Springs and Conservation of Energy

4.7

Most drivers try to avoid collisions, but not at a demolition derby like the one shown in **Figure 1**. The point of a demolition derby is to crash your car into as many other cars as possible. Each car tries to damage the other cars so much that they will stop working. The harder the crash, the more damage you are likely to do. The last car running is the winner.

Figure 1 In a demolition derby, the cars can be crashed, but the drivers must remain safe.

How can the drivers of demolition cars avoid serious injury? What types of safety equipment do they use? Like most drivers, they wear seat belts to hold themselves securely in their seat. They have shoulder straps to prevent lurching forward. Padding inside the driver's-side door might provide cushioning from side impacts. Most cars on the road, however, have safety features that are missing or unimportant in demolition cars. Cars you ride in probably have airbags to cushion the passengers during a crash. They may have anti-lock brakes or other computer-controlled systems that act during emergency situations. In this section, you will explore the physics behind safety equipment and other systems in which energy is stored and transformed. CAREER LINK

Conservation of Mechanical Energy

Systems that make a car safe use either springs or elastic materials, so they have elastic potential energy. You have read about the conservation of energy in an isolated system. The law of conservation of energy includes elastic potential energy: Energy is neither created nor destroyed in an isolated system, but it can be transformed between kinetic energy, gravitational potential energy, elastic potential energy, and other forms of energy. In this section, we will explore interactions of systems in which mechanical energy is conserved; that is, the total amount of kinetic, gravitational potential, and elastic potential energy remains constant. Energy losses due to effects such as friction, air resistance, thermal energy, and sound can be ignored as negligible.

UNIT TASK BOOKMARK

You can apply what you learn about springs and conservation of energy to the Unit Task on page 270.

Suppose, for example, a student jumps up from a diving board. Assuming air friction is negligible, the mechanical energy of the diver will be conserved. Use the diving board as the reference point, $y = 0$, for measuring the gravitational potential energy. At the maximum height h above the diving board, the diver has only gravitational potential energy equal to $mg\Delta y$. He then starts to fall toward the board, gaining kinetic energy because of his motion.

The diver's distance above the reference point is decreasing, so his gravitational potential energy is decreasing. His total mechanical energy does not change. Halfway to the board, his gravitational potential energy has decreased by half, so it exactly equals his kinetic energy. At the moment just before the diver hits the diving board, the gravitational potential energy is zero, and he has kinetic energy that equals his starting gravitational potential energy.

Conservation of mechanical energy still applies after the diver hits the diving board. He applies a downward force on the board, displacing it a distance x. This work transfers the diver's kinetic energy to the board. The energy is stored in the board as elastic potential energy. As the board dips down, the diver drops below $y = 0$, so his gravitational potential energy becomes negative. The total elastic potential energy increases to offset the decrease in gravitational potential energy. In reality, some energy is lost as friction, sound, and vibrations of the diving board. If we ignore these effects, mechanical energy is conserved. You will apply the conservation of mechanical energy in the following Tutorial.

Tutorial 1 Applying the Law of Conservation of Energy

In this Tutorial, we will analyze the transformations of gravitational potential, kinetic, and elastic potential energies of various systems.

Sample Problem 1: Analyzing Energy Transformations

In this problem, you will model a collision and apply conservation of energy to analyze the outcome. A model car of mass 5.0 kg slides down a frictionless ramp into a spring with spring constant $k = 4.9$ kN/m (**Figure 2**).

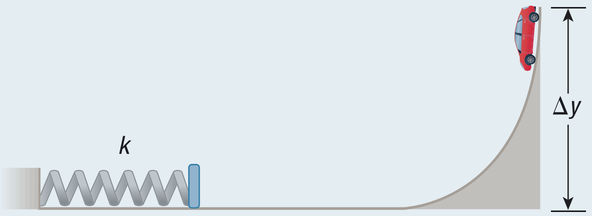

Figure 2

(a) The spring experiences a maximum compression of 22 cm. Determine the height of the initial release point.
(b) Calculate the speed of the model car when the spring has been compressed 15 cm.
(c) Determine the maximum acceleration of the car after it hits the spring.

