 1 moves away with a velocity of 4.7 m/s at an angle of $\theta = 60.0°$ from its original path and ball 2 moves away at an unknown angle ϕ. Determine the magnitude and direction of velocity of ball 2 after the collision. K/U T/I

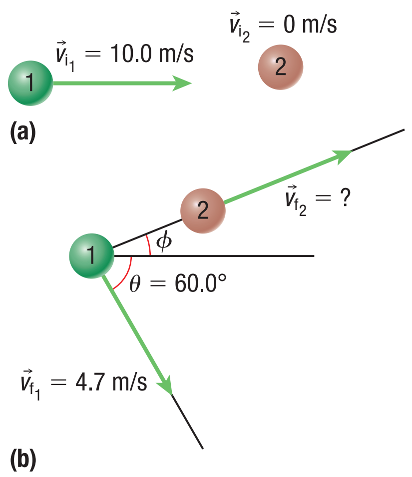

Figure 6

2. A hockey puck of mass 0.16 kg, sliding on a nearly frictionless surface of ice with a velocity of 2.0 m/s [E], strikes a second puck at rest with a mass of 0.17 kg. The first puck has a velocity of 1.5 m/s [N 31° E] after the collision. Determine the velocity of the second puck after the collision. T/I A
3. Two hockey pucks of equal mass approach each other. Puck 1 has an initial velocity of 20.0 m/s [S 45° E], and puck 2 has an initial velocity of 15 m/s [S 45° W]. After the collision, the first puck is moving with a velocity of 10.0 m/s [S 45° W]. K/U T/I C
   (a) Determine the final velocity of the second puck.
   (b) Is this collision elastic, perfectly inelastic, or (non-perfectly) inelastic? Explain your reasoning.
4. An automobile collides with a truck at an intersection. The car, of mass 1.4×10^3 kg, is travelling at 32 km/h [S]; the truck has a mass of 2.6×10^4 kg and is travelling at 48 km/h [E]. The collision is perfectly inelastic. Determine their velocity just after the collision. T/I
5. Two balls of equal mass m undergo a collision. One ball is initially stationary. After the collision, the velocities of the balls make angles of 25.5° and −45.9° relative to the original direction of motion of the moving ball. T/I C
   (a) Draw and label a diagram to show the balls before and after the collision. Label the angles θ and ϕ.
   (b) Calculate the final speeds of the balls if the initial ball had a speed of 3.63 m/s.
6. A carbon-14 nucleus, initially at rest, undergoes a nuclear reaction known as beta decay. The nucleus emits two particles horizontally: one with momentum 7.8×10^{-21} kg·m/s [E] and another with momentum 3.5×10^{-21} kg·m/s [S]. T/I A
   (a) Calculate the direction of the motion of the nucleus immediately following the reaction.
   (b) Determine the final momentum of the nucleus.
   (c) The mass of the residual carbon-14 nucleus is 2.3×10^{-26} kg. Determine its final velocity.
7. A neutron of mass 1.7×10^{-27} kg, travelling at 2.2 km/s, hits a stationary helium nucleus of mass 6.6×10^{-27} kg. After the collision, the velocity of the helium nucleus is 0.53 km/s at 52° to the original direction of motion of the neutron. Determine the final velocity of the neutron. T/I
8. Your classmate makes the following statement: “For a head-on elastic collision between two objects of equal mass, the after-collision velocities of the objects are at right angles to each other.” Evaluate the accuracy of this statement. K/U T/I A