5.1 Momentum and Impulse

Objects in motion are a big part of everyday life. It is important that we understand how to put objects in motion, how to change their direction, and how to bring them to a stop. These changes in motion often occur as a result of collisions between two or more objects. A collision between a baseball and a bat, for example, brings about a sudden change in velocity of the ball but also has an effect on the bat (**Figure 1**). A collision between a car and a tree can have negative impacts for both; however, a well-designed collision between the driver and an airbag can save a life.

Why does a puck propelled by a slap shot travel faster than a puck propelled by a wrist shot? How do modern tennis racquets allow today's players to hit the ball with much greater speed than was possible with older wooden racquets? Why do golf courses have to be lengthened from time to time to remain challenging? The concepts of momentum and impulse will help you understand the science of collisions and answer these questions. CAREER LINK

Figure 1 When you hit a ball with a bat, the resulting collision has an effect on both the ball and the bat.

Momentum

Two variables, velocity and acceleration, describe the motion of a single object. An additional quantity, linear momentum, is useful for dealing with a collection of objects. The **linear momentum**, $\vec{p}$, of a single object of mass m moving with velocity $\vec{v}$ is

linear momentum ($\vec{p}$) a quantity that describes the motion of an object travelling in a straight line as the product of its mass and velocity

$$\vec{p} = m\vec{v}$$

Note that for the rest of this section, the term *momentum* refers to linear momentum. The momentum $\vec{p}$ of an object is directly proportional to the object's velocity, so the momentum vector is along the same direction as the velocity. Note also that $\vec{p}$ is proportional to the mass of the object. In the following Tutorial, you will learn more about how to calculate momentum.

Tutorial 1 Calculating Momentum

The following Sample Problem shows you how to use the equation $\vec{p} = m\vec{v}$ to calculate the momentum of an object.

Sample Problem 1: The Vector Nature of Momentum

(a) Calculate the momentum of a 2.5 kg rabbit travelling with a velocity of 2.0 m/s [E].

(b) Calculate the momentum of a 5.0 kg groundhog travelling with a velocity of 1.0 m/s [S].

(c) Compare the momentum and kinetic energies of the rabbit and the groundhog.

Given: $m_{rabbit} = 2.5$ kg; $\vec{v}_{rabbit} = 2.0$ m/s [E]; $m_{groundhog} = 5.0$ kg; $\vec{v}_{groundhog} = 1.0$ m/s [S]

Required: $\vec{p}_{rabbit}$; $E_{k\ rabbit}$; $\vec{p}_{groundhog}$; $E_{k\ groundhog}$

Analysis: $\vec{p} = m\vec{v}$; $E_k = \frac{1}{2}mv^2$

Solution:

(a)
$$\vec{p} = m\vec{v}$$
$$\vec{p}_{rabbit} = (2.5\text{ kg})(2.0\text{ m/s [E]})$$
$$\vec{p}_{rabbit} = 5.0\text{ kg·m/s [E]}$$

(b)
$$\vec{p} = m\vec{v}$$
$$\vec{p}_{groundhog} = (5.0\text{ kg})(1.0\text{ m/s [S]})$$
$$\vec{p}_{groundhog} = 5.0\text{ kg·m/s [S]}$$

(c) For the rabbit:

$$E_k = \frac{1}{2}mv^2$$
$$E_{k\ rabbit} = \frac{1}{2}(2.5\text{ kg})(2.0\text{ m/s})^2$$
$$E_{k\ rabbit} = 5.0\text{ J}$$

For the groundhog:

$$E_{k\ groundhog} = \frac{1}{2}(5.0\text{ kg})(1.0\text{ m/s})^2$$
$$E_{k\ groundhog} = 2.5\text{ J}$$

Statement:

(a) The momentum of the rabbit is 5.0 kg·m/s [E].

(b) The momentum of the groundhog is 5.0 kg·m/s [S].

(c) The momenta of the two animals are equal in magnitude but in different directions. Although the momenta of the rabbit and the groundhog are of the same magnitude, the kinetic energy, E_k, of the rabbit is twice that of the groundhog. Note that, unlike momentum, kinetic energy is a scalar quantity and does not have a direction.