Solution

(a) **Given:** $m = 5.0$ kg; $k = 4.9$ kN/m $= 4.9 \times 10^3$ N/m; $\Delta x = 22$ cm $= 0.22$ m

Required: Δy

Analysis: $\Delta E_g = mg\Delta y$; $E_e = \frac{1}{2}k(\Delta x)^2$

Since energy is conserved, the change in potential energy of the model car must equal the change in elastic potential energy when the spring is compressed.

Solution: If we choose the bottom of the ramp to be the $y = 0$ reference point, the car will have no gravitational potential energy at the bottom of the ramp. The initial gravitational potential energy has been converted into kinetic energy. When the spring is fully compressed, the kinetic energy has been converted to elastic potential energy. Therefore, the spring's initial gravitational potential energy must equal its final elastic potential energy:

$$E_g = E_e$$

$$mg\Delta y = \frac{1}{2}k(\Delta x)^2$$

$$\Delta y = \frac{k(\Delta x)^2}{2\,mg}$$

$$= \frac{(4.9 \times 10^3\ \text{N/m})(0.22\ \text{m})^2}{2(5.0\ \text{kg})(9.8\ \text{m/s}^2)}$$

$$\Delta y = 2.42\ \text{m (one extra digit carried)}$$

Statement: The initial height of the model car is 2.4 m.

(b) **Given:** $m = 5.0$ kg; $k = 4.9$ kN/m $= 4.9 \times 10^3$ N/m; $\Delta x = 15$ cm $= 0.15$ m; $\Delta y = 2.42$ m

Required: v

Analysis: $\Delta E_g = mg\Delta y$; $E_e = \frac{1}{2}k(\Delta x)^2$; $E_k = \frac{1}{2}mv^2$

Since energy is conserved, the sum of the kinetic energy and the elastic potential energy when the spring is compressed must equal the initial gravitational potential energy.

Solution: The initial gravitational potential energy is

$$E_g = mg\Delta y$$
$$= (5.0 \text{ kg})(9.8 \text{ m/s}^2)(2.42 \text{ m})$$
$$E_g = 119 \text{ J}$$

When the spring is compressed to $\Delta x = 15$ cm, the elastic potential energy is

$$E_e = \frac{1}{2}k(\Delta x)^2$$
$$= \frac{1}{2}(4.9 \times 10^3 \text{ N/m})(0.15 \text{ m})^2$$
$$E_e = 55.1 \text{ J}$$

The kinetic energy when $x = 15$ cm must equal the difference between the initial gravitational potential energy and the final elastic potential energy:

$$E_k = E_g - E_e$$
$$= 119 \text{ J} - 55.1 \text{ J}$$
$$E_k = 63.9 \text{ J}$$

Finally, use E_k to solve for v:

$$\frac{1}{2}mv^2 = E_k$$
$$v^2 = \frac{2E_k}{m}$$
$$v = \sqrt{\frac{2E_k}{m}}$$
$$= \sqrt{\frac{2(63.9 \text{ J})}{5.0 \text{ kg}}}$$
$$v = 5.1 \text{ m/s}$$

Statement: The speed of the model car when the spring is compressed 15 cm is 5.1 m/s.

(c) **Given:** $m = 5.0$ kg; $k = 4.9$ kN/m $= 4.9 \times 10^3$ N/m; $\Delta x = 22$ cm $= 0.22$ m

Required: $\vec{a}$

Analysis: $\vec{F}_e = -k\Delta\vec{x}$; $\vec{F}_{net} = m\vec{a}$

The maximum acceleration occurs when the maximum force is acting, and this occurs when the spring is at the maximum compression of 22 cm.

Solution: Combining Hooke's law and Newton's second law gives

$$\vec{F}_{net} = \vec{F}_x$$
$$m\vec{a} = -k\Delta\vec{x}$$
$$\vec{a} = -\frac{k\Delta\vec{x}}{m}$$
$$= \frac{(4.9 \times 10^3 \text{ N/m})(-0.22 \text{ m})}{5.0 \text{ kg}}$$
$$\vec{a} = 2.2 \times 10^2 \text{ m/s}^2 \text{ [toward ramp]}$$

Statement: The maximum acceleration of the model car is 2.2×10^2 m/s^2 directed toward the ramp.

Sample Problem 2: Using Elastic Potential, Kinetic, and Gravitational Potential Energies

A 48 kg child bounces on a pogo stick. At the lowest point of one bounce, the compressed spring in the stick has 120 J of elastic potential energy as it compresses 0.19 m. Assume that the pogo stick is light enough that we can ignore its mass.

(a) Determine the child's maximum height during the jump following the bounce.
(b) Determine the child's maximum speed during the jump.

Solution

(a) **Given:** $m = 48$ kg; $E_e = 120$ J

Required: Δy

Analysis: $\Delta E_g = mg\Delta y$

Solution: Choose the lowest point of the bounce as the $y = 0$ reference point. At the maximum height, Δy, all elastic potential energy has converted to gravitational potential energy.

$$E_g = E_e$$
$$mg\Delta y = E_e$$
$$\Delta y = \frac{E_e}{mg}$$
$$= \frac{120 \text{ J}}{(48 \text{ kg})(9.8 \text{ m/s}^2)}$$
$$\Delta y = 0.26 \text{ m}$$

Statement: The child rises 0.26 m from the lowest point of the bounce.

(b) **Given:** $m = 48.5$ kg; $E_e = 120$ J; $h = 0.19$ m

Required: v

Analysis: The point of maximum speed is the point at which the spring is at its equilibrium position. At this point, all of the elastic potential energy has been converted to gravitational potential energy and kinetic energy.

$$\Delta E_g = mg\Delta y;\ E_k = \frac{1}{2}mv^2$$

Solution: If we choose the lowest part of the bounce as the $y = 0$ reference point, then at the equilibrium position,

$$E_g = mg\Delta y$$
$$= (48\text{ kg})(9.8\text{ m/s}^2)(0.19\text{ m})$$
$$E_g = 89.4\text{ J (one extra digit carried)}$$

The kinetic energy is the difference between the initial elastic potential energy and the gravitational potential energy:

$$E_k = E_e - E_g$$
$$= 120\text{ J} - 89.4\text{ J}$$
$$E_k = 30.6\text{ J (one extra digit carried)}$$

Now solve for v:

$$\frac{1}{2}mv^2 = E_k$$
$$v = \sqrt{\frac{2E_k}{m}}$$
$$= \sqrt{\frac{2(30.6\text{ J})}{48\text{ kg}}}$$
$$v = 1.1\text{ m/s}$$

Statement: The child's maximum speed is 1.1 m/s.

Sample Problem 3: A Block Pushed Up a Frictionless Ramp by a Spring

A block with a mass of 2.0 kg is held against a spring with spring constant 250 N/m. The block compresses the spring 22 cm from its equilibrium position. After the block is released, it travels along a frictionless surface and then up a frictionless ramp. The ramp's angle of inclination is 30.0°, as shown in **Figure 3**.

(a) Determine the elastic potential energy stored in the spring before the mass is released.
(b) Calculate the speed of the block as it travels along the horizontal surface.
(c) Determine how far along the ramp the block will travel before it stops.

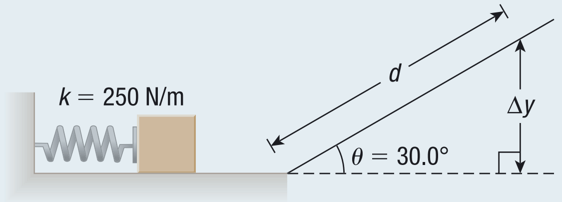

Figure 3

(a) **Given:** $k = 250$ N/m; $x = 22$ cm $= 0.22$ m

Required: E_e

Analysis: Before the block is released, the entire mechanical energy of the block–spring system is in the form of elastic potential energy stored in the compressed spring. We can use the given information to determine the amount of stored energy, $E_e = \frac{1}{2}k(\Delta x)^2$.