Practice

1. Calculate the momentum and kinetic energy of a hockey puck with a mass of 160 g travelling with a velocity of 40.0 m/s [E]. K/U T/I A [ans: 6.4 kg·m/s; 130 J]
2. Compare the momentum of a bowling ball with a mass of 6.2 kg travelling with a velocity of 1.6 m/s [E] to that of a hockey puck with a mass of 160 g travelling with a velocity of 40.0 m/s [E]. What is the difference in their momenta? T/I A [ans: 3.5 kg·m/s]

Impulse

A golf ball resting on a tee has mass but zero velocity. Its momentum, therefore, is zero. Although the golfer may not think of it in these terms, the goal is to use the golf club to change the momentum of the ball. If the golfer is successful, the golf ball will fly through the air with considerable momentum an instant after colliding with the golf club. What happens during this transition?

Newton's first law states that the velocity of an object is constant unless acted on by an external force. So, in the absence of an external force, an object with constant mass must also have constant linear momentum. If a net force is applied to the object, its velocity will change and, therefore, its momentum will also change. Consider a force, $\vec{F}$, acting on a golf ball with a mass of 45 g. You can use linear momentum and Newton's second law, $\vec{F} = m\vec{a}$, to calculate the change in momentum.

The object's acceleration, $\vec{a}$, is related to the change in the velocity according to $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$. Suppose that the object has an initial velocity $\vec{v}_i$ just before the force is applied, and a final velocity $\vec{v}_f$ after a time Δt.

$$\begin{aligned}\vec{F} &= m\vec{a}\\ &= m\frac{\Delta \vec{v}}{\Delta t}\\ \vec{F} &= m\frac{\vec{v}_f - \vec{v}_i}{\Delta t}\end{aligned}$$

From the definition of momentum, the initial momentum of the object is $\vec{p}_i = m\vec{v}_i$ and the final momentum is $\vec{p}_f = m\vec{v}_f$. Rearrange the previous equation and then substitute these expressions.

$$\begin{aligned}\vec{F}\Delta t &= m(\vec{v}_f - \vec{v}_i)\\ &= m\vec{v}_f - m\vec{v}_i\\ &= \vec{p}_f - \vec{p}_i\\ \vec{F}\Delta t &= \Delta \vec{p}\end{aligned}$$

The product $\vec{F}\Delta t$ is called the **impulse** and is the change in the momentum of an object:

$$\vec{F}\Delta t = \Delta \vec{p}$$

impulse the product of force and time that acts on an object to produce a change in momentum

Impulse is a vector; its direction is the same as the direction of the total force on the object. You can see from this equation that applying a large force for a short time could produce the same change in momentum as applying a smaller force for a longer time. Dimensional analysis shows that the SI units for impulse (newton seconds, or N·s) are the same as the units for momentum (kg·m/s):

$$[\mathrm{N \cdot s}] = \left[\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}^2}\right][\mathrm{s}]$$

$$[\mathrm{N \cdot s}] = \left[\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}^{\cancel{2}}}\right][\cancel{\mathrm{s}}]$$

$$[\mathrm{N \cdot s}] = \left[\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}}\right]$$

Impulse in Sports

How do modern tennis racquets allow players to hit the ball with much greater speed than was possible with older wooden racquets (**Figure 2**)? You can investigate this using the equations for momentum and impulse. Assume the mass of the tennis ball is still the same; if its velocity is greater, its momentum must also be greater. So the tennis player is providing a greater impulse, or change in momentum, to the ball. Assuming also that the force the player uses does not change, then the change in impulse can only come from a change in Δt. For the most part, the greater speeds are the result of new materials and designs that allow more contact time between the ball and the racquet.

Figure 2 A modern tennis racquet can transfer a greater impulse to a tennis ball than older wooden racquets.

Impulse and Force–Time Graphs

In situations where the force applied to an object varies over time, you can use a force–time graph to estimate the impulse. A force–time graph shows force as a function of time during a collision within a time interval Δt. **Figure 3** shows the force–time graph of a struck tennis ball. The area under the force–time graph is equal to the impulse, because this area represents the product of force and time over the course of the collision.