Solution:

$$E_e = \frac{1}{2}k(\Delta x)^2$$
$$= \frac{1}{2}(250\text{ N/m})(0.22\text{ m})^2$$
$$E_e = 6.05\text{ J (one extra digit carried)}$$

Statement: The elastic potential energy stored in the spring before the mass is released is 6.0 J.

(b) **Given:** $E_k = 6.05$ J; $m = 2.0$ kg

Required: v

Analysis: As the block travels along the flat surface, all of the elastic potential energy is converted to kinetic energy. Use this to determine the speed of the block using the equation for kinetic energy: $E = \frac{1}{2}mv^2$.

Solution:

$$E = \frac{1}{2}mv^2$$
$$\frac{2E}{m} = v^2$$
$$v = \sqrt{\frac{2E}{m}}$$
$$= \sqrt{\frac{2(6.05\text{ J})}{(2.0\text{ kg})}}$$
$$v = 2.46\text{ m/s}$$

Statement: The block will travel at a constant speed of 2.5 m/s along the frictionless horizontal surface.

(c) **Given:** $E_g = 6.05$ J; $m = 2.0$ kg; $g = 9.8$ m/s^2

Required: Δy, d

Analysis: As the block travels up the ramp, kinetic energy is gradually converted to gravitational potential energy. When the block reaches its maximum height, all energy will be in potential form. Use this to determine the vertical height attained, Δy, and then use trigonometry to calculate the distance travelled along the ramp, d, as shown in Figure 3.

$$E_g = mg\Delta y;\ \frac{\Delta y}{d} = \sin\theta$$

Solution: Mechanical energy is conserved throughout this problem because there are no energy losses due to friction.

The total potential energy at the top of the block's path is therefore 6.05 J.

$$E_g = mg\Delta y$$

$$\Delta y = \frac{E_g}{mg}$$

$$= \frac{6.05\ \text{J}}{(2.0\ \text{kg})(9.8\ \text{m/s}^2)}$$

$$\Delta y = 0.308\ \text{m}$$

Now use the sine ratio to determine how far along the ramp the block travels, *d*.

$$\frac{\Delta y}{d} = \sin\theta$$

Rearrange this equation to express *d* in terms of Δy and θ.

$$\Delta y = d\sin\theta$$

$$d = \frac{\Delta y}{\sin\theta}$$

$$= \frac{0.308\ \text{m}}{\sin 30.0°}$$

$$d = 0.62\ \text{m}$$

Statement: The block will travel a distance of 0.62 m, or 62 cm, along the ramp.

Practice

1. A block slides down a ramp from a fixed height and collides with a spring, compressing the spring until the block comes to rest. Compare the amount of compression in the case that the ramp is frictionless to the case where the ramp is not frictionless. Explain your answer. K/U T/I
2. A 3.5 kg mass slides from a height of 2.7 m down a frictionless ramp into a spring. The spring compresses 26 cm. Calculate the spring constant. T/I [ans: 2.7×10^3 N/m]
3. A 43 kg student jumps on a pogo stick with spring constant 3.7 kN/m. On one bounce, he compresses the stick's spring by 37 cm. Calculate the maximum height he reaches on the following jump. T/I [ans: 0.60 m above the compressed point]
4. A 0.35 kg branch falls from a tree onto a trampoline. If the branch was initially 2.6 m above the trampoline, and the trampoline compresses 0.14 m, calculate the spring constant of the trampoline. T/I [ans: 9.6×10^2 N/m]
5. Consider the block in Sample Problem 3. Suppose that the mass of the block is doubled at the top of its path of motion before returning down the frictionless ramp. K/U T/I
   (a) Determine the speed of the block as it returns along the horizontal surface. [ans: 2.5 m/s]
   (b) Does the block have the same kinetic energy as before along the horizontal surface? Explain your answer.
   (c) Will the block compress the spring twice as far as it did before? Explain your answer. If your answer is no, determine the new value for *x*.
   (d) Suppose the coefficient of friction of the ramp is 0.15. Does your answer to (c) change? Explain your answer. If your answer is yes, determine the new value for *x*.

Perpetual Motion Machines

An ideal spring would never lose energy and would continue with SHM forever, or as long as you did not disturb it. A machine that can continue to operate for an unlimited amount of time without outside help is a **perpetual motion machine**. To be a true perpetual motion machine, the machine must be able to run forever without restarting or refuelling. A grandfather clock, for example, is not a perpetual motion machine, since you must wind it up every now and then.

perpetual motion machine a machine that can operate forever without restarting or refuelling

Figure 4 shows a device called Newton's cradle. The leftmost ball has gravitational potential energy with respect to the other balls. Once released, the ball will interact with the other balls in such a way as to imitate perpetual motion. You will have the opportunity to explore the physics behind Newton's cradle in Chapter 5.

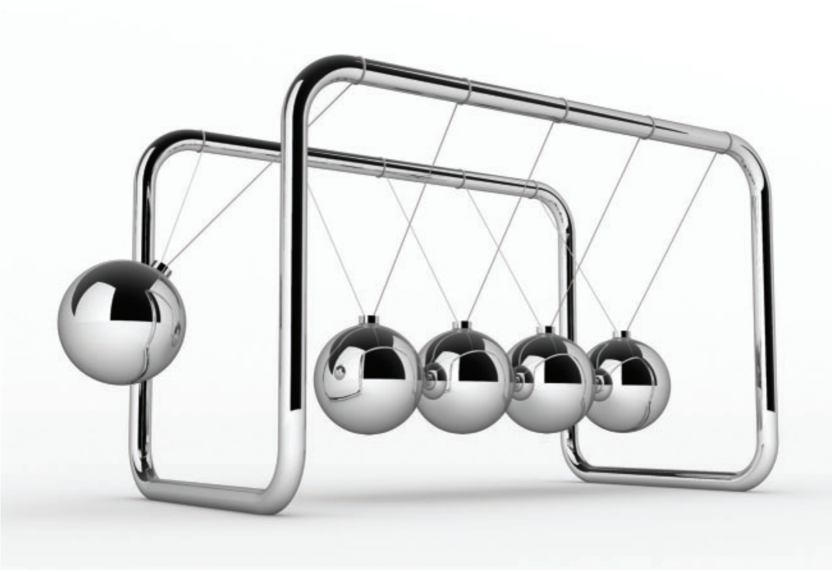

Figure 4 An ideal version of Newton's cradle would be a perpetual motion machine because it would never lose energy.

An ideal version of Newton's cradle would never lose energy, and the cycle of falling, colliding, and rising would continue forever. It would then be a perpetual motion machine. Can you build such a machine?

The answer is no. In real-world machines, some mechanical energy will always be lost from the system as thermal energy, sound energy, or other forms of energy. This loss of energy can be useful. For example, the purpose of shock absorbers in cars, which we mentioned in Section 4.6, is to use friction to stop the SHM of the car's springs.

Investigation 4.7.1

Energy and Springs (page 211)
You have learned about the spring constant and how the movement of springs is related to the conservation of energy. Now you are ready to conduct an investigation to observe the conservation of energy.

Research This

Perpetual Motion Machines

Skills: Researching, Communicating — SKILLS HANDBOOK A4.1

Hobbyists and serious researchers alike have attempted to design perpetual motion machines. They have not been successful, but some of their ideas have useful applications.

1. Choose one machine, such as an analog watch, a metronome, a flywheel, or a child's swing, that relies on ongoing, consistent motion to work properly.
2. Research the design principles that have been incorporated into modern versions of the machine to make it work more efficiently.