One way to estimate impulse from the force–time graph is to count the number of squares and partial squares under the variable force–time graph, as shown in Figure 3(a). The total will represent the area under the curve, and therefore the impulse due to the applied force. Another way is to consider the average force exerted on the ball over the duration of the collision. A constant average force will produce a horizontal straight-line graph, as shown in Figure 3(b). Note that the rectangular area under this graph is approximately equal to the area under the variable force–time graph found by counting squares. In many situations it is easier to estimate the impulse produced by a variable force by assuming a constant average force acting over the same time interval.

UNIT TASK **BOOKMARK**

You can apply what you learn about momentum and impulse to the Unit Task on page 270.

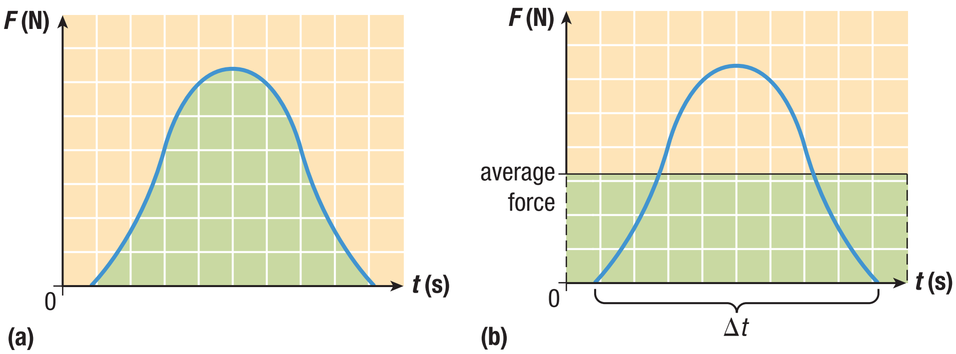

Figure 3 (a) A variable force–time graph and (b) its corresponding constant average force–time graph. The area under the curve is equal to approximately 30 square units, which corresponds to 30 N·s.

In the following Tutorial, you will learn how to calculate impulse and use force–time graphs.

Tutorial 2 Calculating Impulse

The Sample Problems in this Tutorial demonstrate various ways to calculate impulse.

Sample Problem 1: Impulse as a Change in Momentum

A 0.160 kg puck is travelling at 5.0 m/s [N]. A slapshot produces a collision that lasts for 0.0020 s and gives the puck a velocity of 40.0 m/s [S].

(a) Calculate the impulse imparted by the hockey stick.

(b) Determine the average force applied by the stick to the puck.

Solution

(a) **Given:** $m = 0.160$ kg; $\vec{v}_i = 5.0$ m/s [N]; $\vec{v}_f = 40.0$ m/s [S]

Required: $\Delta\vec{p}$

Analysis: $\Delta\vec{p} = m(\vec{v}_f - \vec{v}_i)$

Solution:

$$\begin{aligned}\Delta\vec{p} &= m(\vec{v}_f - \vec{v}_i)\\ &= 0.160\text{ kg}\,[40\text{ m/s [S]} - 5\text{ m/s [N]}]\\ &= 0.160\text{ kg}\,[40\text{ m/s [S]} - (-5\text{ m/s [S]})]\\ &= 0.160\text{ kg}\,(40\text{ m/s [S]} + 5\text{ m/s [S]})\\ &= 0.160\text{ kg}\,(45\text{ m/s [S]})\\ &= 7.2\text{ kg}\cdot\text{m/s [S]}\\ \Delta\vec{p} &= 7.2\text{ N}\cdot\text{s [S]}\end{aligned}$$

Statement: The impulse imparted by the hockey stick is 7.2 N·s [S].

(b) **Given:** $\Delta t = 0.0020$ s; $\Delta\vec{p} = 7.2$ N·s [S]

Required: $\vec{F}$

Analysis: $\vec{F}\Delta t = \Delta\vec{p}$

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

Solution:

$$\begin{aligned}\vec{F} &= \frac{\Delta\vec{p}}{\Delta t}\\ &= \frac{7.2\text{ N}\cdot\cancel{\text{s}}\text{ [S]}}{0.0020\ \cancel{\text{s}}}\\ \vec{F} &= 3600\text{ N [S]}\end{aligned}$$

Statement: The average force applied by the stick to the puck is 3.6×10^3 N [S].