A. What scientific principles explain how the machine operates? A

B. How has the design of the machine been improved over time? K/U

C. Have improvements been the result of the development of new materials, new technology, or new scientific discoveries? T/I

D. Prepare a short presentation that summarizes your findings. C

WEB LINK

Damped Harmonic Motion

So far, we have ignored the effect of friction on the motion of a simple harmonic oscillator. The friction in a real periodic system is referred to as damping, and the harmonic motion of a system affected by friction is called **damped harmonic motion**. The presence of friction means that the mechanical energy of the system will be transformed into thermal energy, and the system's motion will not continue perpetually. We can classify damped motion into three categories: underdamped, overdamped, and critically damped (**Figure 5**).

damped harmonic motion periodic motion affected by friction

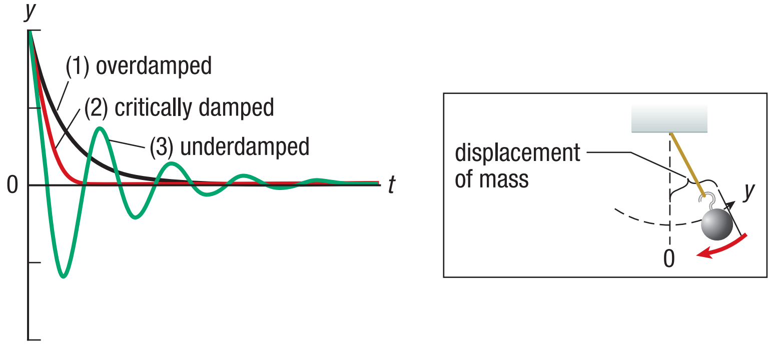

Figure 5 When a damped oscillator is given a non-zero displacement at $t = 0$ and then released, it can exhibit three different types of behaviour: (1) overdamped, (2) critically damped, and (3) underdamped.

Consider a pendulum. Curve 3 in Figure 5 shows how the displacement varies with time when the damping is weak, that is, when there is only a small amount of friction. This curve applies to any weakly damped harmonic oscillator. It describes the back-and-forth swinging of a pendulum or the motion of a mass attached to a spring on a horizontal surface when the surface is quite slippery. The system still oscillates because the displacement alternates between positive and negative values, but the amplitude of the oscillation gradually decreases with time. The amplitude eventually goes to zero, but the system undergoes many oscillations before damping brings it to rest. This type of motion is an underdamped oscillation.

When quite a bit of friction exists, the oscillator is overdamped. The resulting displacement as a function of time in this case is shown as curve 1 in Figure 5. This type of motion happens when the mass moves through a very thick fluid, like the hydraulic fluid inside the closing mechanism on many doors. If you pull the mass of an overdamped oscillator to one side and then release it, the mass moves extremely slowly back to the equilibrium.

Critically damped motion falls in between the two extreme cases. In underdamped motion, displacement always passes through zero—the equilibrium point—at least once, and usually many times, before the system comes to rest. In contrast, an overdamped system released from rest moves just to the equilibrium point, but not beyond. In critically damped motion, displacement falls to zero as quickly as possible without moving past the equilibrium position. Displacement as a function of time for the critically damped case is illustrated by curve 2 in Figure 5.

These different categories of damping have different applications. For example, a car's shock absorbers provide damping for springs that support the car's body (**Figure 6**).

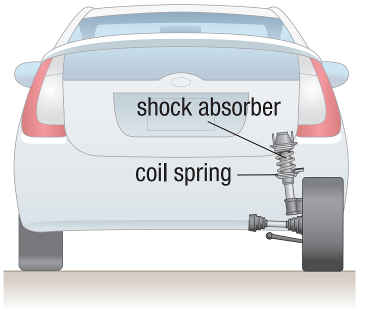

Figure 6 The shock absorbers on a car serve to dampen its coil springs. The goal is usually to have a car respond to bumps in the road as a critically damped oscillator.

Shock absorbers enable the tires to move up and down over bumps in the road without directly passing vibrations to the car's body or passengers. When the car hits a bump, the springs compress. To make the ride as comfortable as possible, the shock absorbers critically damp the motion of the springs. The critically damped motion means the body of the car returns to its original height as quickly as possible. Worn-out shock absorbers lead to underdamped motion, and the car bounces up and down more. Overdamped shock absorbers give a soft "spongy" ride with poor steering response and handling. CAREER LINK

4.7 Review

Summary

- For an isolated mass–spring system, the total mechanical energy—kinetic energy, elastic potential energy, and gravitational potential energy—remains constant.
- A perpetual motion machine is a machine that can operate forever without restarting or refuelling.
- Damped harmonic motion is periodic motion in which friction causes a decrease in the amplitude of motion and the total mechanical energy.

Questions

1. A mass hangs from a vertical spring and is initially at rest. A person then pulls down on the mass, stretching the spring. Does the total mechanical energy of this system (the mass plus the spring) increase, decrease, or stay the same? Explain. K/U
2. A mass rests against a spring on a horizontal, frictionless table. The spring constant is 520 N/m, and the mass is 4.5 kg. The mass is pushed against the spring so that the spring is compressed by 0.35 m, and then it is released. Determine the velocity of the mass when it leaves the spring. T/I
3. A toy airplane ejects its 8.4 g pilot using a spring with a spring constant of 5.2×10^2 N/m. The spring is initially compressed 5.2 cm. T/I
   (a) Calculate the elastic potential energy of the compressed spring.
   (b) Calculate the speed of the pilot as it ejects upward from the airplane.
   (c) Determine the maximum height that the pilot will reach.
4. In a pinball game, a compressed spring with spring constant 1.2×10^2 N/m fires an 82 g pinball. The pinball first travels horizontally and then travels up an inclined plane in the machine before coming to rest. The ball rises up the ramp through a vertical height of 3.4 cm. Determine the distance of the spring's compression. T/I
5. A bungee jumper of mass 75 kg is standing on a platform 53 m above a river. The length of the unstretched bungee cord is 11 m. The spring constant of the cord is 65.5 N/m. Calculate the jumper's speed at 19 m below the bridge on the first fall. T/I
6. A spring with a spring constant of 5.0 N/m has a 0.25 kg box attached to one end such that the box is hanging down from the string at rest. The box is then pulled down another 14 cm from its rest position. Calculate the maximum height, the maximum speed, and the maximum acceleration of the box. T/I
7. A 0.22 kg block is dropped on a vertical spring that has a spring constant of 280 N/m. The block attached to the spring compresses it by 11 cm before momentarily stopping. Determine the height from which the block was dropped. T/I
8. A block of 1.0 kg with speed 1.0 m/s hits a spring placed horizontally, as shown in **Figure 7**. The spring constant is 1000.0 N/m. T/I
   (a) Calculate the maximum compression of the spring.
   (b) How far will the block travel before coming to rest? Assume that the surfaces are frictionless.

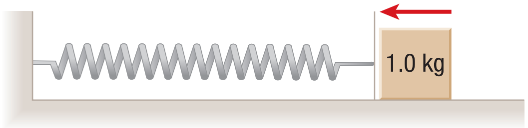

Figure 7

9. A wooden box of mass 6.0 kg slides on a frictionless tabletop with a speed of 3.0 m/s. It is brought to rest by a compressing spring. The spring constant is 1250 N/m. T/I
   (a) Calculate the maximum distance the spring is compressed.
   (b) Determine the speed and acceleration of the block when the spring is compressed a distance of 14 cm.
10. A tennis coach uses a machine to help with tennis practice. The machine uses a compressed spring to launch tennis balls. The spring constant is 440 N/m, and the spring is initially compressed 45 cm. A 57 g tennis ball leaves the machine horizontally at a height of 1.2 m. Calculate the horizontal distance that the tennis ball can travel before hitting the ground. T/I