Sample Problem 2: Impulse as the Product of Force and Time

A volleyball player starts a serve by throwing the ball vertically upward. The 260 g volleyball comes to rest at its maximum height. The server then hits it and exerts an average horizontal force of magnitude 6.5 N on the ball.

(a) Determine the speed of the ball after the player hits it if the average force is exerted on the ball for 615 ms.

(b) On the next serve, the volleyball player hits the ball with the same amount of horizontal force, but the time interval is 875 ms. Determine the speed of the ball.

Solution

(a) **Given:** $m = 260$ g $= 0.260$ kg; $\vec{F} = 6.5$ N; $\Delta t_a = 615$ ms $= 0.615$ s

Required: $\vec{v}_f$

Analysis: $\vec{F}\Delta t = \Delta\vec{p}$

$$\Delta\vec{p} = m(\vec{v}_f - \vec{v}_i)$$

$$\vec{F}\Delta t = m(\vec{v}_f - \vec{v}_i)$$

$$\vec{v}_f = \frac{\vec{F}\Delta t}{m} + \vec{v}_i$$

Solution:

$$\begin{aligned}\vec{v}_f &= \frac{\vec{F}\Delta t}{m} + \vec{v}_i\\ &= \frac{6.5\ \cancel{\text{kg}}\cdot\frac{\text{m}}{\text{s}^{\cancel{2}}}(0.615\ \cancel{\text{s}})}{0.260\ \cancel{\text{kg}}} + 0\text{ m/s}\\ \vec{v}_f &= 15\text{ m/s}\end{aligned}$$

Statement: The speed of the volleyball when the force is exerted for 615 ms is 15 m/s.

(b) **Given:** $m = 260$ g $= 0.260$ kg; $\vec{F} = 6.5$ N; $\Delta t_b = 875$ ms $= 0.875$ s

Required: $\vec{v}_f$

Analysis: Use the same equation for $\vec{v}_f$ we derived in (a), substituting the different time interval:

$$\vec{v}_f = \frac{\vec{F}\Delta t}{m} + \vec{v}_i$$

Solution:

$$\begin{aligned}\vec{v}_f &= \frac{\vec{F}\Delta t}{m} + \vec{v}_i\\ &= \frac{6.5\ \cancel{\text{kg}}\cdot\frac{\text{m}}{\text{s}^{\cancel{2}}}(0.875\ \cancel{\text{s}})}{0.260\ \cancel{\text{kg}}} + 0\text{ m/s}\\ \vec{v}_f &= 22\text{ m/s}\end{aligned}$$

Statement: The speed of the volleyball when the force is exerted for 875 ms is 22 m/s. (Note: By following through on the serve, the player increases the time interval of the applied force, resulting in a faster serve.)

Sample Problem 3: Impulse as Area under a Force–Time Curve

Two figure skaters approach each other in a straight line. They meet hand to hand and then push off in opposite directions. The increase and decrease of force are both linear, which produces a force–time curve that is in the shape of a triangle. The force–time curve for this interaction is shown in **Figure 4**. Determine the impulse for this interaction.

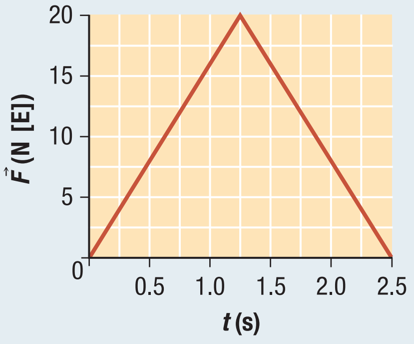

Figure 4

Given: Force–time graph of the skaters' interaction

Required: $\vec{F}\Delta t$

Analysis: Determine the impulse by calculating the area under the force–time curve of the collision. Use the equation for the area of a triangle: $A = \frac{1}{2}bh$; $A = \vec{F}\Delta t$.

Solution:
$$\vec{F}\Delta t = \frac{1}{2}bh$$
$$= \frac{1}{2}(2.5\text{ s})(20.0\text{ N})$$
$$\vec{F}\Delta t = 25\text{ N}\cdot\text{s}$$

Statement: The impulse of the interaction of the two skaters is 25 N·s away from each other.

Practice

1. A hockey player passes a puck with an average force of 250 N. The hockey stick is in contact with the puck for 0.0030 s, and the mass of the puck is 180 g. The puck is not moving before the player passes it. T/I A
   (a) Determine the impulse imparted by the hockey stick. [ans: 0.75 kg·m/s [forward]]
   (b) Calculate the velocity of the puck as a result of this collision. [ans: 4.2 m/s [forward]]
2. A hockey player collides with a wall, and then pushes away from it. The collision occurs over 2.9 s and the average force applied by the player in the collision is 468 N. Draw a force–time graph similar to the one in Figure 3(b) and use it to determine the impulse of the collision. T/I C [ans: 1400 N·s [away from the wall]]

As you have seen, the concepts of linear momentum and impulse are relevant to an understanding of motion in sports and in the design of improved sports gear. You can also apply the concepts of momentum and impulse in many other areas, ranging from the analysis of motor vehicle collisions to the motion of rockets. You will learn more about applications of momentum and impulse later in the chapter. CAREER LINK

5.1 Review

Summary

- Linear momentum is the product of an object's mass and its velocity, expressed in units of kilograms times metres per second (kg·m/s): $\vec{p} = m\vec{v}$.
- Impulse is the change in momentum caused by the application of a force over a time interval, expressed in units of newton seconds (N·s): $\vec{F}\Delta t = \Delta\vec{p}$.
- The magnitude of an impulse can be found by measuring the area under a force–time curve.

Questions

1. Calculate the momentum of each of the following: K/U
   (a) a male moose of mass 4.25×10^2 kg running at 6.9 m/s [N]
   (b) a city bus of mass 9.97×10^3 kg moving at 5 km/h [forward]
   (c) a flying squirrel of mass 995 g gliding at 16 m/s [S]
2. In your own words, describe what impulse is. K/U C
3. A bicycle and rider have a combined mass of 79.3 kg and a momentum of 2.16×10^3 kg·m/s [W]. Determine the velocity of the bicycle. K/U
4. A projectile travelling at 9.0×10^2 m/s [W] has a momentum of 4.5 kg·m/s [W]. What is the mass of the projectile? K/U
5. A downhill skier travelling at a constant velocity of 29.5 m/s [forward] has a momentum of 2.31×10^3 kg·m/s [forward]. Determine the mass of the skier. T/I
6. Explain how increasing the time interval over which a force is applied can affect performance in sports. Use a sport not discussed in this section in your answer. K/U A
7. A teacher drops a tennis ball and a basketball from the same height onto the floor. The force from the floor produces an impulse on each ball. If the basketball is heavier than the tennis ball, which impulse is larger? Explain your answer. T/I C A
8. A hockey player passes a puck that is initially at rest. The force exerted by the stick on the puck is 1100.0 N [forward], and the stick is in contact with the puck for 5.0 ms. T/I
   (a) Determine the impulse imparted by the stick to the puck.
   (b) If the puck has a mass of 0.12 kg, calculate the speed of the puck just after it leaves the hockey stick.
9. You accidentally drop a cellphone, which has a mass of 225 g, from a height of 74 cm. T/I A
   (a) Calculate the cellphone's momentum at the moment of impact with the sidewalk.
   (b) If the cellphone lands on a grassy lawn, is its momentum less, the same, or greater? Explain your answer.
10. A rubber ball with a mass of 0.25 kg is dropped from a height of 1.5 m onto the floor. Just after bouncing from the floor, the ball has a velocity of 4.0 m/s [up]. T/I
    (a) Determine the impulse imparted by the floor to the ball.
    (b) If the average force of the floor on the ball is 18 N [up], for how long is the ball in contact with the floor?
11. An archer shoots an arrow with a mass of 0.030 kg. The arrow leaves the bow with a horizontal velocity of 88 m/s. T/I
    (a) Determine the impulse imparted to the arrow.
    (b) If the arrow is in contact with the bowstring for 0.015 s after the archer releases, what is the approximate average force of the bowstring on the arrow?
12. A tennis player hits a serve at a speed of 63 m/s [W], and the opponent returns the 0.057 kg tennis ball to the server with a speed of 41 m/s [E]. T/I
    (a) Calculate the magnitude of the impulse imparted to the ball by the opponent.
    (b) Calculate the approximate average force on the ball if the opponent's racquet is in contact with the ball for 0.023 s.